Outage Performance Analysis of Two-Way Relay System with Multi-Antenna Relay Node

Rui Wang and Meixia Tao
Dept. of Electronic Engineering
Shanghai Jiao Tong University, Shanghai, 200240, P. R. China
Email: {liouxingrui, mxtao}@sjtu.edu.cn

***Abstract*—This paper presents an analytical study on the outage performance of amplify-and-forward (AF) two-way relay system with multi-antenna relay node (RN). Two major bidirectional protocols, i.e., two time slots multiple access broadcast (MABC) protocol and three time slots time division broadcast (TDBC) protocol, are considered. For both considerations, we first assume that instantaneous channel-state-information (CSI) is unavailable at RN, thus RN just simply uses the fixed relay gain derived from statistical CSI to scale the received signals before forwarding. We then consider the scenario where RN can obtain the instantaneous CSI to perform the zero-forcing (ZF) relay precoding. The closed-form expressions of outage probability are derived for all cases. Based on these expressions, the diversity-multiplexing tradeoff (DMT) is further obtained for the MABC protocol. The analytical results show that, for non-precoding MABC scheme, the diversity order is only 1, which is independent to the relay antenna number M. While for the ZF-precoding case, the diversity order of $M-1$ can be obtained.**

I. Introduction

Relay-assisted cooperative transmission has been shown able to offer significant benefits, such as throughput enhancement, coverage extension and power reduction, in wireless communications. However, due to the half-duplex constraint, the spectral efficiency of traditional one-way relaying is highly constrained. Recently, two-way relaying has been proposed as a more efficient protocol to complete the bidirectional transmission of two users by applying the physical layer network coding (PLNC). Based on the required time slots to complete the bidirectional communication, two categories of protocols have been widely studied in the literature. The first one is called multiple access broadcast (MABC) which consists of two time slots. In the first time slot, two sources send their signals to the relay node (RN) simultaneously. Then in the second time slot, the RN broadcasts the mixed signals to two destinations. The second one is referred to as the time division broadcast (TDBC) where a round of information exchange is completed within three time slots. In the first and second time slot, two sources transmit their signals to the RN separately. Then, the RN broadcasts the mixed signals to two destinations in the third time slot. Note both transmission protocols have their own merits. The MABC is more spectral efficient for needing less time slots, while TDBC can achieve more reliable transmission by exploiting extra direct-path link.

A few analytical studies have been reported to evaluate the promising gain of the two-way relaying. For example, the authors in [1] compare the performance of amplify-and-forward (AF) bidirectional transmission over different schemes. Besides the studies on two-way relaying with single RN, the multiple RNs case, which can be used to increase the transmission diversity, has recently drawn significant attention, especially combined with relay selection technique. In [2], the relay selection is investigated based on max-min criterion to minimize the outage probability under AF strategy. In [3], the relay selection is studied based on both max-min and max-sum criteria under decode-and-forward (DF) strategy.

An alternative approach to combat the fading is to use the multi-antenna RN. In [4], the authors analyze the diversity-multiplexing tradeoff (DMT) at finite signal-to-noise ratio (SNR) for multi-antenna RN (MAR) two-way relay system under DF strategy. In this work, we aim to provide an analytical study for the MAR two-way relay system under AF strategy. Here the AF strategy is chosen for its low complexity in implementation. The two major bidirectional transmission protocols, i.e., MABC and TDBC, are considered. For both considerations, we first assume that the RN is unable to obtain the required CSI to perform complex processing and just uses the fixed relay gain based on the statistical CSI to scale the received signals to satisfy the power constraint at RN in long-term. Such approach can reduce the relay computational complexity and system feedback overhead. Second, we consider a more complicated scenario where the instantaneous CSI is available at the RN. Thus the zero-forcing (ZF) relay precoding is conducted to improve the system performance. The closed-form expressions of outage probability are derived for all cases. Based on the derived expressions, the diversity-multiplexing tradeoff is obtained for the MABC protocol.

II. System Model

Consider a two-way relay system where two single-antenna source nodes, denoted as S_1 and S_2, want to exchange messages through a RN, denoted as R and equipped with M antennas. Let x_i, with $\mathcal{E}[|x_i|^2] = E_i$, denote the transmit signal from S_i. The channel vector from S_i to R and R to S_i are denoted by $\mathbf{h}_i$ and $\mathbf{g}_i$, respectively. They are modeled as

This work is supported by the Doctoral Fund of Ministry of Education of China (200802481002), the National Natural Science Foundation of China under grant 60902019 and the Innovation Program of Shanghai Municipal Education Commission under grant 11ZZ19.

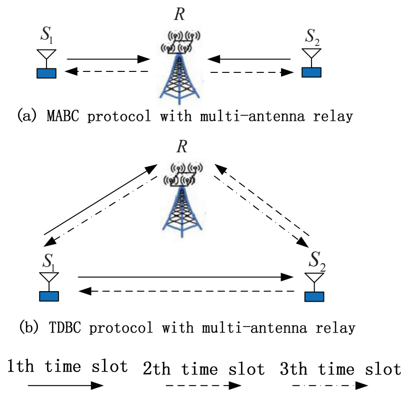

Fig. 1. Illustration of two-way relay system with multi-antenna RN.

independent and identically distributed (i.i.d) Rayleigh-fading channels with $\mathbf{h}_i \sim \mathcal{CN}(0, \sigma_{h_i}^2 \mathbf{I}_M)$ and $\mathbf{g}_i \sim \mathcal{CN}(0, \sigma_{g_i}^2 \mathbf{I}_M)$.

