Pairwise Check Decoding for LDPC Coded Two-Way Relay Fading Channels

Jianquan Liu[†], Meixia Tao[†], Youyun Xu[†*], and Xiaodong Wang[‡]
[†]Dept. of Electronic Engineering, Shanghai Jiao Tong University, Shanghai, 200240, China
[*]PLA University of Science and Technology, Nanjing 210007, China
[‡]Dept. of Electrical Engineering, Columbia University, New York, NY 10027, USA
Emails: {jianquanliu, mxtao, xuyouyun}@sjtu.edu.cn, wangx@ee.columbia.edu

***Abstract*— We present a novel partial decoding method at the relay, called *pairwise check decoding* (PCD), for two-way relay fading channels. The proposed PCD method forms a so-called check-relationship table for the superimposed Low-Density Parity-Check (LDPC)-coded packet pair during the multiple access phase. Meanwhile, it incorporates adaptive network coding by using closest-neighbor clustering mapping (CNCM) to compensate the phase deviation of the fading channels. The proposed PCD method is a practical and efficient realization of the promising denoise-and-forward relay strategy with advanced channel coding and non-linear network coding. Simulation results show that under the same LDPC-coded two-way relay system, our proposed PCD considerably outperforms the case where the relay performs only adaptive network coding without channel decoding. It also performs better than the case where the relay adopts the belief propagation decoding along with conventional XOR-based network coding under certain regions.**

I. Introduction

Two-way relaying with physical layer network coding (PLNC) is an efficient technique to increase the spectral efficiency of wireless cooperative networks [1]–[4]. A few works have been reported on the achievable rate region of two-way relay channels [5], [6]. Recently, Schnurr *et al* in [7] and Gunduz *et al* in [8] have attempted to study the new achievable rate region by relaxing the unnecessary burden of full decoding at the intermediate node. Although the capacity region of two-way relaying with partial decoding is still an open topic, it is now well recognized that partial decoding has the potential to achieve a larger capacity as opposed to full decoding. In particular, the denoise-and-forward (DNF) protocol, a type of partial decoding, has demonstrated performance gain over conventional amplify-and-forward (AF) and decode-and-forward (DF) protocols.

It remains to be a fundamental and challenging task to realize the benefits of partial decoding in two-way relay channels through practical coding and modulation techniques with low complexity. Nowadays, two realizable methods can be used to apply channel coding in this case. One is that the relay node decodes the network coded packet (binary addition or modulo addition) from the received superimposed packet pairs, where the network-coded packet is also a valid codeword. In this method, however, both two source nodes are required to use the same linear channel codes (e.g., LDPC code and lattice code, which are linear in binary addition and modulo addition, respectively) [9], [10]. The other is that the relay node separate sections of packet pairs (the Euclidean distance between which is lager comparatively) in channel decoding period while operate network coding through hard decision. In this method, Zhang *et al* proposed a channel-decoding network-coding (CNC) process based on RA (repeat accumulate) codes in [11]. This scheme transforms the superimposed channel-coded packets received during the multiple-access (MA) phase to a network-coded combination (i.e., XOR) for transmission in the broadcast (BC) phase with minimum decoding loss. Note that both two methods are designed specifically for binary or modulo-addition based linear network coding and for Gaussian channels. For fading channels, on the other hand, Koike-Akino *et al* in [12] found that the conventional network coding does not always work well due to the undesired phase offset between the two channels in the MA phase. Based on the DNF protocol, they proposed a closest-neighbor clustering based adaptive network coding for high-level modulation. To further ensure reliable communication, the authors extended the similar concept to a convolutional coded two-way relay system in [13]. Therein, the main idea is still trying to optimize the clustering and mapping based on shortest error-path distance. Therefore, it is highly complex and difficult to generalize to more advanced channel coding schemes, such as Turbo codes and LDPC codes.

