Converge-Cast: On the Capacity and Delay Tradeoffs

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Abstract—In this paper, we define an ad hoc network where multiple sources transmit packets to one destination as Converge-Cast network. We will study the capacity delay tradeoffs assuming that n wireless nodes are deployed in a unit square. For each session[1], k nodes are randomly selected as active sources and transmit one packet to a particular destination node, which is also randomly selected. We first consider the stationary case, where capacity is mainly discussed and delay is entirely dependent on the average number of hops. We find that the per-node capacity is $\Theta(1/\sqrt{n \log n})^2$, which is the same as that of unicast, presented in [3]. Then node mobility is introduced to increase network capacity, for which our study is performed in two steps. The first step is to establish the delay in single-session transmission. We find that the delay is $\Theta(n \log k)$ under 1-hop strategy, and $\Theta(n \log k/m)$ under 2-hop redundant strategy, where m denotes the number of replicas for each packet. The second step is to find delay and capacity in multi-session transmission. We reveal that the per-node capacity and delay for 2-hop non-redundancy strategy are $\Theta(1)$ and $\Theta(n \log k)$ respectively. The optimal delay is $\Theta(\sqrt{n \log k} + k)$ with redundancy, corresponding to a capacity of $\Theta(\sqrt{\frac{1}{n \log k} + \frac{k}{n \log k}})$. Therefore we obtain that the capacity delay tradeoff satisfies $delay/rate \geq \Theta(n \log k)$ for both strategies.

Index Terms—ConvergeCast, Capacity, Delay

1 INTRODUCTION

Converge-cast network is a special type of wireless ad hoc network, which concerns many-to-one transmission. In each session, k sources and one destination are randomly selected from n nodes in the network, and the remaining nodes act as relays. These k distinct sources transmit different packets to the destination, either directly or via multiple relays. A successful transmission is completed when all the k packets reach the destination.

Converge-cast network is of significant practical value and has drawn much attention recently. One example of stationary converge-cast network is the Monitoring

1. The session is a dataflow from k different source nodes to 1 destination node.

2. Given non-negative functions f(n) and g(n): f(n) = O(g(n)) means there exist positive constants c and m such that f(n) $\leq$ cg(n) for all n $\geq$ m; $f(n) = \Omega(g(n))$ means there exist positive constants c and m such that $f(n) \geq cg(n)$ for all $n \geq m$; $f(n) = \Theta(g(n))$ means that both $f(n) = \Omega(g(n))$ and $f(n) = O(g(n))$ hold.

and Alarming System, where sensors are deployed in a certain area to gather information. Usually we have to concern a group of sensors simultaneously to achieve a much wider view. In MANET, one such example is the Exploring System, where mobile sensors are required to explore in a certain area and send information back periodically. Moreover, converge-cast network and multicast network can be combined to form a full duplex network that permits both one-to-many and many-to-one transmissions, which is very useful in many situations.

Converge-Cast network has a very similar configuration with multicast network in that the information dissemination can both be modeled as a spanning tree. Yet there still exist some differences. The first difference is the transmission direction. Secondly, k sources transmit different packets to their destination in Converge-Cast network, while k destinations receive the same packets duplicated from the only source in multicast network. In the Converge-Cast network, all the packets are entirely different and indispensable so that they are forced to reach the destination node. Thirdly, there are more interferences in Converge-Cast network than multicast network when we use large redundancy to reduce the delay.

While unicast and multicast networks have been extensively investigated, there has been few works dealing with congerge-cast networks. Just like unicast to multicast, the generalization from unicast to congerge-cast requires new techniques. In this paper, we will concentrate on converge-cast capacity and delay in wireless ad hoc networks. In the stationary network, capacity is a major issue because the delay is only related to the number of hops, which is determined by the node positions. However, packet delay deserves to be well investigated in MANET because it is quite different from that of multicast in MANET. There are various applications of converge-cast MANET [25], [26], [27]. In [25], a ZebraNet is constructed to monitor and study animal migrations and each zebra is equipped with a wireless antenna and pairwise communication is used to transmit data when two zebras are close to each other. Since converge-cast

network requires k different packets to be delivered to the destination when there are exactly k source nodes, delay is defined as the longest delay among all source-destination pairs. Intuitively, the delay of Converge-Cast network should be larger than that of multicast network, but less than direct extension of unicast network, namely k times of unicast strategy. Another intuition is that delay increases as the number of replicas decreases, which will actually increase per-node capacity for the better use of bandwidth. Therefore, there should be tradeoffs between capacity and delay in MANET for converge-cast traffic pattern.

Our main results are presented as follows: For stationary converge-cast network, the per-node capacity is $\Theta(1/\sqrt{n \log n})$, which is identical to the result obtained under unicast network, mentioned in [3]. That is mainly because these two types of network require the same number of hops, which is the essential key that controls the capacity between a source-destination pair. For the *i.i.d* mobile converge-cast network, we investigate the 2-hop algorithm with and without redundancy. By adjusting the number of replicas from one packet to a group of relays, the per-node capacity and delay are $\Theta(1/m)$ and $\Theta((n \log k)/m)$, respectively, where m stands for the number of replicas for each packets. This result implies that a smaller delay should be tolerated if more copies are exploited with delay capacity tradeoff $delay/capacity \geq \Theta(n \log k)$. This result is better than that of k unicasts, namely $\Theta(kn)$, mentioned in [1]. For 2-hop algorithm without redundancy, we prove that adding relay cannot improve the network performance with per-node capacity and delay $\Theta(1)$ and $\Theta(n \log k)$, respectively. The optimal delay for 2-hop algorithm with redundancy is $\Theta(\sqrt{n \log k} + k)$ and the corresponding capacity is $\Theta(\sqrt{\frac{1}{n \log k}} + \frac{k}{n \log k})$.

The rest of the paper is organized as follows. Section 2 discusses some related works. Section 3 describes the network model. Section 4 analyzes the capacity of both static and mobile converge-cast networks. Section 5 studies delay in the single-session transmission under 1-hop and 2-hop algorithms. In Section 6, we study delay and capacity with 2-hop protocol with and without redundancy. Section 7 discusses our results and compare them to that under unicast and multicast. Tradeoffs between capacity and delay are also discussed in this section. Conclusions and future works are presented in Section 8.

2 RELATED WORK

Capacity and delay tradeoff in both static and mobile ad hoc networks is a hot topic in recent years. Gupta and Kumar [3] find that the overall network throughput is $\Theta(\frac{n}{\sqrt{\log n}})$ in the static network. Some proofs are simplified in [4] and [11]. In [12], they show that the optimal per-node capacity $\Theta(1/\sqrt{n})$ can be achieved by nearest neighbor multihop scheme. And in [16], the capacity is derived when each node is equipped with

directional antenna. To improve it, Grossglauser and Tse [6] demonstrate that per-node capacity can reach $\Theta(1)$ by introducing mobility to the network, but incurring a disadvantage of large delay. Then various types of mobility models are investigated in [13] [15] [22] [23] [24][18][21]. Neely et al. [1], for example, present a strategy utilizing redundant packets transmissions along multiple paths in a cell partitioned MANET to reduce delay with a sacrifice on the capacity. They establish the following necessary tradeoff: $delay/capacity \geq O(n)$ and develop schemes that can achieve $\Theta(1), \Theta(\frac{1}{\sqrt{n}})$, $\Theta(\frac{1}{n \log n})$ per-node capacity, when the delay constraint is $\Theta(n), \Theta(\sqrt{n})$ and $\Theta(\log n)$, respectively. In [20], the authors establish the optimal delay constrained unicast capacity. Afterwards, in [7], Lin and Shroff search the optimal capacity-delay tradeoff and identify the limiting factors of the existing scheduling schemes in MANETs. Recently, M. Garetto et al. [30], [30] study the supper-critical and subcritical case in the heterogeneous mobile network and in their following work [31], they find that correlated mobility can increase the capacity delay tradeoffs.