A. MABC Protocol

As depicted in Fig. 1(a), S_1 and S_2 simultaneously transmit their signals x_1 and x_2 to RN in the first time slot. Thus, the received signals at the RN is denoted as

$$\mathbf{y}_R^{MA} = \mathbf{h}_1 x_1 + \mathbf{h}_2 x_2 + \mathbf{n}_R, \tag{1}$$

where $\mathbf{n}_R$ denotes the noise vector with $\mathbf{n}_R \sim \mathcal{CN}(\mathbf{0}, \sigma_R^2 \mathbf{I}_M)$. Upon receiving $\mathbf{y}_R^{MA}$, the relay amplifies it by multiplying it with a linear processing matrix $\mathbf{F} \in \mathbb{C}^{M \times M}$ and then forwards it in the second time slot. Therefore, the $M \times 1$ transmit signal vector from RN can be expressed as

$$\mathbf{x}_R^{MA} = \mathbf{F}\mathbf{y}_R^{MA} = \mathbf{F}\mathbf{h}_1 x_1 + \mathbf{F}\mathbf{h}_2 x_2 + \mathbf{F}\mathbf{n}_R. \tag{2}$$

Then the received signal at S_i during the BC phase can be written as, after the self-interference subtraction

$$\mathbf{y}_i^{MA} = \mathbf{g}_i^T \mathbf{F}\mathbf{h}_{\bar{i}} x_{\bar{i}} + \mathbf{g}_i^T \mathbf{F}\mathbf{n}_R + n_i,$$

where $\bar{i} = 2$ if $i = 1$ and $\bar{i} = 1$ if $i = 2$, n_i denotes the Gaussian noise at S_i with $n_i \sim \mathcal{CN}(0, \sigma_i^2)$. Superscript $(\cdot)^T$ denotes the transpose. Thus, the received signal-to-noise-ratio (SNR) at each destination is expressed as

$$\mathrm{SNR}_i^{MA} = \frac{E_i |\mathbf{g}_{\bar{i}}^T \mathbf{F}\mathbf{h}_i|^2}{\sigma_R^2 ||\mathbf{g}_{\bar{i}}^T \mathbf{F}||^2 + \sigma_{\bar{i}}^2}, \; i = 1, 2. \tag{3}$$

B. TDBC Protocol

As depicted in Fig. 1(b), the received signal through the direct-path link at S_i after the first two time slot transmission is denoted as

$$y_{1,0}^{TD} = h_{21} x_2 + n_{1,0}, \; y_{2,0}^{TD} = h_{12} x_1 + n_{2,0}, \tag{4}$$

where $h_{i\bar{i}}$ denotes the direct-path channel from S_i to $S_{\bar{i}}$ with $h_{i\bar{i}} \sim \mathcal{CN}(0, \sigma_{h_{i\bar{i}}}^2)$, $n_{i,0}$ denotes the destination noise at S_i with $n_{i,0} \sim \mathcal{CN}(0, \sigma_i^2)$, $i = 1, 2$. For the relay-path link, since the signals from two sources are transmitted in orthogonal channels, the received signals at the RN after the first and second time slots are denoted as

$$\mathbf{y}_{R,1}^{TD} = \mathbf{h}_1 x_1 + \mathbf{n}_{R,1}, \; \mathbf{y}_{R,2}^{TD} = \mathbf{h}_2 x_2 + \mathbf{n}_{R,2}, \tag{5}$$

where $\mathbf{n}_{R,i}$ denotes the additive noise vector at RN with $\mathbf{n}_{R,i} \sim \mathcal{CN}(0, \sigma_R^2 \mathbf{I}_M)$. The transmit signal vectors from RN is expressed as

$$\mathbf{x}_{R,1}^{TD} = \mathbf{F}_1 \mathbf{h}_1 x_1 + \mathbf{F}_1 \mathbf{n}_{R,1}, \; \mathbf{x}_{R,2}^{TD} = \mathbf{F}_2 \mathbf{h}_2 x_2 + \mathbf{F}_2 \mathbf{n}_{R,2},$$

where $\mathbf{F}_i$ denotes the linear processing for $\mathbf{y}_{R,i}^{TD}$ as in (2). Then, in the third time slot, the RN broadcasts the superimposed signal and the received signal at each destination node can be expressed as, after subtracting the self-interference

$$y_{i,3}^{TD} = \mathbf{g}_i^T \mathbf{F}_{\bar{i}} \mathbf{h}_{\bar{i}} x_{\bar{i}} + \mathbf{g}_i^T \mathbf{F}_i \mathbf{n}_{R,i} + \mathbf{g}_i^T \mathbf{F}_{\bar{i}} \mathbf{n}_{R,\bar{i}} + n_{i,3}, \tag{6}$$

where $n_{i,3}$ denotes the noise at S_i in the third time slot with $n_{i,3} \sim \mathcal{CN}(0, \sigma_i^2)$. Since two copies of signal $x_{\bar{i}}$ can be exploited at S_i as expressed in (4) and (6), the maximal-ratio combining (MRC) technique is used to combine them and the resultant SNR at S_i is depicted as

$$\mathrm{SNR}_i^{TD} = \frac{E_i |h_{i\bar{i}}|^2}{\sigma_{\bar{i}}^2} + \frac{E_i |\mathbf{g}_{\bar{i}}^T \mathbf{F}_i \mathbf{h}_i|^2}{\sigma_R^2 ||\mathbf{g}_{\bar{i}}^T \mathbf{F}_i||^2 + \sigma_R^2 ||\mathbf{g}_{\bar{i}}^T \mathbf{F}_{\bar{i}}||^2 + \sigma_{\bar{i}}^2}. \tag{7}$$