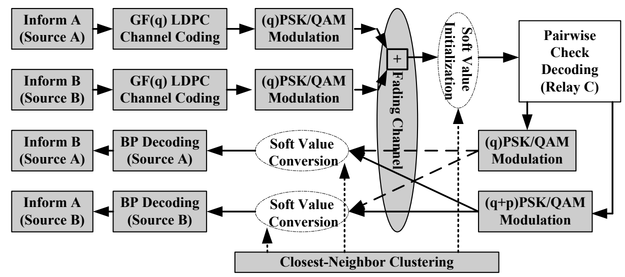

Fig. 1: Channel coding model for two-way relay fading channels

In this paper, we propose a new channel decoding method at the relay, called *pairwise check decoding* (PCD), for two-

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way relay fading channels with partial decoding of LDPC codes. The novelty is to form a check relationship table (check-relation-tab) for the superimposed LDPC coded packet pair in the MA phase in conjunction with adaptive clustering in the BC phase. On the one hand, it generalizes the channel-decoding-network coding process in [9]–[11] to non-binary codes and over fading channels. On the other hand, the proposed method is a more efficient realization of DNF protocol with advanced channel coding as compared with [13]. Simulation results demonstrate that the new PCD algorithm considerably outperforms the schemes proposed previously under certain conditions.

II. System Model

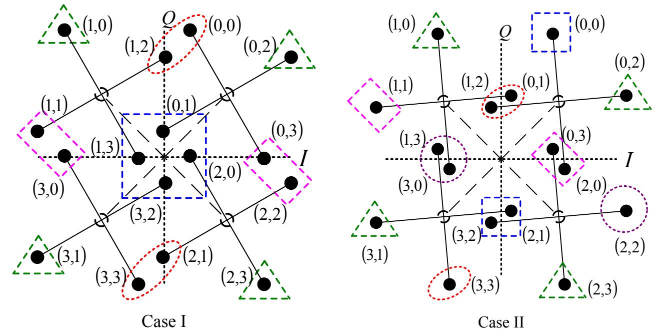
We consider a two-way relay channel where two source nodes, denoted as A and B, wish to exchange information with the help of a relay node, denoted as C. We assume that all the nodes operate in the half-duplex mode. The channel on each communication link is assumed to be corrupted with Rayleigh fading and additive white Gaussian noise.

The proposed channel coding model of two-way relaying is shown in Fig. 1, where the communication takes place in two phases. First, the information packet from each source, denoted as S_i, for $i \in \{A, B\}$, is encoded individually by a GF(q) LDPC encoder, where $q \in \{2^2, 2^3, \ldots\}$. The encoded packet, C_i, is modulated by (q)PSK/QAM, generating X_i, and then transmitted simultaneously to the relay node. The n-th symbol of each packet is denoted as $S_i(n) \in \mathcal{Z}_q, \mathcal{Z}_q = \{0, 1, \ldots, q-1\}$, $C_i(n) \in \mathcal{Z}_q$, and $X_i(n) \in \mathcal{Q}_q$, respectively. Here, $\mathcal{Q}_q$ denotes the set of constellation points. The superimposed packet received by the relay over the fading channel, denoted as Y_C is given by

$$Y_C = H_{AC}X_A + H_{BC}X_B + Z_C, \qquad (1)$$

where $H_{gl} = h_{gl}e^{j\phi_{gl}}$ denotes the channel gain of link from node g to node l, and Z_l denotes complex additive white Gaussian noise with variance σ_l^2 of node l. Therein, $\{g, l\} \in \{A, B, C\}$. We assume perfect symbol synchronization and accurate channel gain are estimated at the relay. After receiving the superimposed packet over the fading channel, the relay initializes the soft value for PCD decoder (to be presented in Section III) by taking into account the closest-neighbor clustering mapping (CNCM) [12] (to be briefed in Section III). The output of the PCD decoder, i.e., network-coded codeword packet, $C_C = M(C_A, C_B)$, is then modulated to (q)PSK/QAM or $(q + p)$PSK/QAM accordingly to obtain X_C, where $p \in \mathcal{Z}_q$ represents the possible expanding of modulation size ($p = 1$ for the 5QAM when QPSK is used at two sources). M denotes a kind of CNCM scheme. Here, no joint decoding of C_A and C_B is needed as an intermediate step. Note there is no extra channel encoding at the relay. Then, the relay broadcasts the modulated symbols and the index of selected mapper to two source nodes. The received signals at the nodes A and B are respectively written as

$$Y_A = H_{CA}X_C + Z_A; Y_B = H_{CB}X_C + Z_B. \qquad (2)$$



(a) Cluster to four constellations (b) Cluster to five constellations

Fig. 2: Received signal constellation with closest-neighbor clustering mapping when $q = 2^2$.