Muticast network is a crucial generalization of the unicast network, which is applicable in many real world scenarios. In [17] and [5], they show that the multicast capacity per session is $O(\frac{1}{\sqrt{n_s \log n_s p}})$ in static network. In [19], [14], the multicast capacity of delay tolerant networks is studied. After that, Shakkottai [8] proposes an architecture for multicast routing which achieves the upper bound for capacity in order sense. In [2], Wang et al. shows that by 2-hop relay strategy, the expected delay is no less than $\Omega(\sqrt{n \log k})$ when mobility is introduced and delay of $O(\sqrt{n \log k})$ is attainable in his proposed scheme with per-node capacity of $\Omega(\frac{1}{k\sqrt{n \log k}})$.

3 NETWORK MODEL

In this section, we introduce the network model for converge-cast network and some definitions, which are important for further analysis.

3.1 Network model for stationary converge-cast network

When the network is stationary, the network structure is very similar to that of multicast and we directly follow the model introduced in [5]. The only difference lies in that we revert the roles of source and destinations in [5] and every branch in the spanning tree is direction-reversed.

Suppose there are a total of n nodes randomly and uniformly deployed in a unit square area, where the transmission range for each node is r. The protocol interference model [3] is adopted, and the critical transmission range $r = \Omega(\sqrt{\frac{\log n}{n}})$, making the whole network fully connected [3].

The two main differences between converge-cast and multicast networks are the structure of spanning tree and the definition of throughput. Although the shapes of the

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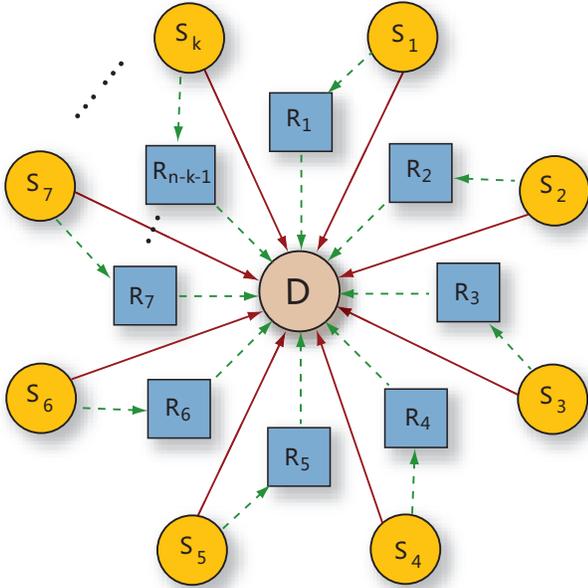IEEE TRANSACTIONS ON MOBILE COMPUTING

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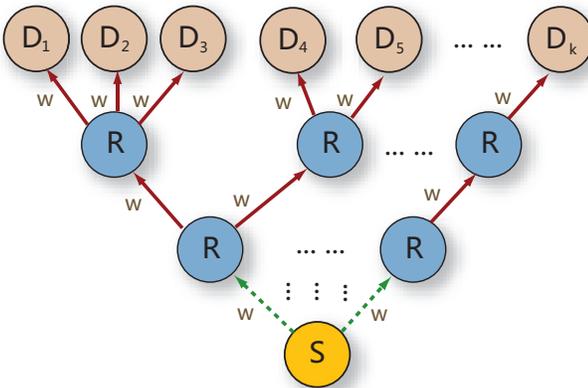
Fig. 1. Traffic pattern

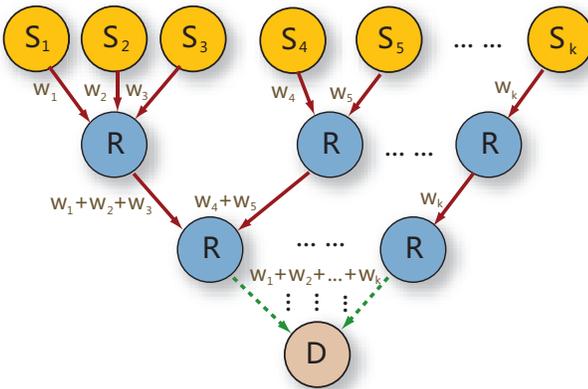Fig. 2. Multicast data flow diagram

Fig. 3. Converge-cast data flow diagram

spanning tree for these two traffic patterns are identical, the average data rate on each branch is different. Each branch of the spanning tree holds the same data rate in multicast traffic pattern as in Figure (2). However, in converge-cast network, the data rate on each branch is no longer the same because all the packets are entirely different. Since the data stream travels from leaves to the root, each intermediate relay has several inputs but only one output. The output rate for this relay should be larger than the sum of all the input rate as in Figure 3. Therefore, a single session cannot make full use of the network capacity due to the overhead of the root in the spanning tree. The second difference is the definition of throughput. In multicast scheme, all the k destinations in a session receive the same packet, and only one packet can be treated as valid and the other $k - 1$ packets are redundancies. In converge-cast scheme, all the k packets transmitted to the designated destination are entirely different and cannot be treated as redundancy. In other words, the number of packets for each session is k times larger than that of multicast scheme. However, under practical conditions, only the complete combination of k packets are useful, and we can also treat all these k packets as one large packet. Both of these two definitions are reasonable and we will treat them differently in this paper.

3.2 Network model for mobile converge-cast network

A cell partitioned model is proposed as interference model, identical to the one in [1]. The unit square is divided into c non-overlapping cells. Two nodes are permitted to transmit if and only if they are in the same cell. Such model can be implemented by using FDMA, allocating 9 different channels to the 9 adjacent cells. In each cell, only one source-destination pair is permitted to be active to avoid data collision. $d = n/c$ is defined as the average number of nodes in a cell and Neely [1] showed that $d^* = 1.7933$ is an optimal value that achieves the maximized capacity.

The basic unit of transmission is a session, which is defined as a whole procedure which begins when k source nodes sample their packets and ends when all the k different packets reach the designated destination. In each new time slot, a new session will be initiated with equal probability. When a new session starts, k sources and one destination will be randomly chosen from n nodes uniformly and independently. Then the chosen k packets will be produced and put in the sending pool, with required information such as the current time, the source ID and the destination ID, and etc., waiting to be sent out.

The time axis is divided into time slots with equal length, which can support the transmission of one packet. When a new time slot begins, all the nodes move to a new cell according to a i.i.d. mobility model.

Buffer model: There are three pools with unlimited size

in each node: a sending pool, a relaying pool and a receiving pool.

- The sending pool is for packets that is sampled by the node itself. When a new session starts, a new packet is produced and duplicated m times if the node is selected as source. Then if the node is permitted to transmit its packet, it will randomly choose a packet in the pool and delete it after the transmission.
- The relaying pool is for packets that are sampled by other node and will be sent to other destinations. When this node N_{send} meets another node $N_{receive}$ which is the destination of one of the packets P in the relaying pool, and this pair is selected according to the scheduling strategy, the packet P will be delivered to the receiving pool in node $N_{receive}$. Then the packet P will be deleted from the relaying pool in node N_{send}.
- The receiving pool is for packets that are designated for itself. When this node $N_{receive}$ meets another node N_{send} which contains a packet P whose destination ID is $N_{receive}$, and this pair is selected according to the scheduling strategy, the packet P will be delivered to the receiving pool in node $N_{receive}$. And then the packet P will be deleted from the relaying pool in node N_{send}.

2-hop algorithm: In 2-hop algorithm, there are two types of transmissions: Source-Relay(S-R) transmission and Relay-Destination(R-D) transmission. Thus, when a particulary pair is selected, there will be two conditions: S-R pair or R-D pair.

- If node N_{send} contains packet P in its relaying pool to be sent to $N_{receive}$, and N_{send} is in the same cell as $N_{receive}$, we call N_{send} and $N_{receive}$ a R-D pair.
- If node N_{send} does not contain packet P in its relaying pool to be sent to $N_{receive}$, while node $N_{receive}$ does not contain packet P in its relaying pool to be sent to N_{send}, and N_{send} is in the same cell as $N_{receive}$, we call N_{send} and $N_{receive}$ a S-R pair.