C. Outage Performance Metric

Based on the SNR given in (3) and (7), the corresponding rate is yielded as

$$\gamma_i^{MA} = \frac{1}{2} \log_2(1 + \mathrm{SNR}_i^{MA}), \; \gamma_i^{TD} = \frac{1}{3} \log_2(1 + \mathrm{SNR}_i^{TD}).$$

Here, the pre-log factors $\frac{1}{2}$ and $\frac{1}{3}$ result from the fact that the MABC is operated in two time slots, while TDBC in three time slots. Since the two-way relay system involves two users, we define the system in outage if any user is in outage, Thus, the outage probability of two-way relay system can be defined as

$$P_{out}^{MA/TD} = \Pr[\gamma_1^{MA/TD} < R_1 \; or \; \gamma_2^{MA/TD} < R_2],$$

where R_1 and R_2 are the pre-set rate thresholds. This definition can also be equivalently written as

$$\begin{aligned} P_{out}^{MA/TD} = \Pr[&\mathrm{SNR}_1^{MA/TD} < \tau_1^{MA/TD} \\ &or \; \mathrm{SNR}_2^{MA/TD} < \tau_2^{MA/TD}], \end{aligned} \tag{8}$$

where $\tau_i^{MA} = 2^{2 * R_i} - 1$ and $\tau_i^{TD} = 2^{3 * R_i} - 1$, $i = 1, 2$.

III. Outage Performance Analysis

In this section, two different relaying schemes, i.e., non-precoding scheme and ZF-precoding scheme, are considered for each protocol. In the first scheme, the instantaneous CSI is unavailable at RN. Thus, the "blind" relay just scales the received signals based on the statistical CSI to satisfy the power constraint in a long term. Note this scheme can significantly reduce the relay computational complexity and feedback overhead. While in the ZF-precoding scheme, the RN is assumed to have the knowledge of instantaneous CSI which will be used to performe the ZF-precoding at RN. Although this scheme needs the RN to have high processing ability and more feedback overheads, we will see that the relay precoding can significantly improve the system performance.

A. Non-precoding Scheme for MABC

Here, $\mathbf{F}$ in (2) should be expressed as $\alpha^{MA}\mathbf{I}_M$ with $\alpha^{MA} = (\frac{E_R}{E_1\mathcal{E}[||\mathbf{h}_1||^2]+E_2\mathcal{E}[||\mathbf{h}_2||^2]+\mathcal{E}[||\mathbf{n}_R||^2]})^{0.5} = (\frac{E_R}{M(E_1\sigma_{h_1}^2+E_2\sigma_{h_2}^2+\sigma_R^2)})^{0.5}$, where E_R is the maximum power at RN. Thus, the expression of outage probability in (8) can be rewritten as

$$P_{out}^{MA} = \Pr[\rho_1 < \tau_1^{MA}] + \Pr[\rho_2 < \tau_2^{MA}] - \Pr[\rho_1 < \tau_1^{MA}]\Pr[\rho_2 < \tau_2^{MA}], \quad (9)$$

where $\rho_i = \frac{\alpha^{MA^2}E_i|\mathbf{g}_{\bar{i}}^T\mathbf{h}_i|^2}{\alpha^{MA^2}\sigma_R^2||\mathbf{g}_{\bar{i}}^T||^2+\sigma_{\bar{i}}^2}$. Here we have used the fact that $\mathbf{h}_1$, $\mathbf{h}_2$, $\mathbf{g}_1$ and $\mathbf{g}_2$ are mutually independent.

Theorem 1[1]: The outage probability of the MABC AF two-way relaying protocol without precoding is given as $P_{out}^{MA} = P_{out_1}^{MA} + P_{out_2}^{MA} - P_{out_1}^{MA}P_{out_2}^{MA}$ with

$$P_{out_i}^{MA} = 1 - \frac{\exp\left(-\tau_i^{MA}\sigma_R^2/(E_i\sigma_{h_i}^2)\right)}{2^{-1}\sigma_{g_{\bar{i}}}^M\Gamma(M)} \left(\frac{\tau_i^{MA}\sigma_{\bar{i}}^2}{E_i\alpha^{MA^2}\sigma_{h_i}^2}\right)^{M/2} \mathcal{K}_M\left(2\sqrt{\frac{\tau_i^{MA}\sigma_{\bar{i}}^2}{E_i\alpha^{MA^2}\sigma_{g_{\bar{i}}}^2\sigma_{h_i}^2}}\right).$$

Here, $\Gamma(\cdot)$ is the Gamma function and $\mathcal{K}_M(\cdot)$ is the Bessel function of the second kind with order M [5].

Based on Theorem 1, we further obtain the asymptotic performance of the system in terms of DMT as follows.

Corollary 1: The achievable DMT for each user in MABC AF two-way relaying without precoding can be expressed as $d(r) = 1 - 2r,\ 0 \le r \le 1/2$.