For simplicity, we assume the channel gains are reciprocal and unchanged during a whole packet transmission. Each source node computes the soft value of the desired information $C_A(C_B)$ from the received symbols $Y_B(Y_A)$ by using the CNCM index sent from the relay with the help of its self-information $C_B(C_A)$. Lastly, the traditional belief propagation (BP) decoding algorithm is operated and the output of which is the desired information flow $S_A(S_B)$.

III. Proposed pairwise check decoding

As confirmed by Zhang, *et al.* [11] that full decoding before PLNC mapping induce to performance loss, whereas clustering neighboring symbol pairs to one symbol in soft decoding period is expected to overcome such performance loss. Moreover, Koike-Akino, *et al.* [12] found that, a necessary condition of successful decoding at the destinations in fading channels is to select PLNC mapping satisfying the exclusive law:

$$M(C_A, C_B) \neq M(C'_A, C_B), \text{for all } \{C_A \neq C'_A, C_B\} \in \mathcal{Z}_q,$$
$$M(C_A, C_B) \neq M(C_A, C'_B), \text{for all } \{C_A, C_B \neq C'_B\} \in \mathcal{Z}_q.$$

It then remains to find a channel coding scheme that can utilize this mapping. The existing two solutions such as direct XOR [9], [10] and arithmetic-sum [11] could not arbitrarily map the neighboring symbol pairs to one symbol only subject to the exclusive law due to the linearity constraint. For example, we consider the two representative received signal constellations at the relay node shown in Fig. 2, where $H_{BC} \approx H_{AC}e^{j\pi/2}$ in case I and $H_{BC} \approx H_{AC}e^{j\pi/4}$ in Case II. Based on the CNCM principle [12], the symbol pairs $(0,1), (1,3), (2,0), (3,2)$ in Case I should be mapped to one symbol, while $(0,1), (1,2)$ in Case II should be mapped together. However, $0 \oplus 1 \neq 1 \oplus 3$, $0 \oplus 1 \neq 1 \oplus 2$ (using direct XOR) and $0 + 1 \neq 1 + 3$, $0 + 1 \neq 1 + 2$ (using arithmetic-sum) while these symbol pairs should be mapped together. In the rest of this section, we shall propose the pairwise check decoding algorithm, which can solve aforementioned puzzle.

A. Encoder at source nodes A and B

We assume nodes A and B use the traditional encoder of LDPC codes. Unlike the existing work, we do not impose the constraint that H^A and H^B must be the same. Instead, we only require that they have the same location of non-zero elements.

It is further assumed that the encoder is operated in GF(q) in this paper. In general, we should obtain the check-relation-tab at relay node based on q-ary check matrix while using q-ary modulation at two source nodes, where $q \in \mathcal{Z}_q$. Note that q-ary coding can improve the performance instead of inducing loss compared with binary coding. More details can be found in [14] and references therein.

B. Decoder at relay node C

As we know, the traditional BP decoding algorithm can be applied effectively once the check matrix of a LDPC code is given. However, in two-way relaying, given alone the individual check matrices of the LDPC codes applied at both source nodes, the packet pairs cannot be decoded directly. Instead, the constraint relationship regarding symbol pairs in the two packets should be derived. In the following, we introduce a so-called check-relation-tab to describe such symbol pair constraints.

1) Check-relation-tab: At first, we set forth some denotations. Let h^A_{mn} and h^B_{mn} denote the elements at the m-th row and n-th column of H^A and H^B, respectively. Two LDPC codes have the same locations of non-zero's. $M_m = \{n : h_{mn} \neq 0\}$ denote the set of column locations of the non-zero's in the m-th row; $M_{m\backslash n} = \{n' : h_{mn'} \neq 0\}\backslash\{n\}$ denotes the set of column locations of the non-zero's in the m-th row, excluding location n; $N_n = \{m : h_{mn} \neq 0\}$ denotes the set of row locations of the non-zero's in the n-th column; $N_{n\backslash m} = \{m' : h_{m'n} \neq 0\}\backslash\{m\}$ denotes the set of row locations of the non-zero's in the n-th column, excluding location m. Note that in this subsection the arithmetic operations are all in GF(q) field unless specified otherwise. Considering the encoding characteristics of H^A and H^B, we have the constraint equations for each $m(\forall m)$ as follows:

$$\sum_{n\in M_m} C_A(n) \times h^A_{mn} = 0, \sum_{n\in M_m} C_B(n) \times h^B_{mn} = 0. \quad (3)$$