Consider a particular cell in a time slot, if there are no pairs in the cell (no nodes or only one node), no transmission will happen there; if there are pairs but no R-D pairs in the cell, then two nodes as a S-R pair are randomly selected and permitted to transmit; if there are at least one R-D pair in the cell, then one of the R-D pairs is selected and permitted to transmit. All nodes that are not selected by scheduling strategy will stay idle. One with the earliest birth time is chosen to transmit if more than one packet can be sent from N_{send} to $N_{receive}$.

We therefore define the life period for a single packet as follows: When a packet P is sent by node S_i, we must duplicate it m times and put them in the sending pool in node S_i. When node S_i meets another node R_j in a particulary cell, the packet P will be sent to relaying pool in node R_j if the following conditions are satisfied:

- There happens to be no R-D pairs in such cell.

- This node happens to be selected as a transmission pair.
- There happens to be no packet of previous session left in the sending pool.

The packet P stays in relaying pool of node R_j when it is transmitted from node R_j to node D. The packet delay is considered as the current time minus the birth of P when the whole session completes. Then all the other $m-1$ copies of P are deleted at this time and the life period is finished.

Definition of delay: when all the k packets of a same session have reached the destination, the delay will be calculated as the difference between the time when the last packet reaches the destination and the time when the packet is sampled. (Notice that k packets' birth times are the same.)

Definition of capacity: the total capacity of the network is defined as the maximum average total data rate, which disregards the amount of redundancy in each time slot. Note that an upper bound of capacity exists to guarantee the stability of the network.

4 USEFUL LEMMAS

Lemma 4.1: The average possibility that every two nodes are selected if they are in a same cell is on the order $\Theta(1)$, which is independent with respect to n.

Proof: Let N_{send} and $N_{receive}$ denote the sender and the receiver , respectively. And r denotes the total number of nodes which can also be selected as transmission pair if one of which are in the same cell as N_{send} and $N_{receive}$ do in the whole network. Obviously, $r < n$. So the worst situation happened when all nodes in the network can interfere with the transmission between N_{send} and $N_{receive}$, and we will prove that the lemma still holds in such situation.

According to the assumption that the number of cell c is on the same order as the number of node n. Let network density $d = n/c = \Theta(1)$.

When N_{send} and $N_{receive}$ are in the same cell in a certain timeslot, consider the other $n-2$ nodes that may interfere with the transmission. We have the following expression (although there are $n-2$ other nodes in the network, we use n instead to make the equation simpler, which does not change the result)

$$
\begin{aligned}
E \quad & (\Pr(N_{send} \text{ is selected as a sender})) \\
= \quad & \Pr(\text{No other nodes in the cell}) \\
& + \sum_{x=1}^{n} \frac{1}{x+1} \Pr(x \text{ other nodes in the cell}) \\
= \quad & (1-\frac{1}{c})^n + \sum_{x=1}^{n} \frac{1}{x+1} C_n^x (\frac{1}{c})^x (1-\frac{1}{c})^{n-x} \\
> \quad & (1-\frac{1}{c})^n \\
\rightarrow \quad & e^{-d} \quad\quad\quad\quad\quad\quad\quad\quad\quad\quad (1)
\end{aligned}
$$

So the average possibility to be chosen as a sender will not diminish to zero when n goes to infinity. Similar

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result also holds for the average possibility to be chosen as a receiver. Thus, the possibility to be chosen as a sender-receiver pair is still on the order $\Theta(1)$ and will not diminish to zero.

□

Lemma 4.2: $\sum_{i=0}^{k} \frac{(-1)^i}{i} C_k^i = \ln(k+1) + r$, where $k \geq 1$ and r is an Euler constant. This lemma is proved in [2].

Proof: Denote the left-hand side of the equation by $A(k)$, then we have $A(k-1) = \sum_{i=0}^{k-1} \frac{(-1)^i}{i} C_{k-1}^i$. Notice that $C_k^i = C_{k-1}^i + C_{k-1}^{i-1}$, it follows

$$
\begin{aligned}
A(k) - A(k-1) &= \sum_{i=0}^{k} \frac{(-1)^i}{i} C_{k-1}^{i-1} \\
&= \frac{1}{k} \sum_{i=0}^{k} (-1)^{i-1} C_k^i \quad (2)
\end{aligned}
$$

Notice that $(1-1)^k = \sum_{i=0}^{k} (-1)^i C_k^i = 0$, hence we obtain

$$
\sum_{i=1}^{k} (-1)^{i-1} C_k^i = -\sum_{i=1}^{k} (-1)^i C_k^i = -\left[\sum_{i=0}^{k} (-1)^i C_k^i - 1 \right] = 1,
$$

combine with Equation 2, we get $A(k) - A(k-1) = \frac{1}{k}$, then

$$
\begin{aligned}
A(k) &= A(1) + \sum_{i=2}^{k} [A(k) - A(k-1)] \\
&= 1 + \sum_{i=2}^{k} \frac{1}{k} = \sum_{i=1}^{k} \frac{1}{k} \quad (3)
\end{aligned}
$$

Since the right-hand side of Equation 3 is the harmonic series, this lemma holds. □

Lemma 4.3: The total capacity for random stationary ad hoc network with one-hop strategy is $\Theta(\frac{n}{\log n})$, and the per-node throughput is $\Theta(\frac{1}{\log n})$.

Proof: We label n nodes $1, 2, 3, \cdots, n$, and they randomly choose their partners labeled $P_1, P_2, \cdots, P_n$, where $1 \leq P_i \leq n$. Then the maximum number of pairs is equal to the number of different numbers within $P_1, P_2, \cdots, P_n$.

The question above is equivalent to estimating the average number of replicas of each number within $P_1, P_2, \cdots, P_n$. For a certain integer within $[1, n]$, the expectation of its replicas among n uniformly distributed random sampling is

$$
\begin{aligned}
E(X) &= \sum_{i=1}^{n} i \cdot \Pr(X = i) \\
&= \sum_{i=1}^{n} i C_n^i \left(\frac{1}{n}\right)^i \left(1 - \frac{1}{n}\right)^{n-i} \\
&= \left(1 - \frac{1}{n}\right)^n \sum_{i=1}^{n} i C_n^i \rho^i \\
&\to \frac{1}{e} \sum_{i=1}^{n} i C_n^i \rho^i \\
&= \frac{1}{e} \cdot A(n) \quad (4)
\end{aligned}
$$

where $\rho = \frac{1}{n-1}$. Notice that $C_n^i = C_{n-1}^i + C_{n-1}^{i-1}$, and it follows

$$
\begin{aligned}
A(n) - A(n-1) &= \frac{1}{e} \sum_{i=1}^{n} i C_n^i \rho^i - \sum_{i=1}^{n-1} i C_{n-1}^i \rho^i \\
&= \sum_{i=1}^{n} i (C_{n-1}^i + C_{n-1}^{i-1}) \rho^i - \sum_{i=1}^{n-1} i C_{n-1}^i \rho^i \\
&= \sum_{i=1}^{n} i C_{n-1}^{i-1} \rho^i \quad (5)
\end{aligned}
$$

Integral the series of Equation (5), we get

$$
\begin{aligned}
\sum_{i=1}^{n} i C_{n-1}^{i-1} \rho^i &= \rho \sum_{i=1}^{n} i C_{n-1}^{i-1} \rho^{i-1} \\
&= \rho \left(\int \sum_{i=1}^{n} i C_{n-1}^{i-1} \rho^{i-1} d\rho \right)' \\
&= \rho \left(\sum_{i=1}^{n} C_{n-1}^{i-1} \rho^i \right)' \\
&= \rho \left(\rho \sum_{i=1}^{n} C_{n-1}^{i-1} \rho^{i-1} \right)' \\
&= \rho \left(\rho (1 + \rho)^{n-1} \right)' \\
&\to \rho (\rho e)' \\
&= \rho e \\
&= \frac{e}{n-1} \quad (6)
\end{aligned}
$$