Corollary 1 demonstrates that each user can only get the maximum diversity order of one, though multiple antennas are equipped at the RN. This is expected as no precoding is applied at the RN.

B. Non-precoding Scheme for TDBC

For this scheme, the matrices $\mathbf{F}_1$ and $\mathbf{F}_2$ are reduced to $\alpha_1^{TD}\mathbf{I}_M$ and $\alpha_2^{TD}\mathbf{I}_M$, respectively, and $\alpha_i^{TD} = (\frac{a_iE_R}{E_i\mathcal{E}(||\mathbf{h}_i||^2)+\mathcal{E}(||\mathbf{n}_{R,i}||^2)})^{0.5} = (\frac{a_iE_R}{M(E_i\sigma_{h_i}^2+\sigma_R^2)})^{0.5}$, where a_1 and a_2 are the relay power allocation parameters for x_1 and x_2, respectively. The optimal a_i is expected to minimize the final system outage probability. Here we simply choose $a_1 = a_2 = 0.5$[2]. Substituting $\mathbf{F}_1$ and $\mathbf{F}_2$ into (7), the outage probability in (8) for TDBC protocol can be denoted as

$$P_{out}^{TD} = \Pr[\epsilon_1 < \tau_1^{TD}] + \Pr[\epsilon_2 < \tau_2^{TD}] - \Pr[\epsilon_1 < \tau_1^{TD}]\Pr[\epsilon_2 < \tau_2^{TD}], \quad (10)$$

where $\epsilon_i = \frac{E_i|h_{i\bar{i}}|^2}{\sigma_{\bar{i}}^2} + \frac{\alpha_i^{TD^2}E_i|\mathbf{g}_{\bar{i}}^T\mathbf{h}_i|^2}{2\alpha_i^{TD^2}\sigma_R^2||\mathbf{g}_{\bar{i}}^T||^2+\sigma_{\bar{i}}^2}$, $i = 1, 2$.

Theorem 2: The outage probability of the TDBC AF two-way relaying protocol without precoding is given as $P_{out}^{TD} = P_{out_1}^{TD} + P_{out_2}^{TD} - P_{out_1}^{TD}P_{out_2}^{TD}$ with

$$P_{out_i}^{TD} = 1 - \exp\left(-\frac{\tau_i^{TD}}{\beta^i}\right) - \frac{2^{M-1}A_2^iA_3^i\exp\left(-\tau_i^{TD}/\beta^i\right)}{\beta^i {A_5^i}^M} \sum_{k=0}^{\infty}\frac{(\frac{1}{\beta^i}-A_1^i)^k{\tau_i^{TD}}^{k+M/2+1}}{k!}G_{13}^{21}\left(\frac{{A_5^i}^2}{4}\middle|\begin{matrix}-k\\M,0,-k-1\end{matrix}\right).$$

Here, $A_1^i = \frac{\sigma_R^2}{E_i\sigma_{h_i}^2}$, $A_2^i = \frac{2}{\sigma_{g_{\bar{i}}}^M\Gamma(M)}$, $A_3^i = (\frac{\sigma_{\bar{i}}^2}{E_i\alpha_i^{TD^2}\sigma_{h_i}^2})^{M/2}$, $A_4^i = \frac{\sigma_{\bar{i}}^2}{E_i\alpha_i^{TD^2}\sigma_{g_{\bar{i}}}^2\sigma_{h_i}^2}$, $A_5^i = 2\sqrt{A_4^i\tau_i^{TD}}$ and $\beta^i = \frac{E_i\sigma_{h_{i\bar{i}}}^2}{\sigma_{\bar{i}}^2}$. $G_{13}^{21}(\cdot)$ is the Meijers G-Function defined in [5].

Remark 1: Although the number k in $P_{out_i}^{TD}$ should range from 0 to infinite, it is shown that $P_{out_i}^{TD}$ converges very fast and setting the maximum k equal to 10 is enough.

Remark 2: From Theorem 2, it is not easy to perform the DMT analysis as in Corollary 1. However, since the TDBC protocol exploits the direct-path link, it is reasonable to derive that the diversity order of 2 can be obtained for each user when $\epsilon_i \to \infty$. This result will be confirmed in Section IV.

C. ZF-precoding Scheme for MABC

We rewrite (1) in vector form as $\mathbf{y}_R^{ZF-M} = \mathbf{Hx} + \mathbf{n}_R$, where $\mathbf{H} = [\mathbf{h}_1, \mathbf{h}_2]$ and $\mathbf{x} = [x_1, x_2]^T$ with $\mathcal{E}(\mathbf{xx}^H) = \mathbf{E} = \mathrm{Diag}(E_1, E_2)$. Superscript $(\cdot)^H$ denotes the conjugate transpose. The transmit signal at RN is denoted as $\mathbf{x}_R^{ZF-M} = \alpha^{ZF-M}\mathbf{FHx} + \alpha^{ZF-M}\mathbf{Fn}_R$. Similarly, the received signals at two destination nodes can be expressed as

$$\mathbf{y}^{ZF-M} = \alpha^{ZF-M}\mathbf{GFHx} + \alpha^{ZF-M}\mathbf{GFn}_R + \mathbf{n}, \quad (11)$$

where $\mathbf{y}^{ZF-M} = [y_2, y_1]$, $\mathbf{G} = [\mathbf{g}_2, \mathbf{g}_1]^T$ and $\mathbf{n} = [n_2, n_1]^T$ is the destination noise vector with $\mathcal{E}(\mathbf{nn}^H) = \mathrm{Diag}(\sigma_2^2, \sigma_1^2)$. Note that the ZF relay precoder aims to eliminate the multi-user interference. Thus, $\mathbf{F}$ can be expressed as

$$\mathbf{F} = \mathbf{G}^\dagger\mathbf{H}^\dagger, \quad (12)$$

where $\mathbf{G}^\dagger = \mathbf{G}^H(\mathbf{GG}^H)^{-1}$ and $\mathbf{H}^\dagger = (\mathbf{H}^H\mathbf{H})^{-1}\mathbf{H}^H$.