The m-th check-relation-tab (also referred as the joint constraint of the symbol pairs $\{C_A(n), C_B(n)\}$ at locations $n \in M_m$) is only constrained by the row weight and non-zero elements of two correlative m-th rows. It consists of two parts, one for virtual encoder, and the other for PCD decoder. In virtual encoder, we randomly assume that the symbol pair $\{C_A(n), C_B(n) \in \mathcal{Z}_q\}$ at only one location n is not known. Thus, we obtain the possible values for $\{C_A(n), C_B(n)\}$ at location n base on (3) through ergoding all values for $\{C_A(n), C_B(n)\}$ at locations $n \in M_{m\backslash n}$. Then, the symbol pairs $\{C_A(n), C_B(n)\}$ at all locations $n \in M_m$ are mapped to $C_C(n) \in \mathcal{Z}_{q+p}$. Since the number of symbol pairs $\{C_A(n), C_B(n)\}$ mapped to each element $C_C(n)$ may not be the same, the probability of occurrence for each element $C_C(n)$ should be computed separatively based on the number of symbol pairs $\{C_A(n), C_B(n)\}$ included. On the contrary, we assume that the element $C_C(n)$ at location n is known in PCD decoder. Then, we should compute the probability of occurrence for the corresponding possible values $C_C(n)$ at locations $n \in M_{m\backslash n}$ by classifying the aforementioned probability of each element which is generated by the virtual encoder. In theory, we should generate each check-relation-tab when the row weight or the elements of this row are not the same as each other. However, only one check-relation-tab need to be obtained for one or more selected CNCM schemes when regular LDPC codes are applied and the non-zeros in every row follows some special pattens like $\{i, \ldots, i\}, i \in \mathcal{Z}_q$.

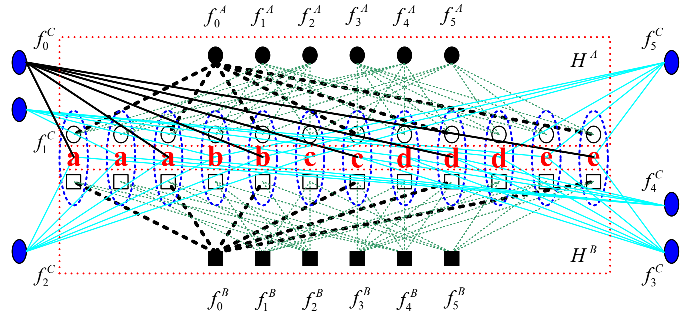


Fig. 3: Tanner graph of the check functions versus symbol pairs for PCD algorithm at relay node

As an example, we present the Tanner Graph of a virtual LDPC code at the relay in Fig. 3, where the code length is 12 and the row weight and column weight are 6 and 3 respectively. The solid circles and squares denote the check functions at each source node, while non-solid circles and squares denote the transmitted symbols of a code. In like manner, the solid and non-solid ellipses denote the check functions and the corresponding received symbol pairs. f^s_r denote the r-th check functions of the LDPC code at node s, where $r \in [0, 5], s \in \{A, B, C\}$. If we know the check functions of symbol pairs, the traditional BP decoding algorithm will be suitable for use after modification to a certain extent. Next, we will derive the check function of f^C_0 from f^A_0 and f^B_0 using the above example. For simplicity, if let $\{i, \ldots, i \in \mathcal{Z}_4\}$ and $\{j, \ldots, j \in \mathcal{Z}_4\}$ denote the non-zero elements of the 0-th row of H^A and H^B, we have $f^s_r = f^s_{r'}, r \neq r'$. Thus, we have the constraint equations as follows:

$$\sum_{n\in\{0,2,4,6,8,11\}} C_A(n) \times i = 0, \quad \sum_{n\in\{0,2,4,6,8,11\}} C_B(n) \times j = 0. \quad (4)$$