Return to the Equation (5), since the right-hand side of Equation (4) is harmonic series, $E(X)$ holds the scale of $\Theta(\log n)$. This result indicates that there are on average $\Theta(\log n)$ replicas for each node within n random samplings. Thus, the average number of transmission pairs is on the scale of $\Theta(\frac{n}{\log n})$, and the average per-node throughput is $\Theta(\frac{1}{\log n})$. □

Lemma 4.4: The delay of single session with 2-hop strategy is

$$
E(D) = \Theta\left(\frac{n \log k}{k} \right).
$$

Proof: First we have $D_i = \min(D_{S_i R_j} + D_{R_j D} | j = 1 \cdots m)$ Then the Probability Distribution Function

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$\Pr(D_i \le x)$ can be expressed by that of every Source-Relay-Destination groups. Our aim is to split the S-R-D delay into the combination of S-R delay and R-D delay.

$$
\begin{aligned}
\Pr \quad & (D_i \le x) \\
= \quad & \Pr(\min(D_{S_iR_j} + D_{R_jD}, j = 1 \cdots m) \le x) \\
= \quad & 1 - \Pr(\min(D_{S_iR_j} + D_{R_jD}, j = 1 \cdots m) > x) \\
= \quad & 1 - (1 - \Pr(D_{S_iR_j} + D_{R_jD} \le x))^m \quad (7)
\end{aligned}
$$

Therefore the term $\Pr(D_{S_iR_j} + D_{R_jD} \le x)$ in Equation 7 becomes expandable. According to Total Probability Formula, we get

$$
\begin{aligned}
\Pr \quad & (D_{S_iR_j} + D_{R_jD} \le x) \\
= \quad & \sum_{i=0}^{x} \Pr(D_{S_iR_j} \le i) \cdot \Pr(D_{R_jD} = x - i) \\
= \quad & \sum_{i=0}^{x} [1 - (1 - P_{SR})^{i+1}] \cdot P_{RD} \cdot (1 - P_{RD})^{x-i} \\
& \text{(Similarly to Equation 18)}
\end{aligned}
$$

P_{RD} stands for the possibility that a certain relay makes a successful transmission to the destination, and P_{SR} stands for such transmission from a certain source node to a certain delay. In order to simplify the expression, let $\tau = 1 - P_{SR}$ and $\sigma = 1 - P_{RD}$, put them in the equation above

$$
\begin{aligned}
\Pr \quad & (D_{S_iR_j} + D_{R_jD} \le x) \\
= \quad & \sum_{i=0}^{x} [1 - (1 - P_{RD})^{i+1}] \cdot P_{SR} \cdot (1 - P_{SR})^{x-i} \\
= \quad & \sum_{i=0}^{x} [1 - \tau^{i+1}] \cdot (1 - \sigma) \cdot \sigma^{x-i} \\
& \text{(Geometric series summation)} \\
= \quad & 1 - \left(\tau^{x+1} + \sigma(1-\tau) \cdot \frac{\tau^{x+1} - \sigma^{x+1}}{\tau - \sigma}\right) \quad (8)
\end{aligned}
$$

Return to Equation 7, we get

$$
\begin{aligned}
\Pr \quad & (D_i \le x) \\
= \quad & 1 - (1 - \Pr(D_{S_iR_j} + D_{R_jD} \le x))^m \\
= \quad & 1 - \left(\tau^{x+1} + \sigma(1-\tau) \cdot \frac{\tau^{x+1} - \sigma^{x+1}}{\tau - \sigma}\right)^m \quad (9)
\end{aligned}
$$

Equation 9 is quite complex, so proper approximation is needed. Consider τ, σ first, according to Lemma 4.1

$$
\tau = 1 - P_{SR} = 1 - \frac{M_1}{c} \quad (10)
$$

and similarly

$$
\sigma = 1 - P_{RD} = 1 - \frac{M_2}{c} \quad (11)
$$

where M_1 and M_2 stands for the possibility for the two certain nodes be selected as S-R pair and R-D pair, respectively. Notice that $M_1 > M_2$.

Then recall Equation 9, and maintaining characteristics of PDF, we get

$$
\begin{aligned}
\Pr \quad & (D_i \le x) \\
= \quad & 1 - \left(\tau^{x+1} + \sigma(1-\tau) \cdot \frac{\tau^{x+1} - \sigma^{x+1}}{\tau - \sigma}\right)^m \\
= \quad & 1 - \left(\tau^{x+1} + \left(1 - \frac{M_2}{c}\right) \cdot \right. \\
& \left. \frac{M_1/c}{(M_2 - M_1)/c} \cdot (\tau^{x+1} - \sigma^{x+1})\right)^m \\
\approx \quad & 1 - \left(\tau^{x+1} + \frac{M_1}{M_2 - M_1}(\tau^{x+1} - \sigma^{x+1})\right)^m \quad (12)
\end{aligned}
$$

Let $T(x)$ denote the term

$$
T(x) \stackrel{\Delta}{=} \tau^x + \frac{M_1}{M_2 - M_1}(\tau^x - \sigma^x),
$$

and see Equation 16

$$
\begin{aligned}
E(D) \quad = \quad & \sum_{x=0}^{+\infty} x \cdot [\Pr(D_i \le x)^k - \Pr(D_i \le x - 1)^k] \\
= \quad & \sum_{x=0}^{+\infty} x \cdot ([1 - T^m(x+1)]^k - [1 - T^m(x)]^k) \\
= \quad & \sum_{x=0}^{+\infty} x \cdot \sum_{i=0}^{k} (-1)^k C_k^i [T^{mi}(x+1) - T^{mi}(x)] \\
& \text{(Binomial expansion)} \quad (13)
\end{aligned}
$$

We first simplify the scale of differential of $T(x)$

$$
\begin{aligned}
T' \quad = \quad & \left(\frac{M_2}{M_2 - M_1}\tau^x - \frac{M_1}{M_2 - M_1}\sigma^x\right)' \\
= \quad & \frac{M_2}{M_2 - M_1}\tau^x \ln \tau - \frac{M_1}{M_2 - M_1}\sigma^x \ln \sigma \\
= \quad & \frac{M_2}{M_2 - M_1}\tau^x \ln \left(1 - \frac{M_1}{c}\right) - \\
& \frac{M_1}{M_2 - M_1}\sigma^x \ln \left(1 - \frac{M_2}{c}\right)
\end{aligned}
$$

Since the term $\frac{M_1}{c}$ and $\frac{M_2}{c}$ become infinitesimal when n goes to infinity, we use Taylor's expansion and approximation $\ln(1 + x) \approx x$, we get

$$
\begin{aligned}
T' \quad \approx \quad & -\frac{M_2}{M_2 - M_1}\tau^x \cdot \frac{M_1}{c} + \frac{M_1}{M_2 - M_1}\sigma^x \cdot \frac{M_2}{c} \\
= \quad & \frac{M_1 M_2}{M_1 - M_2} \cdot \frac{1}{c} \cdot (\tau^x - \sigma^x) \\
= \quad & \frac{M_0}{c}(\tau^x - \sigma^x) \quad (14)
\end{aligned}
$$

According to Lagrange's Differential intermediate

value theorem, we get

$$
\begin{aligned}
E(D) &= \sum_{x=0}^{+\infty} x \cdot \sum_{i=0}^{k} (-1)^i C_k^i [T^{mi}(x+1) - T^{mi}(x)] \\
&= \sum_{x=0}^{+\infty} x \cdot \sum_{i=0}^{k} (-1)^i C_k^i [T^{mi}(\xi)]' \\
&\qquad \text{(where } x \le \xi \le x+1 \text{)} \\
&= \sum_{x=0}^{+\infty} x \cdot \sum_{i=0}^{k} (-1)^i C_k^i \cdot mi \cdot T^{mi-1}(\xi) \cdot T'(\xi) \\
&= m \sum_{i=0}^{k} (-1)^i i C_k^i \sum_{x=0}^{+\infty} x \cdot T^{mi-1}(\xi) \cdot T'(\xi) \\
&\qquad \text{(Exchange of summing label)} \\
&\approx \frac{mM_0}{c} \sum_{i=0}^{k} (-1)^{i-1} i C_k^i \sum_{x=0}^{+\infty} x \tau^{xmi} \\
&= \frac{mM_0}{c} \sum_{i=0}^{k} (-1)^{i-1} i C_k^i \cdot \frac{1}{(1-\tau^{mi})^2} \\
&\qquad \text{(Refer to Equation 17)}
\end{aligned}
$$