Next, we try to determine the power scalar α^{ZF-M}. Although the instantaneous CSI is available at RN, we assume that α^{ZF-M} is still decided based on the statistical CSI as in [6]. Such decision approach can reduce the computational complexity at RN since it does not need to compute the scalar for each channel realization. On the other hand, it is helpful in deriving the closed-form analytical results. From (11), it is not hard to obtain α^{ZF-M} as $\alpha^{ZF-M} = (\frac{E_R}{\mathcal{E}\{\mathrm{Tr}(\mathbf{G}^\dagger\mathbf{EG}^{\dagger H})\}+\sigma_R^2\mathcal{E}\{\mathrm{Tr}(\mathbf{G}^\dagger\mathbf{H}^\dagger\mathbf{H}^{\dagger H}\mathbf{G}^{\dagger H})\}})^{0.5} = (\frac{E_R}{\mathrm{Tr}\{\mathcal{E}[(\mathbf{GG}^H)^{-1}]\mathbf{E}\}+\sigma_R^2\mathrm{Tr}\{\mathcal{E}[(\mathbf{H}^H\mathbf{HGG}^H)^{-1}]\}})^{0.5}$ where $\mathrm{Tr}(\cdot)$ denotes the trace operator. To obtain the explicit value of α^{ZF-M}, we have the following Lemma.

Lemma 1: For two-way relaying channel, we have $\mathrm{Tr}\{\mathcal{E}[(\mathbf{GG}^H)^{-1}\mathbf{E}]\} = \frac{E_1/\sigma_{g_2}^2+E_2/\sigma_{g_1}^2}{M-2}$. Let $\varphi = \mathrm{Tr}\{\mathcal{E}[(\mathbf{H}^H\mathbf{HGG}^H)^{-1}]\}$, if $\sigma_{h_1}^2\sigma_{g_2}^2 = \sigma_{h_2}^2\sigma_{g_1}^2 = \sigma_{hg}^2$, we have $\varphi = \frac{1}{\sigma_{hg}^2\prod_{l=1}^2(M-l)!(2-l)!(M-2)!}\sum_{k=1}^2\det(\mathbf{D}^k)$ with

$$\mathbf{D}_{i,j}^k = \begin{cases}(M-2+i-2)!(M-2+i+j-3)!, & j = k\\(M-2+i-1)!(M-2+i+j-2)!, & j \ne k\end{cases}$$

[1]Note that all the proofs in the paper are omitted for space limitation.

[2]When bidirectional channels are statistical symmetrical, i.e., $\sigma_{h_1}^2 = \sigma_{h_2}^2$ and $\sigma_{g_1}^2 = \sigma_{g_2}^2$, $a_1 = a_2 = 0.5$ is actually optimal.

While if $\sigma_{h_1}^2\sigma_{g_2}^2 \neq \sigma_{h_2}^2\sigma_{g_1}^2$, we have $\varphi = \frac{\sigma_1\sigma_2^{-1}-\sigma_2\sigma_1^{-1}}{(M-2)^2(\sigma_1-\sigma_2)}$, where $\sigma_1 = \max\{\sigma_{h_1}^2\sigma_{g_2}^2, \sigma_{h_2}^2\sigma_{g_1}^2\}$ and $\sigma_2 = \min\{\sigma_{h_1}^2\sigma_{g_2}^2, \sigma_{h_2}^2\sigma_{g_1}^2\}$.

Substituting (12) into (11), we have

$$\mathbf{y}^{ZF-M} = \alpha^{ZF-M}\mathbf{x} + \alpha^{ZF-M}\mathbf{H}^{\dagger}\mathbf{n}_R + \mathbf{n}.$$

Thus, the SNR at each destination is yielded as

$$\zeta_i = \frac{\alpha^{ZF-M^2}E_i}{\alpha^{ZF-M^2}\sigma_R^2[(\mathbf{H}^H\mathbf{H})^{-1}]_{i,i} + \sigma_{\bar{i}}^2},\ i = 1,2$$

where $[\cdot]_{i,i}$ denotes the i-th diagonal entry of a matrix. Then, the outage probability of ZF-precoding MABC is obtained as

$$P_{out}^{ZF-M} = \Pr[\zeta_1 < \tau_1^{MA} \ or\ \zeta_2 < \tau_2^{MA}]. \tag{13}$$

To compute (13), we need to know the relationship between $[(\mathbf{H}^H\mathbf{H})^{-1}]_{1,1}$ and $[(\mathbf{H}^H\mathbf{H})^{-1}]_{2,2}$. Next we assume that $[(\mathbf{H}^H\mathbf{H})^{-1}]_{1,1}$ and $[(\mathbf{H}^H\mathbf{H})^{-1}]_{2,2}$ are independent with each other to simplify the derivation. As confirmed by the simulation, this approximation almost leads no gap between analytical and Monte-Carlo results.