Take the $\mathcal{C}_2$ mapper of Table I in [12] for example, which is displayed as Case II in Fig. 2. Each symbol pair $(C_A(n), C_B(n))$ is mapped to one of the five elements based on fading condition. Here, to avoid confusion, we let $\{a, b, c, d, e\}$ indicate $\{0, 1, 2, 3, 4\}$ of Table I in [12]. All the possible mappings are listed below: a:(0,2),(1,0),(2,3),(3,1); b:(0,0),(2,1),(3,2); c:(0,1),(1,2),(3,3); d:(0,3),(1,1),(2,0); e:(1,3),(2,2),(3,0); where $n \in \{0, 2, 4, 6, 8, 11\}$.

As an example, the symbol pairs, $(C_A(0), C_B(0))$ and $(C_A(2), C_B(2))$ are mapped to the same cluster-mapping symbol $\{a\}$ as shown in Fig. 3. It is important to note that each element constrained by relay check functions f^C_n corresponds to one symbol pair in $\{(C_A(n), C_B(n)), n \in \{0, 2, 4, 6, 8, 11\}\}$, the range of which is in $\{a, b, c, d, e\}$. Now, the so-called check-relation-tab could be generated, as shown in Tables I and II.

TABLE I: Check-relation-tab of f_0^C for virtual encoder

$(0,0)$	$(2,2)$	$(4,4)$	$(6,6)$	$(8,8)$	$(11,11)$
a	a	a	a	a	a
a	a	a	a	b	5b,5c,d,e
a	a	a	a	c	5b,5c,d,e
⋮	⋮	⋮	⋮	⋮	⋮
e	e	e	e	d	20b,61d
e	e	e	e	e	20c,61e

TABLE II: Check-relation-tab of f_0^C for PCD decoder

$(11,11)$	F_W	$(0,0)$	$(2,2)$	$(4,4)$	$(6,6)$	$(8,8)$
a	1	a	a	a	a	a
a	0.5556	a	a	a	a	b
a	0.5556	a	a	a	a	c
⋮	⋮	⋮	⋮	⋮	⋮	⋮
e	0.2469	e	e	e	e	c
e	0.7531	e	e	e	e	e

2) Pairwise check decoding algorithm: Since we have obtained the check-relation-tab, the PCD algorithm can be operated smoothly. Note that the locations of non-zeros in the q-ary LDPC codes used at two source nodes are still valid here. We only change the check functions of symbol pairs from $\{f_r^A, f_r^B\}$ to f_r^C by the derived check-relation-tab. Let $u_n^k = Pr(C_C(n) = k|Y(n))$; $v_m^k = Pr(f_m^C satisfied|C_C(n) = k, Y(n))$; $t_{nm} = (u_n^k, k \in \mathcal{Z}_{q+p})$ denotes the messages to be passed from symbol node $C_C(n)$ to check node f_m^C; $w_{mn} = (v_m^k, k \in \mathcal{Z}_{q+p})$ denotes the messages to be passed from check node f_m^C to symbol node $C_C(n)$. Suppose that $\{m_k\}, k \in \mathcal{Z}_{q+p}$, denote the index-set of rows of check-relation-tab for PCD decoding in which the element at (n,n) is generated as $\{k\}$ and m_{tab} means the index of row.

Compute for each $[m,n]$ that satisfies $h_{mn} \neq 0$.

1. Initialization

Compute the initial value of each u_n^k based on

$$\begin{aligned} &u_n^{'k} = \sum_{(C_A(n),C_B(n)):C_C(n)=k} Pr((C_A(n),C_B(n))|Y_C(n)); \\ &t_{nm} = (u_n^k, k \in \mathcal{Z}_{q+p}) = ((1/\beta_1)u_n^{'k}, k \in \mathcal{Z}_{q+p}); \\ &\beta_1 = \sum_{k\in\mathcal{Z}_{q+p}} u_n^{'k}. \end{aligned} \tag{5}$$

where $Pr((C_A(n), C_B(n))|Y_C(n))$ is the probability of $(C_A(n), C_B(n))$ given $Y_C(n)$ is received.