Notice that

$$
1 - \tau^{mi} = 1 - \left(1 - \frac{M_1}{c}\right)^{mi} \approx \frac{M_1 mi}{c}
$$

We have

$$
\begin{aligned}
E(D) &= \frac{mM_0}{c} \sum_{i=0}^{k} (-1)^{i-1} i C_k^i \cdot \frac{c^2}{M_1^2 m^2 i^2} \\
&= \frac{M_0 c}{M_1^2 m} \sum_{i=0}^{k} \frac{(-1)^{i-1}}{i} C_k^i \\
&= \Theta\left(\frac{n}{m} \log k\right) \\
&\qquad \text{(Lemma 4.2)}
\end{aligned} \tag{15}
$$

$\square$

5 CAPACITY OF STATIONARY CONVERGE-CAST NETWORK

In this section, we mainly study the capacity of stationary converge-cast networks. The capacity of stationary ad hoc network has already been discussed in a lot of papers previously. Recall that Gupta and Kumar studied stationary unicast ad hoc network with per-node capacity $\Theta(1/\sqrt{n \log n})$[3], and Li et al. showed that the per-node capacity of multicast network is $\Theta(1/\sqrt{kn \log n})$[5], which degenerates by the factor of $\sqrt{k}$. The reason for such degeneration is that multicast traffic pattern requires more redundancy since the whole transmission is a spanning tree, while unicast pattern only needs a chain, whose length only depends on the number of hops.

Then we know that an efficient way to estimate the upper bound in static converge-cast networks is to study its redundancy under the pattern.

Theorem 5.1: The total capacity for random stationary ad hoc network with one-hop strategy is $\Theta(\frac{n}{\log n})$ with per-node throughput $\Theta(\frac{1}{\log n})$.

Proof: See details in Section 4.3. $\square$

However, relays are necessary according to protocol model, yet Theorem 5.1 only shows the capacity of static ad hoc network with no relays. Then the whole transmission route can be treated as a chain whose length can be treated as redundancy because the whole transmission utilizes the same number of transmissions. In [3], the authors showed that the average number of hops in static ad hoc network is $\Theta(\sqrt{\frac{n}{\log n}})$ and therefore leads to $\Theta(\sqrt{\frac{n}{\log n}})$ redundancy.

Theorem 5.2: The maximum total capacity for a random stationary ad hoc network with n nodes and redundancy m is $O(\frac{n}{m \log n})$, while the per-node throughput is $O(\frac{1}{m \log n})$.

Proof: We first consider the case where there is no redundancy and every packet is transmitted from source directly to the destination. We have proved in Theorem 7.1 that the per-node capacity is $O(\frac{1}{\log n})$. When redundancy is introduced, all the packets will be transmitted m times on average in its session. However, the total number of transmission pairs in each time slot is $O(\frac{n}{\log n})$. If there exists an algorithm that we can be informed all the transmission pairs in the future T timeslots and write them in a table (notice that the pair with same nodes but in different time slots are treated as different pairs), we can deploy them for certain packets. Notice that each packet needs m pairs to finish its transmission and in this case $m = O(\frac{nT}{\log n})$. Therefore the maximum number of packets that can be transmitted is $O(\frac{nT}{m \log n})$. When T goes to infinity, the capacity can be treated as the average transmission rate, namely $O(\frac{n}{m \log n})$. $\square$

Theorem 5.2 can be used to explain the results in both unicast [3] and multicast [5]. For unicast, the average redundancy is $m = \Theta(\sqrt{\frac{n}{\log n}})$, then the total capacity is $O(\frac{n}{m \log n}) = O(\sqrt{\frac{n}{\log n}})$; For multicast, the average number of edges of the spanning tree is $m = \Theta(\sqrt{k} \cdot \sqrt{\frac{n}{\log n}})$ then the total capacity is $O(\frac{n}{m \log n}) = O(\sqrt{\frac{n}{k \log n}})$ when $k = O(\frac{n}{\log n})$.

Theorem 5.3: There exists an algorithm that can make full use of the network capacity.

Proof: Suppose we know all the transmission pairs in the future T timeslots and write them in a table. And we also know every mission that may happen, sorted by the starting time, labeled $S_1, S_2, \cdots$. Then we will build a scheduling strategy to make full use of transmission capacity.

Take S_1, and look up on the table. Catch and remove the pairs that are needed in S_1 transmission. The earlier the time, the higher the priority. Then we take S_2, with the same procedure, and look up on the table. Catch and remove the pairs that is needed in S_2 transmission. The procedure continues. $\square$

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As we have mentioned, the structure of spanning tree is the same as that of multicast according to [5], the average number of edges in a random spanning tree is $\Theta(\sqrt{k} \cdot \sqrt{\frac{n}{\log n}})$. We know that the result obtained in previous work does not vary in converge-cast network.

Theorem 5.4: The average redundancy of stationary converge-cast network is $\Theta(\sqrt{\frac{n}{\log n}})$, which is independent of the number of sources k.

Proof: Consider a spanning tree, the average length of k routes from k leaves to the root is $\Theta(\sqrt{\frac{n}{\log n}})$. Let $\lambda_1, \lambda_2, \cdots, \lambda_k$ denote the data rates of the k sources. Then the total valid transmission rate is $\lambda = \lambda_1 + \lambda_2 + \cdots + \lambda_k$. Notice that this rate remains the same in each layer of the tree, and the average number of layers is $\Theta(\sqrt{\frac{n}{\log n}})$. The total transmission rate of the spanning tree is $\Theta(\sqrt{\frac{n}{\log n}}) \cdot \lambda$, and therefore the number of redundancy is $\Theta(\sqrt{\frac{n}{\log n}})$, which is independent of k. □

According to Theorem 5.2 and Theorem 5.4, the total capacity for stationary converge-cast network is $\Theta(\sqrt{\frac{n}{\log n}})$ with per-node throughput $\Theta(\sqrt{\frac{1}{n \log n}})$. The result is the same as that of unicast, which indicates that the converge-cast network is essentially a unicast network.

6 DELAY OF SINGLE SESSION IN MOBILE CONVERGE-CAST NETWORK

In this section, we study the delay of single session in mobile converge-cast network. Denote the corresponding delays of k packets as $D_1, D_2, D_3, \cdots, D_k$. Note that the delay consists of two types of delays: transmission delay and queuing delay. However, transmission delay can be ignored since there are at most 2 hops in this model and such assumption does not change the order of the result. The total delay is defined as the maximum of k delays:

$$D = \max(D_1, D_2, \cdots, D_k)$$

The order of delay can be analyzed by taking its expectation:

$$
\begin{aligned}
E(D) &= \sum_{x=0}^{+\infty} x \cdot \Pr(D = x) \\
&= \sum_{x=1}^{+\infty} x \cdot [\Pr(D \leq x) - \Pr(D \leq x - 1)]
\end{aligned}
$$

Since all the sources are equally treated and mutually independent,

$$
\begin{aligned}
\Pr(D \leq x) &= \Pr(\max(D_1, D_2, \cdots, D_k) \leq x) \\
&= \Pr(D_1 \leq x \wedge D_2 \leq x \wedge \cdots \wedge D_k \leq x) \\
&= \Pr(D_i \leq x)^k
\end{aligned}
$$

Finally, we obtain the expression of delay as

$$E(D) = \sum_{x=0}^{+\infty} x \cdot [\Pr(D_i \leq x)^k - \Pr(D_i \leq x - 1)^k]. \quad (16)$$

6.1 The estimation of delay by using 1-hop algorithm

We first consider a simple case where only Source-Destination transmission is allowed and the delay is simply the Source-Destination transmission delay. Thus we have $D_i = D_{S_i D}$. To begin with, we introduce an important equation

$$\sum_{x=0}^{+\infty} x T^x = \frac{T}{(1-T)^2}. \quad (17)$$

Let $\Pr(D_{S_i D} = x)$ denote the probability that the time required for a successful transmission from source S_i to D is x. Obviously, such probability follows a geometric distribution with $\Pr(D = x) = (1 - P_{SD})^x P_{SD}$.