Theorem 3: When $\tau_1^{MA} < \frac{\alpha^{ZF-M^2}E_1}{\sigma_2^2}$ and $\tau_2^{MA} < \frac{\alpha^{ZF-M^2}E_2}{\sigma_1^2}$ (otherwise $P_{out}^{ZF-M} = 1$), the outage probability of MABC two-way relaying with ZF-precoding can be approximated as $P_{out}^{ZF-M} = P_{out_1}^{ZF-M} + P_{out_2}^{ZF-M} - P_{out_1}^{ZF-M}P_{out_2}^{ZF-M}$ with

$$P_{out_i}^{ZF-M} = \frac{\gamma\left(M-1, \frac{\tau_i^{MA}\alpha^{ZF-M^2}\sigma_R^2}{\sigma_{h_i}^2(E_i\alpha^{ZF-M^2}-\tau_i^{MA}\sigma_{\bar{i}}^2)}\right)}{\Gamma(M-1)},$$

where $\gamma(\cdot,\cdot)$ is the incomplete gamma function [5].

Corollary 2: The achievable DMT for each user in MABC AF two-way relaying with ZF-precoding can be expressed as $d(r) = (M-1)(1-2r),\ 0 \leq r \leq 1/2$.

Corollary 2 implies that, different from the non-precoding case, the ZF-precoding at relay node can benefit from increasing the relay antenna number, and the diversity order is linear with the number of relay antennas.

D. ZF-precoding Scheme for TDBC

For ZF-precoding TDBC protocol, we rewrite (5) as $\mathbf{y}_R^{ZF-T} = \mathbf{H}\mathbf{x} + \mathbf{n}_{R,1} + \mathbf{n}_{R,2}$. Since x_1 and x_2 are received separately in different time slots at RN, the relay precoding matrix $\mathbf{F}_1$ and $\mathbf{F}_2$ can be designed differently to improve the performance. Here for simplicity, we let $\mathbf{F}_1 = \mathbf{F}_2 = \mathbf{F}$ as presented in (12), and the power allocation parameter α^{ZF-T} is also determined as α^{ZF-M} only by replacing σ_R^2 with $2\sigma_R^2$. Therefore, we have $\mathbf{x}_R^{ZF-T} = \alpha^{ZF-T}\mathbf{F}\mathbf{H}\mathbf{x} + \alpha^{ZF-T}\mathbf{F}(\mathbf{n}_{R,1} + \mathbf{n}_{R,2})$. By combining the direct-path link presented in (5), we obtain the destination SNR after using the MRC technique as

$$\kappa_i = \frac{E_i}{\sigma_{\bar{i}}^2}|h_{i\bar{i}}|^2 + \frac{\alpha^{ZF-T^2}E_i}{2\alpha^{ZF-T^2}\sigma_R^2[(\mathbf{H}^H\mathbf{H})^{-1}]_{i,i} + \sigma_{\bar{i}}^2}.$$

Due to the independence of h_{12} and h_{21}, we can approximate the outage probability as

$$\begin{aligned} P_{out}^{ZF-T} =& \Pr[\kappa_1 < \tau_1^{TD}] + \Pr[\kappa_2 < \tau_2^{TD}] \\ &- \Pr[\kappa_1 < \tau_1^{TD}]\Pr[\kappa_2 < \tau_2^{TD}]. \end{aligned} \tag{14}$$

Theorem 4: The outage probability of TDBC two-way relaying with ZF-precoding can be approximated as $P_{out}^{ZF-T} = P_{out_1}^{ZF-T} + P_{out_2}^{ZF-T} - P_{out_1}^{ZF-T}P_{out_2}^{ZF-T}$ with $P_{out_i}^{ZF-T}$ being given in the following two cases:

Case 1: when $\tau_i^{TD} \geq \frac{\alpha^{ZF-T^2}E_i}{\sigma_{\bar{i}}^2}$: $P_{out_i}^{ZF-T} = P_{out_i}^{ZF-T'} + P_{out_i}^{ZF-T''}$, where $P_{out_i}^{ZF-T'} = 1 - \exp[-(\frac{\tau_i^{TD}\sigma_{\bar{i}}^2}{E_i\sigma_{h_{i\bar{i}}}^2} - \frac{\alpha^{ZF-T^2}}{\sigma_{h_{i\bar{i}}}^2})]$ and

$$P_{out_i}^{ZF-T''} = \exp(-\frac{C_1^i}{\sigma_{h_{i\bar{i}}}^2}) - \exp(-\frac{C_2^i}{\sigma_{h_{i\bar{i}}}^2}) - \frac{B_2^i\exp(-B_1^i)}{\sigma_h^2E_i\sigma_{h_{i\bar{i}}}^2}$$

$$\exp(\frac{B_3^i}{\sigma_h^2E_2\sigma_{h_{i\bar{i}}}^2})\sum_{l=0}^{\infty}\frac{(-B_4^i)^l}{l!}\sum_{m=0}^{M-2}\frac{1}{m!}\sum_{k=0}^{m}\binom{m}{k}{B_1^i}^{m-k}$$

$$\begin{cases} \exp(-C_3^i) & \text{if } s=0 \\ \Gamma(s+1, C_3^i) & \text{if } s>0 \\ -\mathrm{Ei}(-C_3^i) & \text{if } s=-1 \\ D^i(\sum_{j=1}^{-s-1}\frac{(-1)^{-j}(j-1)!}{(-s-1)!{C_3^i}^j} - \mathrm{Ei}(-C_3^i)) & \text{if } s<-2 \end{cases}$$