2. First half round iteration

$$\begin{aligned} &v_m^k = \sum_{m_{tab}\in\{m_k\}} F_W(m_{tab}) \prod_{n'\in M_{m\backslash n}} t_{n'm}(m_{tab}); \\ &w_{mn} = (v_m^k, k \in \mathcal{Z}_{q+p}). \end{aligned} \tag{6}$$

where $t_{n'm}(m_{tab})$ denotes that k of $(u_n^k, k \in \mathcal{Z}_{q+p})$ is the designated elements at the m_{tab}-th row in the second table of the check-relation-tab.

3. Second half round iteration

$$\begin{aligned} &u_n^{'k} = u_n^k \prod_{m'\in N_{n\backslash m}} w_{m'n}; \\ &t_{nm} = (u_n^k, k \in \mathcal{Z}_{q+p}) = ((1/\beta_2)u_n^{'k}, k \in \mathcal{Z}_{q+p}); \\ &\beta_2 = \sum_{k\in\mathcal{Z}_{q+p}} u_n^{'k}. \end{aligned} \tag{7}$$

4. Soft decision

$$\begin{aligned} &U_n^{'k} = u_n^k \prod_{m\in N_n} w_{mn}; \\ &T_{nm} = (U_n^k, k \in \mathcal{Z}_{q+p}) = ((1/\beta_3)U_n^{'k}, k \in \mathcal{Z}_{q+p}); \\ &\beta_3 = \sum_{k\in\mathcal{Z}_{q+p}} U_n^{'k}. \end{aligned} \tag{8}$$

5. Hard decision

$$\hat{C}_C(n) = \arg \max_{k\in\mathcal{Z}_{q+p}} U_n^k. \tag{9}$$

If C_C satisfies the check-relation-tab for virtual encoder or the number of iterations exceeds a certain value, then the algorithm stops, otherwise we go to Step 2. So far, the whole PCD algorithm for arbitrary CNCM scheme is presented.

C. Decoder at source nodes A and B

The decoder at each source is same to traditional point to point (PTP) channel decoding except that the soft channel information of the symbol pair needs to be converted to that of an individual symbol. Through substituting the probability of its own symbols based on one CNCM scheme, we obtain the soft channel information for desired symbols easily.

IV. Simulation results

Suppose that the channel gains on all links follow Rayleigh distribution and are independent. We assume $E[|H_{AC}|^2] = E[|H_{BC}|^2]$ and $\sigma_A = \sigma_B = \sigma_C$, where notation $E[.]$ denotes expectation function. Define an average SNR per bit of the system as $E[|H_{AC}|^2 + |H_{BC}|^2]/(4R\sigma_C^2)$, where R is the code rate. The selection for CNCM scheme is based on instantaneous realization of the channel gain pair $\{H_{AC}, H_{BC}\}$ using Fig. 10 of [12]. Moreover, replacing $\{1,\ldots,1\}$ by $\{i,\ldots,i\}, i \in \mathcal{Z}_4$, at every row, we generate two 4-ary LDPC codes from the binary regular LDPC code ”504.504.3.504”, produced by MacKay in [15]. Code length, code rate, row weight and column weight are 1008, 0.5, 6 and 3, respectively. We adopt the PCD decoding at relay and conventional decoding at each destination.

For comparison, three benchmark systems are considered. One, marked as “uncoded”, is the uncoded case where QPSK is applied at the sources and the relay demodulates using CNCM. The second, marked as “coded-PTP”, is the coded case where the same 4-ary LDPC is applied at the sources but the relay does not perform channel decoding and only demodulates using CNCM. The last one, marked as “BP-XOR” differs from the proposed scheme in that the relay performs BP decoding together with traditional XOR-based mapping. Both frame error rate (FER) and bit error rate (BER) are observed. Note that each error probability is computed after observing at least 100 error frames and 20000 error bits. The maximum iteration is 25.