Then according to Geometric Series Summation, $\Pr(D_{S_i D} \leq x)$ can be expressed as

$$
\begin{aligned}
\Pr(D_{S_i D} \leq x) &= \sum_{x=0}^{+\infty} P_{SD}(1 - P_{SD})^x \\
&= 1 - (1 - P_{SD})^{x+1} \quad (18)
\end{aligned}
$$

We label $\gamma = 1 - P_{SD}$ and substitute it into Equation (16)

$$
\begin{aligned}
E(D) &= \sum_{x=0}^{+\infty} x \cdot [(1 - \gamma^{x+1})^k - (1 - \gamma^x)^k] \\
&= \sum_{x=0}^{+\infty} x \cdot \sum_{i=0}^{k} (1 - \gamma^i) \cdot C_k^i \cdot (-1)^{i+1} \cdot \gamma^{ix} \\
&\quad \text{(According to Binomial Expansion)} \\
&= \sum_{i=0}^{k} (1 - \gamma^i) \cdot C_k^i \cdot (-1)^{i+1} \sum_{x=0}^{+\infty} x \gamma^{ix} \\
&= \sum_{i=0}^{k} (1 - \gamma^i) \cdot C_k^i \cdot (-1)^{i+1} \cdot \frac{\gamma^i}{(1 - \gamma^i)^2} \\
&\quad \text{(Equation 17)} \\
&= \sum_{i=0}^{k} \frac{\gamma^i}{(1 - \gamma^i)} \cdot C_k^i \cdot (-1)^{i+1} \\
&= -1 + \sum_{i=0}^{k} \frac{1}{(1 - \gamma^i)} \cdot C_k^i \cdot (-1)^{i+1} \quad (19)
\end{aligned}
$$

In order to obtain γ, we need to calculate P_{SD}, which stands for the probability that a certain source can transmit data to a certain destination. Such an event happens when these two nodes reside in the same cell and are chosen to be the pair that can communicate. For the event that the two nodes meet in a particular cell, we have $P_{meet} = 1/c$. And for the event that the two nodes are chosen as an active pair, we find the average

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probability is a constant and independent of growth of n, according to Lemma 4.1. Thus we have

$$\gamma = 1 - \frac{M}{c}$$

Then we apply the Binomial Expansion and the fact that $i \leq k = o(n)$

$$1 - \gamma^i \approx 1 - (1 - \frac{M}{c})^i$$
$$= \frac{Mi}{c}$$

Then according to Lemma 4.2, $E(D)$ is characterized as follows,

$$E(D) = -1 + \frac{c}{M} \sum_{i=0}^{k} (-1)^{i+1} \cdot \frac{C_k^i}{i}$$
$$= \Theta(\frac{c}{M} \log k)$$
$$= \Theta(n \log k). \qquad (20)$$

6.2 Delay estimation of 2-hop algorithm

In this part, we only take Source-Relay transmission and Relay-Destination transmission into consideration, and ignore direct Source-Destination transmission, which means that all the packets should take a typical 2-hop journey from source to destination. Since we only consider the 2-hop transmission, and each source node will send their packets to the destination via m different relays, the delay of a single node can be expressed as the minimum one among m 2-hop transmission delays. Here we only present the result and the detailed proof is provided in 4.4.

Theorem 6.1: The expectation of delay in 2-hop single session mobile converge-cast network is

$$E(D) = \frac{n \log k}{k}.$$

7 DELAY AND CAPACITY FOR MULTI SESSION IN MOBILE CONVERGE-CAST NETWORK

In this section, we focus on the multi-session transmission, which relies on the results of single-session transmission. Capacity and delay of the converge-cast network will be derived under protocols with and without redundancy.

7.1 Protocol without redundancy

In this part we will discuss non-redundancy scheme. When we turn to multi-session, we have to consider, as usual, the definition of capacity and delay in the network. We focus on the order of delay first. According to the result of the 1-hop scheme of single-session Equation (20), the average delay of 1-hop is $\Theta(n \log k)$ for each session. The following theorem shows that such a result still holds for multi-session.

Theorem 7.1: The average delay for 1-hop converge-cast network is $\Theta(n \log k)$.

Proof: The 1-hop scheme only allows Source-Destination transmission in the network, which indicates that the source node keeps the packet all the time until it meets the destination. Thus, the number of transmissions equals k for each session and no extra transmission exists. Therefore, the only difference between the single session 1-hop and multi session 1-hop is that the number of nodes that have chances to compete for the transmission is larger than k. However, according to Lemma 4.1, the minimum probability of being selected is independent of n. Therefore, all the results for 1-hop single session still holds for 1-hop multi session and the expected delay is $\Theta(n \log k)$. □

In the following theorem, we will continue to show that non-redundancy 2-hop scheme does not improve the performance of the network.

Theorem 7.2: Non-redundancy relay scheme does not improve the delay and capacity in ad hoc networks.

Proof: Obviously, adding relays cannot improve the capacity, as proved in [1]. We need to consider the delay under this scheme. Assume that a certain packet is sent to its destination by two routes: via relay or via direct transmission. For direct transmission, the source waits until it resides in the same cell as the destination. For non-direct transmission, the source will send the packet to one of the relay, and then the relay waits until it meets the destination. By comparing these two strategies, we find that the second step of non-direct transmission is no different from the direct transmission, and therefore follows the same distribution function. Notice that $E(D_{S-R-D}) > E(D_{R-D}) = E(D_{S-D})$, we prove that relay cannot improve the performance of ad hoc network when no redundancy is used. □

Then we come to the capacity. In [6], Grossglauser et al. showed that per-node capacity is $\Theta(1)$ without redundancy in MANET under unicast traffic pattern. In multicast networks with mobility [2], the authors showed that the per-node capacity is $\Theta(1/k)$, with no redundancy. The degeneration rate of k mainly results from the fact that every single packet has k different destinations, and thus the output stream is increased by k times. Intuitively, the per-node capacity for converge-cast network is $\Theta(1)$, identical to that of the unicast network.

Theorem 7.3: The per-node capacity of the mobile converge-cast network without redundancy is $\Theta(1)$.

Proof: According to Theorem 1 in [1], we know that the network can stably and sufficiently support any rate $\lambda < \mu$, where $\mu = \frac{p+q}{2d}$.

For converge-cast networks, the output stream from sources is the same as that of the unicast network, which achieves the per-node capacity of $\Theta(1)$. We mainly concern the input stream to the destinations. Each destination node has to receive k different packets from k different sessions. However, the probability that the session is selected as a destination among these $k + 1$

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nodes is only $\frac{1}{k}$. If the whole network is divided into $\lceil\frac{n}{k+1}\rceil$ groups, with $k+1$ nodes for each group (We assume n is dividable by $k+1$). For each group, the input stream to each destination is $\frac{k}{k+1} = \Theta(1)$. Therefore, the input stream of the whole network can achieve $\Theta(1)$. Since the network is stable with input and output stream balanced, we draw the conclusion that the capacity can still reach $\Theta(1)$. □

7.2 Protocol with redundancy

In previous section, we obtain that the average delay with redundancy for a single session is $\Theta(\frac{n}{m}\log k)$, when every packet is duplicated to m different relays. When the situation changes from single session to multi session, a certain relationship between m and delay can be settled.