Here, $s = k-l-2$, $B_1^i = -\frac{\alpha^{ZF-T^2}\sigma_R^2}{\sigma_{h_i}^2\sigma_{\bar{i}}^2}$, $B_2^i = \frac{E_i\alpha^{ZF-T^4}\sigma_R^2}{\sigma_{\bar{i}}^2}$, $B_3^i = \sigma_{h_i}^2(E_i\alpha^{ZF-T^2} - \tau_i^{TD}\sigma_{\bar{i}}^2)$, $B_4^i = \frac{B_2^i}{\sigma_{h_i}^2E_i\sigma_{h_{i\bar{i}}}^2}$, $C_1^i = \frac{\tau_i^{TD}\sigma_{\bar{i}}^2}{E_2} - \alpha^{ZF-T^2}$, $C_2^i = \frac{\sigma_{\bar{i}}^2\tau_i^{TD}}{E_i}$, $C_3^i = \frac{B_2^i}{B_3^i + \sigma_{h_i}^2E_iC_2^i}$ and $D^i = \frac{\exp(-C_3^i)(-1)^{-s-1}}{(-s-1)!}$.

Case 2: when $\tau_i^{TD} < \frac{\alpha^{ZF-T^2}E_i}{\sigma_{\bar{i}}^2}$:

$$P_{out_i}^{ZF-T} = 1 - \exp(-\frac{C_2^i}{\sigma_{h_{i\bar{i}}}^2}) + \frac{B_2^i\exp(-B_1^i)\exp(\frac{B_3^i}{\sigma_h^2E_i\sigma_{h_{i\bar{i}}}^2})}{\sigma_h^2E_i\sigma_{h_{i\bar{i}}}^2}$$

$$\sum_{l=0}^{\infty}\frac{(-B_4^i)^l}{l!}\sum_{m=0}^{M-2}\frac{1}{m!}\sum_{k=0}^{m}\binom{m}{k}{B_1^i}^{m-k}$$

$$\begin{cases} \exp(-C_3^i)(\sum_{p=0}^{s}-p!\binom{s}{p}{C_3^i}^{s-p}) & \\ -\exp(-C_4^i)(\sum_{p=0}^{s}-p!\binom{s}{p}{C_4^i}^{s-p}) & \text{if } s>0 \\ \exp(-C_4^i) - \exp(-C_3^i) & \text{if } s=0 \\ -\exp(-C_3^i)\sum_{p=1}^{-s-1}\frac{E}{{C_3^i}^{(-s-p)}} + F\mathrm{Ei}(-C_3^i) & \\ +\exp(-C_4^i)\sum_{p=1}^{-s-1}\frac{E}{{C_4^i}^{(-s-p)}} - F\mathrm{Ei}(-C_4^i) & \text{if } s<0 \end{cases}$$

where $C_4^i = \frac{B_2^i}{B_3^i}$, $E = \frac{(-1)^{p-1}(-s-p-1)!}{(-s-1)!}$, $F = \frac{(-1)^{-s-1}}{(-s-1)!}$ and $\mathrm{Ei}(\cdot)$ is the exponential integral function [5].

Remark 3: Similar to Theorem 2, although $P_{out_i}^{ZF-T}$ contains the infinite number of l, it converges very fast and setting the maximal l as 20 is enough.

IV. Numerical results and Discussions

In the numerical evaluation, all the channels are set to be Rayleigh fading. We let $\rho_{iR} = \frac{E_i\sigma_{h_i}^2}{\sigma_R^2}$ and $\rho_{Ri} = \frac{E_R\sigma_{g_i}^2}{\sigma_{\bar{i}}^2}$ define

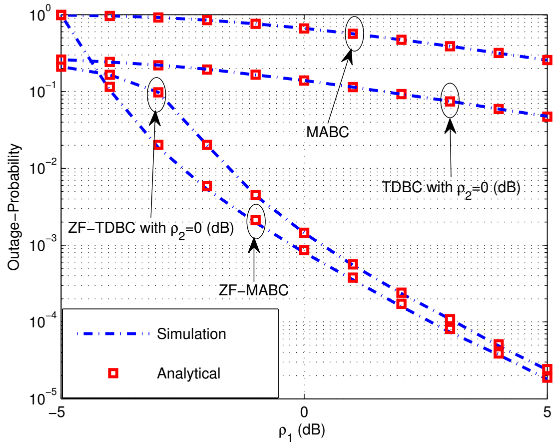

Fig. 2. The outage performance at $M = 4$ with $R = 0.08$.

