Impact of Location Popularity on Throughput and Delay in Mobile Ad Hoc Networks

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Abstract—With the advent of smart portable devices and location-based applications, user’s mobility pattern is found to be highly dependent on varying locations. In this paper, we analyze asymptotic throughput-delay performance of mobile ad hoc networks (MANETs) under a location popularity based scenario, where users are more likely to visit popular locations. This work provides a complementary perspective compared with previous studies on fundamental scaling laws for MANETs, mostly assuming that nodes move uniformly in the network. Specifically, we consider a cell-partitioned network model with cells of known popularity, which follows a Zipf’s law distribution with popularity exponent $\alpha$. We first conduct the analysis under traditional store-carry-forward paradigm, and find that location heterogeneity affects the network performance negatively, which is due to the waste of potential transmission opportunities in popular cells. Motivated by this observation, we further propose a novel store-carry-accelerate-forward paradigm to enhance the network communication, exploiting these potential transmissions. Theoretical results demonstrate that our proposed scheme outperforms all delay-capacity results obtained in conventional scheme for any $\alpha$. In particular, when $\alpha = 1$, it can achieve a constant capacity with an average delay of $\Theta(\sqrt{\pi})$ (except for a polylogarithmic factor), while the delay is $\Theta(n)$ in conventional scheme. And by letting $\alpha = 0$, our results can cover Neely’s scaling laws. Moreover, we show that the delay-capacity tradeoff ratio satisfies $\geq \Theta(\sqrt{\pi})$, revealing that exploiting location popularity can effectively improve the performance in MANETs.

Index Terms—Mobile Ad Hoc Networks (MANETs), location popularity, capacity and delay, buffer analysis

1 INTRODUCTION

As inter-vehicle communications and smart portable devices are becoming increasingly popular, the store-carry-forward paradigm is widely adopted to mimic the communication process in the context of delay-tolerant networking (DTN). DTN has vast applications which include but not limited to vehicular networks [1], pocket switched networks [2], sensor networks [3], and rural kiosks providing Internet access in developing countries [4]. In spite of the disruptive nature and intermittent connectivity of MANETs, it is shown that a constant per-node throughput can still be achieved by exploiting mobility according to Grossglauser and Tse’s work [5]. This is a dramatic improvement compared with $\Theta(n^{1/4})$ in static networks [6], while this nice scalability property comes at the cost of an excessive delay of $\Theta(n)$.

Since then, there is a tremendous interest in studying how to exploit the mobility properties to enhance network communications [7]–[12]. Note that network performance is determined by information delivery paradigm which is highly correlated to mobility patterns. Therefore, various mobility models are extensively studied, ranging from the simple independently and identically distributed (i.i.d.) model [13]–[15], to more complex random mobility models, such as the Brownian motion model [16], random way-point [17], [18], and variants of random walk [19]–[21]. Neely and Modiano [13] study delay-capacity tradeoffs in a cell-partitioned structure under i.i.d. mobility model. They develop a scheduling scheme utilizing redundant packet transmissions to reduce delay at the expense of capacity. A necessary tradeoff between delay and capacity is established, i.e., $\text{delay/capacity} \geq O(n)$. In [14], Toumpis and Goldsmith propose a better scheme that can achieve a per-node capacity of $\Theta(n(d-1)/2/\log^{5/2} n)$ under fading channels when the delay is bounded by $O(n^d)$. Lin and Shroff [15] later study the fundamental capacity-delay tradeoff and identify the limiting factors of the existing scheduling schemes in MANETs. Recently, Ying et al. [22] propose joint coding-scheduling algorithms to improve capacity-delay tradeoffs.

Previous works mentioned above all consider that nodes uniformly move in the entire network area, such an assumption may not hold in many practical settings. Therefore, while these studies have made great contributions in understanding the role of mobility patterns in wireless communications, there are still some additional dimensions needed to be exploited, e.g., location heterogeneity.

As with most related work, Garetto et al. [23] investigate the impact of restricted mobility on network performance. In [23], each node moves around a home point where its occurrence probability decays as the distance from the home point becomes large. For a specific node, different locations represent different occurrence probabilities and this information can be used to construct scheduling scheme. In this regard, this model captures some kind of location heterogeneity. However, from the perspective of the whole network, the node distribution is still uniform over all locations. And as the location-
based applications such as Four Square become more and more popular, there are many traces recording users’ visits. These traces are retrieved by many researches to study users’ mobility patterns. As observed in a number of measurement studies [24]–[26], it is shown that some locations are visited more frequently while some are less. This is also in accordance with intuitiveness, for example, there maybe many more students in classrooms, auditoriums, or libraries than that in a street. Such a location popularity property indicates a quite different mobility pattern and leads to a non-uniform node distribution in the network. This significant difference from prior works suggests that conventional schemes of MANETs may not be directly applied to the performance analysis in MANETs with location popularity. Therefore, one fundamental question is: what is the impact of location popularity on the performance of MANETs? And to the best of our knowledge, there is no previous work considering this issue in theoretical performance analysis of MANETs or delay-tolerant networks.