Let D_m denote the average time for k source nodes sending their packets to m relays and D_r denote the average time for these relays sending their packets to the destination. Then D_m should satisfy $D_m \le D$. Notice that the result about D Equation (15) is based on the assumption that every packet is duplicated to exact m different relays. When the order of D_m is larger than D, most of sessions cannot rely on such an assumption before the whole session is completed and it requires $D_m = O(m)$. The inequality above implies that the total delay cannot be larger than the time required by the sources to complete transmissions to all the relays.

However, when $D_m = o(D)$, almost all the packets still need to wait for the whole session to finish after they have been spread to m different relays and the delay is obviously not optimal in this case. The sources should have chances to duplicate packets to more relays. And it is easy to understand that the larger m is settled, the shorter the achievable delay is. In order to make the delay optimal, we need to keep the balance, which requires $D_m = D_r = \Theta(D)$.

The next step is to determine the value of D_m and D_r, which stand for the average time for k sources to send their packets to m different relays and the average time it takes all these relays to reach the destination, respectively. We have already known the expectation of total delay when the redundancy m is given, according to Equation (15). It is required that D_m and D_r should be of the same order with D to obtain the minimal delay.

When k is small, the handing-out delay D_m is much larger than D_r and we mainly focus on D_m. Obviously $D_m = \Theta(m)$ when k is small and after substituting it into equation $D_m = \Theta(D)$, we obtain the expression of delay for multi session scheme when k is small:

$$D = \Theta(\sqrt{n\log k}). \tag{21}$$

And the average number of replicas

$$m = \Theta(\sqrt{n\log k}). \tag{22}$$

Next, we study the capacity of converge-cast network with redundancy. According to Equation (7.3), the per-node capacity for non-redundant converge-cast network is $\Theta(1)$. Intuitively, the overall capacity will decrease when redundancy is introduced.

Theorem 7.4: The capacity of converge-cast network is $\Theta(\sqrt{\frac{1}{n\log k}})$, if redundancy is introduced.

Proof: Let the rate of the output stream from each source be λ. Since every packet is duplicated for m times, the overall input rate of relays is $m\lambda$. Notice that each relay can only receive one packet from each source, implying that the per-node throughput will not exceed $\Theta(1)$. We have $m\lambda \le \Theta(1)$, and therefore $\lambda \le \Theta(1/m)$. Since the minimum delay can be achieved when $m = \Theta(\sqrt{n\log k})$, we obtain the corresponding capacity $C = \Theta(\sqrt{\frac{1}{n\log k}})$. □

When k is large, the receiving delay D_r becomes magnificent and we will focus on it. Notice that in a k-source converge-cast session there are a total of k packets to be transmitted to the destination. Since only one transmission is allowed in each session for the destination node, $\Theta(k)$ is the lower bound on the receiving delay D_r. Then substituting it into $D_r = \Theta(D)$, we get the expression of delay for multi-session scheme when k is large:

$$D = \Theta(k). \tag{23}$$

The average number of replicas

$$m = \Theta(\frac{n\log k}{k}). \tag{24}$$

By combining Equations (21) and (23), we get the expression of delay for all k

$$D = \Theta(\sqrt{n\log k} + k). \tag{25}$$

Notice that the watershed $k = \Theta(\sqrt{n})$ exists, which is common in the analysis of wireless ad hoc networks. Then based on Theorem 7.4, we obtain the expression of capacity as follows:

$$C = \Theta(\sqrt{\frac{1}{n\log k}} + \frac{k}{n\log k}). \tag{26}$$

8 DELAY/CAPACITY TRADEOFF AND DISCUSSION

In Section 3, we study capacity of stationary converge-cast network . Capacity delay of mobile converge-cast network with and without redundancy is investigated in Sections 4 and 5. In this section, we will focus on discussions of the results we have obtained in the previous sections.

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Network	Per-node Capacity	Delay
unicast	$\Theta(1/\sqrt{n \log n})$	$\Theta(\sqrt{n/\log n})$
multicast	$\Theta(1/\sqrt{nk \log n})$	$\Theta(\sqrt{n/\log n})$
converge-cast	$\Theta(1/\sqrt{n \log n})$	$\Theta(\max(\sqrt{n/\log n}, k))$

TABLE 1
Summary of the results for stationary networks.

8.1 Discussion on stationary network

The capacity of stationary converge-cast network is $\Theta(1/\sqrt{n \log n})$. Some comparison between this result and that of unicast [3], multicast [5] is shown in Table 1.

We can see the capacity of converge-cast is the same as that of unicast. This is because each single packet is still transmitted as unicast transmission in converge-cast pattern, while the multicast pattern requires each packet transmitted to k different receivers. Such replication requires more redundancy and therefore decreases the capacity of multicast by a factor of $\sqrt{k}$.

For the delay, we can see when k is small, the delay of the three traffic patterns are all $\Theta(n/\log n)$. This is because delay is mainly determined by the number of hops required for a packet to finish the transmission and the number of hops of stationary random ad hoc network is bounded by $\Theta(n/\log n)$, regardless of the traffic pattern.

When k is large, the delay of converge-cast will be bounded by k itself, and is no longer bounded by the number of hops. Since the destination can receive only one packet in each time slot, therefore at least k time slots are needed to receive k packets.

8.2 Discussion on mobile network

The capacity and delay tradeoffs between the 2-hop relay algorithm without and with redundancy are summarized in Table 2 and we also make some comparisons between this result and those of unicast [1] and multicast [2].

Compared to unicast without redundancy, we notice that the per-node capacity does not change, namely $\Theta(1)$. This is mainly because k sources and one destination are randomly selected among n nodes in the network. So the transmission pairs are also randomly selected on average in a certain time slot. Therefore the network can be treated as a unicast network. The delay is better than the simple extension of k unicasts $\Theta(kn)$. This is because the destination has more chances to meet a packet among k packets than to meet the only packet in the whole network.

When redundancy is used with 2-hop scheme, the per-node capacity has little increment. The explanation is that the number of replicas is smaller than that of unicast to reach the optimal delay. When the number of replicas decreases, the degree of redundancy also decreases, and thus increases the capacity. The delay is also smaller than direct k unicast, as we have expected.

For comparison with multicast, without redundancy, we notice that the per-node capacity is k times larger. That is mainly because k packets in converge-cast are entirely different while k receivers ask for same packets in multi-cast problem, which can only be treated as one valid packet, and therefore achieves a k-time increment. When considering delay, we notice that they are the same, namely $\Theta(n \log k)$, which indicates that the converge-cast transmission can be treated as a reversed multicast transmission when no redundancy is used. When the degree of redundancy is low, the interference between transmission pairs will be small enough to be treated approximate to that of multicast. When we consider all the transmissions in a single session are changed in direction, the whole session becomes a converge-cast session, and the delay still holds.

When redundancy is used in this case, the converge-cast network can no longer be treated as a reversed multicast network any more. We notice that the per-node capacity is lower than k times of that of the multicast. The explanation is that the converge-cast network creates k times more replicas in each session than the multicast network, and therefore consumes the per-node capacity by interference. As for the delay, the multicast is also better than converge-cast, which can also be explained by interference. All such comparisons coincide with our expectations.

Then we come to the tradeoff between delay and capacity. According to Equation (25) and Theorem 7.4, both delay and capacity have inverse proportion relation with the degree of redundancy m. When m is in the valid interval, we can obtain different capacity and delay by adjusting m. If we eliminate m in both expression by determining the ratio $delay/capacity$, we can see the tradeoff. For converge-cast networks, we have the following tradeoff

$$delay/capacity \geq \Theta(n \log k) \qquad (27)$$

The ratio is equal to k times the tradeoff of multicast $O(nk \log k)$, and better than directly extending k unicast $\Theta(nk)$.