From Fig. 4, it is first observed that the coded-PTP scheme has about 2.5 dB performance loss compared with the uncoded scheme in terms of FER at the relay but achieves about the same performance in terms of end-to-end FER observed at the source. Such performance loss is due to the bit energy reduction by channel coding and is gained back because of

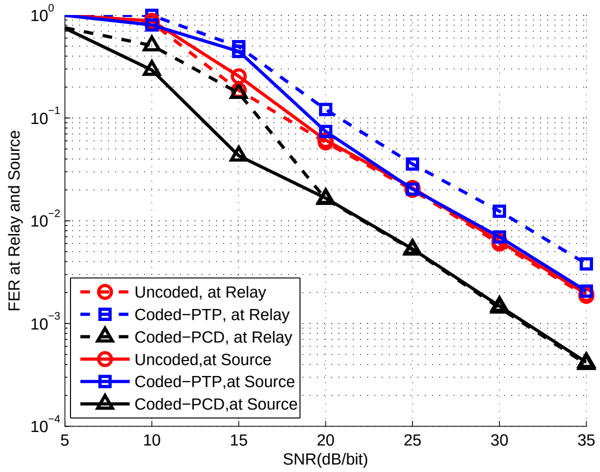


Fig. 4: Performance comparisons among the PCD algorithm, uncoded and PTP coded systems based on the CNCM scheme.

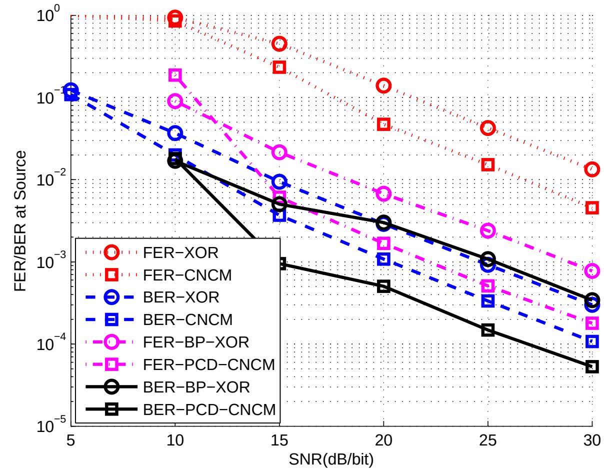


Fig. 5: Performance comparisons between the PCD decoded CNCM (PCD-CNCM) and the BP decoded XOR (BP-XOR).

PTP decoding at the destination receiver. One also observes that the proposed PCD scheme obtains about 6dB gain when FER= 2×10^{-3} over both uncoded and coded-PTP schemes, no matter observed at relay or source. This confirms that decoding at relay is critical.

Fig. 5 compares the end-to-end error rates observed at each source between the proposed scheme (PCD-CNCM) and the third benchmark scheme (BP-XOR). Note that there is almost no performance difference between PCD-CNCM and BP-XOR when averaged over all possible channel realizations. This is because under identically distributed Rayleigh fading, the CNCM based PLNC provides little gain over XOR based linear network coding in uncoded systems [12] and, accordingly, the gain is expected to be further reduced when channel coding is employed. Thus, in Fig. 5, we only plot the results obtained by averaging over partial channel gain realizations where CNCM is preferred in order to highlight the effectiveness of the proposed PCD algorithm. In specific, we impose the constraint that at least one of the real and imaginary part of two relevant channel coefficients (H_{AC}, H_{BC}) during the MA phase should be no less than σ_C. Results in Fig. 5 show that the gain achieved by the PCD-CNCM over BP-XOR is 8dB when BER=3×10^{-4}. It is also shown that our proposed PCD algorithm still achieves about 3dB gain over the uncoded CNCM system when BER=1×10^{-4}.

V. Conclusion

We presented a so-called pairwise check decoding strategy at the relay for two-way relay fading channels. The main idea is to construct a check relationship table, which consists of two parts, one for virtual encoder and one for decoding, for the superimposed LDPC coded packet pair. The scheme is designed specifically for non-binary LDPC codes together with CNCM based physical layer network coding. In addition, the two LDPC codes employed by the sources are only required to have the same locations of non-zero elements in check matrices, rather than being completely identical. Simulation results confirmed the excellent error performance of the proposed scheme over existing schemes.

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