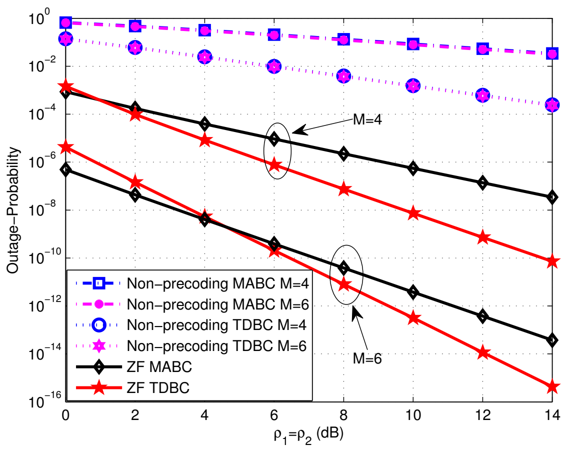

Fig. 3. The outage performance at $M = 4$ and 6 with $R = 0.08$.

the SNR from source S_i to RN R, and RN R to destination node S_i, respectively, and let $\rho_{i\bar{i}} = \frac{E_i \sigma^2_{h_{i\bar{i}}}}{\sigma^2_{\bar{i}}}$ define the SNR from S_i to $S_{\bar{i}}$ in the TDBC scheme. For simplicity, we also assume $\rho_1 = \rho_{1R} = \rho_{2R} = \rho_{R1} = \rho_{R2}$ and $\rho_2 = \rho_{12} = \rho_{21}$.

In Fig. 2, we show both the analytical and simulation results on the outage performance for all the schemes when $M = 4$ and $R = R_1 = R_2 = 0.08$. It is seen that the derived analytical results match the Monte-Carlo results very well, although we have used the approximation for the ZF precoding scheme. Meanwhile, it is observed that precoding at RN can significantly improve the system performance.

Fig. 3 illustrates the outage performance comparison when the number of relay antennas changes. We find that increasing the antenna in non-precoding MABC and TDBC schemes does not bring any notable performance difference. However, for the ZF-precoding two-way relaying scheme, the performance is enhanced significantly. Fig. 3 also shows that the non-precoding MABC scheme obtains the diversity order of one and it is irrelevant to relay antenna number, which confirms our conclusions derived in Corollary 1. Moreover, compared to the non-precoding MABC scheme, the diversity of 2 can be achieved for non-precoding TDBC scheme due to the existence of direct-path link. While for the ZF-precoding MABC

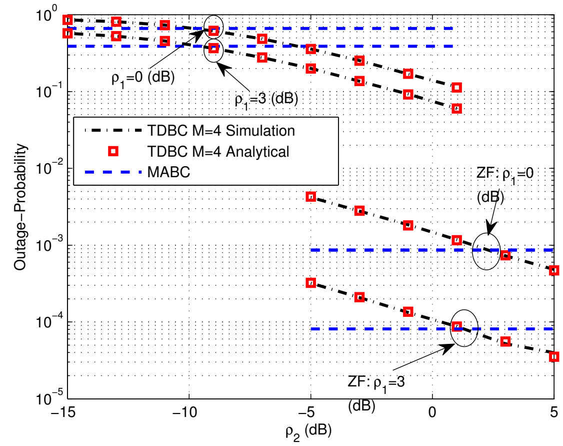

Fig. 4. The outage performance at different ρ_2 and fixed ρ_1 with $R = 0.08$.

scheme, we see that the diversity order is increased with the increasing of relay antenna number which is consistent with our conclusion derived in Corollary 2. Similarly, the obtained diversity order of ZF-precoding TDBC scheme is also increased with the antenna number. However, different from the non-precoding scheme, 2 more diversity order can be achieved compared to the ZF-precoding MABC scheme due to the existence of direct-path link.

In Fig. 4, the outage probability is shown as the function of ρ_2 for fixed ρ_1 at 0 and 3dB. We see that TDBC can always outperform MABC when ρ_2 exceeds some thresholds and usually such threshold is increased if ZF-precoding is applied at the relay node.

V. Conclusions

This paper presented the study on the outage performance of AF two-way relaying with multi-antenna RN. Both two time slots MABC protocol and three time slots TDBC protocol were considered. We derived the closed-form expressions of outage probability for the two protocols under both non-precoding and ZF-precoding schemes. Based on the derived outage probabilities, the DMT was also discussed for MABC protocol. Out analytical results implied that, with the ZF relay precoding, the system can benefit from the increased diversity order by increasing the number of relay antennas.

References

[1] R. H. Y. Louie, Y. Li, and B. Vucetic, "Practical physical layer network coding for two-way relay channels: performance analysis and comparison," *IEEE Trans. Wireless Commun.*, vol. 9, no. 2, pp. 764–777, 2010.

[2] M. Ju and I.-M. Kim, "Relay selection with ANC and TDBC protocols in bidirectional relay networks," *IEEE Trans. Commun.*, vol. 58, no. 12, pp. 3500–3511, 2010.

[3] I. Krikidis, "Relay selection for two-way relay channels with MABC DF: A diversity perspective," *IEEE Trans. Veh. Technol.*, vol. 59, no. 9, pp. 4620–4628, 2010.

[4] X. Lin, M. Tao, and Y. Xu, "Finite-SNR diversity-multiplexing tradeoff for two-way multi-antenna relay fading channels," in *Proc. IEEE Global Telecommunications Conf. GLOBECOM 2010*, 2010, pp. 1–5.

[5] I. Gradshteyn and I. Ryzhik, *Table of Integrals, Series, and Products, Seventh Edition*, 7th ed. San Diego, CA: Academic Press, 2007.

[6] R. Louie, Y. Li, and B. Vucetic, "Zero forcing in general two-hop relay networks," *IEEE Trans. Veh. Technol.*, vol. 59, no. 1, pp. 191–202, 2010.