In summary, the performance of converge-cast networks is better than directly extending result of that of unicast networks, which is quite reasonable as it stands. Since we increase the amount of information by k times, the converge-cast network achieves a larger capacity than multicast network. However, when discarding the increment of k, we see that the multicast network is better than the converge-cast network. This is quite reasonable if we consider all the k packets in the same session combined into one big packet.

8.3 Comparison between stationary and mobile networks

There are two reasons why we introduce mobility into wireless ad hoc network. The first reason is that nodes are required to move in some applications. One such

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Traffic	Scheme	Capacity	Delay
converge-cast	w.o red 2-hop	$\Theta(1)$	$\Theta(n \log k)$
	w. red 2-hop	$\Theta(\sqrt{\frac{1}{n \log k}} + \frac{k}{n \log k})$	$\Theta(\sqrt{n \log k} + k)$
uni-cast	w.o. red	$O(1)$	$O(n)$
	w. red 2-hop	$O(1/\sqrt{n})$	$O(\sqrt{n})$
	w. red k-hop	$O(1/(n \log n))$	$O(\log n)$
multi-cast	w.o. red 2-hop	$\Theta(1/k)$	$\Theta(n \log k)$
	w. red 2-hop	$\Omega(1/(k\sqrt{n \log k}))$	$\Theta(\sqrt{n \log k})$

TABLE 2
Summary of results for mobile networks.

example is the active observing sensor networks. Compared with passive observing sensor networks, the sensors actively travel around to search for information. The second reason is that mobility improves the performance of the network, such as capacity and connectivity. In [6], the author showed that mobility can increase the scale of per-node capacity up to $\Theta(1)$.

Similar to unicast and multicast networks, in converge-cast network, mobility can also be used to increase capacity, as is shown in Tables 1 and 2.

8.4 Discussion about redundancy

Redundancy is an important concept in both static and mobile wireless ad hoc network . In static ad hoc networks, the redundancy is used to establish a transmission route between one node and its remote destination. Such redundancy is necessary since the transmission range cannot be very large due to the need to avoid mutual interference. Under multicast pattern, we can even combine some edges together and build a spanning tree to reduce the redundancy. In mobile networks, redundancy helps reduce the delay by providing greater chances for the destination to catch its packet. Since the total delay also includes the time for the source to make a certain number of replicas, the optimal delay happens when the scale of hand-out delay is the same as the total delay.

On the other hand, redundancy decreases packet delay at the cost of capacity. That is because the overall capacity is fixed when the network is settled. The larger the redundancy, the less the efficient throughput and such relationship exists in both stationary and mobile ad hoc networks. The difference is that the non-redundancy throughput are $\Theta(\frac{1}{\log n})$ and $\Theta(1)$ for stationary and mobile schemes respectively. The factor of $\Theta(\log n)$ results from the randomness of interference.

8.5 Summary

We make a conclusion of the results of hops, redundancy, delay and capacity and their relationship in all kinds of ad hoc network in Table 3. And several summarizations are shown as below.

- **Hops:** The number of hops is controlled by the transmission range in the stationary pattern and

can be artificially determined in the *i.i.d* mobility pattern. Therefore, we find that the number of hops are all $\Theta(\sqrt{n/\log n})$ in stationary networks. For the *i.i.d* mobility pattern, the discussion on 2-hop is enough. 2-hop strategy stands for redundancy strategy, whose transmission depends on both mobility and data copy. This algorithm is first proposed in [1] and extensively deployed in MANET. In [28], its features are well studied under various mobility models and our 2-hop scheme is also similar to the previous case.

- **Redundancy:** The redundancy is the deterministic factor that influences the capacity. In the stationary pattern, the redundancy mainly depends on the number of hops. In the mobile pattern, the redundancy depends on the data copies and the number of replicas is controlled by scheduling scheme to maintain the minimum tolerated delay.

- **Capacity:** The capacity here is per-node capacity, which mainly depends on redundancy. We can see in the stationary pattern that the capacity can be obtained by dividing $\Theta(1/\log n)$ with redundancy. However, in the mobile pattern, the ratio of *redundancy* to *capacity* is $\Theta(1)$, which is better than that of stationary pattern.

- **Delay:** The delay depends on the number of hops in the stationary pattern, and is controlled by redundancy in the mobile pattern, but with inverse proportion relation. The ratio of delay-capacity tradeoff *delay*/*capacity* is in fact the reflection of inverse relation between delay and redundancy.

9 CONCLUSION AND FUTURE WORK

In this paper, we study the capacity, delay and their tradeoffs in k-source converge-cast networks. We consider both stationary and mobile networks. For stationary network, we compare our result with that in unicast networks [3] and multicast networks [5]. We find that the per-node capacity is the same as that of unicast. We also show that the essential key that controls the capacity is the redundancy, with inverse relationship. Converge-cast network has the same redundancy as the unicast, and thus the same capacity.

For mobile networks, we only analyze the i.i.d mobility model. Our analysis is also divided into two parts:

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Mobility	Traffic pattern	Hops	Redundancy	Capacity	Delay	Delay/Capacity
stationary	unicast	$\Theta(\sqrt{\frac{n}{\log n}})$	$\Theta(\sqrt{\frac{n}{\log n}})$	$\Theta(\sqrt{\frac{1}{n\log n}})$	$\Theta(\sqrt{\frac{n}{\log n}})$	$\Theta(n)$
stationary	multicast	$\Theta(\sqrt{\frac{n}{\log n}})$	$\Theta(\sqrt{\frac{kn}{\log n}})$	$\Theta(\sqrt{\frac{1}{nk\log n}})$	$\Theta(\sqrt{\frac{n}{\log n}})$	$\Theta(\sqrt{k}n)$
stationary	converge-cast	$\Theta(\sqrt{\frac{n}{\log n}})$	$\Theta(\sqrt{\frac{n}{\log n}})$	$\Theta(\sqrt{\frac{1}{n\log n}})$	$\Theta(\sqrt{\frac{n}{\log n}})$	$\Theta(n)$
i.i.d	unicast	1	1	$\Theta(1)$	$\Theta(n)$	$\Theta(n)$
i.i.d	unicast	2	$\Theta(\sqrt{n})$	$\Theta(\sqrt{\frac{1}{n}})$	$\Theta(\sqrt{n})$	$\Theta(n)$
i.i.d	unicast	$\Theta(\log n)$	$\Theta(n\log n)$	$\Theta(\frac{1}{n\log n})$	$\Theta(\log n)$	$\Theta(n)$
i.i.d	multicast	1	k	$\Theta(\frac{1}{k})$	$\Theta(n\log k)$	$\Theta(kn\log k)$
i.i.d	multicast	2	$\Theta(\sqrt{n\log k}+k)$	$\Theta(\frac{1}{\sqrt{n\log k}+k})$	$\Theta(\sqrt{n\log k})$	$\Theta(n\log k)$
i.i.d	converge-cast	1	1	$\Theta(1)$	$\Theta(n\log k)$	$\Theta(n\log k)$
i.i.d	converge-cast	2	$\Theta(\min(\sqrt{n\log k},\frac{n\log k}{k}))$	$\Theta(\sqrt{\frac{1}{n\log k}+\frac{k}{n\log k}})$	$\Theta(\sqrt{n\log k}+k)$	$\Theta(n\log k)$

TABLE 3
Comparison of redundancy, delay and capacity

the delay of the single-session and the multi-session. For the single-session problem, we mainly discuss the delay, with either 1-hop or 2-hop protocol, and find that 2-hop without redundancy cannot help to improve delay and bears no difference with the 1-hop protocol. The packet delay can be decreased only when packet redundancy is introduced in the network. For the multi-session problem, we also discuss both schemes with and without redundancy. The capacity delay tradeoff achieved is much better than direct extension of uni-cast problem, but worse than k-destination multi-cast network.

For future work, we will take into account the multi-hop transmission schemes and the effect of different mobility patterns, such as random walk pattern and slow mobility pattern.

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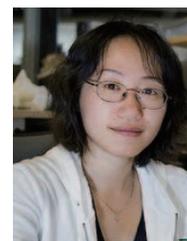

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