In this paper, we analyze asymptotic throughput-delay performance of MANETs under a location popularity based scenario. We adopt a cell-partitioned structure, where each cell is assigned with a known popularity. Mobile users determine their own cell locations according to location popularity, which follows a Zipf’s law [24]–[26]. Under this assumption, the number of nodes may vary dramatically in different cells. We first investigate the delay and capacity under the conventional 2-hop store-carry-forward paradigm, and find that location popularity brings a worse performance compared with that in uniform networks. Intuitively, this is due to a large amount of node resource wasted in popular cells and rather low node density of other cells (called ordinary cells). Motivated by this observation, we try to improve the network performance by proposing a 3-hop store-carry-accelerate-forward paradigm which utilizes potential transmissions in popular cells. It is worth mentioning that a small number of access points (APs) are deployed in this scheme, on the purpose of handling the heavy traffic in popular cells to ensure network stability. This is also in accordance with practical scenarios. For example, in the case of vehicle traffic, the deployed infrastructure can assist to obtain a higher communication rate in concentration spots such as gas stations or parking lots. In this manner, the proposed scheme can make use of the high node density feature of popular cells to help spread packets more quickly. The purpose of this work is not to establish optimal information theoretic results, but to show that there is an additional dimension to be exploited, i.e., location popularity, which has been so far neglected by related studies. In summary, our main contributions are as follows:

- For the first time, we investigate the performance scaling laws in MANETs with location popularity, which is observed in real traces. Our studies may provide some novel insights for the design of location-based mobile networks.
- Under conventional 2-hop scheme, we analyze the capacity and delay. It is found that the per-node throughput will decrease when the network becomes more heterogenous. Meanwhile, the delay remains as $\Theta(n)$ for all $\alpha$. When $\alpha = 0$, our results can recover Neely’s scaling laws [13] which are established under i.i.d. mobility model.
- Observing that popular cells are not fully utilized, we propose a novel store-carry-accelerate-forward paradigm to enhance packet delivery. Our scheme can achieve the same capacity with conventional scheme while achieving a smaller delay of $\Theta(\sqrt{n})$, taking advantage of broadcast in popular cells.
- Compared the two aforementioned schemes, we find that if popular cells have a stronger transmission ability, the delay can be effectively reduced. This sheds light in reality that if we deploy a small number of access points within chosen cells, the packets can be delivered much more quickly throughout the network. Note that in the 3-hop scheme at most $O(1/\sqrt{n})$ fraction of cells are required to be deployed with an AP, indicating a small deployment cost. Moreover, we analyze the buffer needed for each node. It is shown that the node’s buffer of 3-hop scheme is much smaller than that of 2-hop scheme, which is due to the high information delivery efficiency of 3-hop scheme.

The rest of this paper is organized as follows. In section II we present the network model. In section III, we analyze capacity and delay under conventional 2-hop relay algorithm and our 3-hop relay algorithm, respectively. Section IV investigates the impacts of location popularity on network performance. We also discuss how location popularity affects the delay-capacity tradeoff. Finally, we conclude in Section VI.

## 2 NETWORK MODEL AND DEFINITIONS

### 2.1 Network Model

Cell-Partitioned Network Model: Consider a cell partitioned network model as shown in Figure 1. There are $n$ mobile nodes in a unit square. Divide the network into $n$ nonoverlapping cells, each of equal area. Each cell is assigned with a known popularity. Nodes visit a cell independently according to its popularity, leading to an inhomogeneous stationary node distribution. We suppose nodes can communicate with each other only when they are in the same cell, and one transmission is restricted per cell per timeslot. To avoid interference, different frequencies are employed among the neighboring cells. It is well known that only four frequencies are enough for the network.

We consider a Zipf’s law for the location popularity distribution. This distribution is observed by [24]–[26], which found that the mobility patterns of users are
rather heterogeneous over the locations, and the probability of a user to visit the i-th most popular location follows \( f_i \sim i^{-\alpha} \). This law implies that, sorting the cells in decreasing order or popularity, a node moves to cell \( i \) with probability
\[
p_i = \frac{H(n)}{i^\alpha}, \quad 1 \leq i \leq n
\]
where \( \alpha \) is the exponent of location popularity distribution, \( H(n) = \left( \sum_{i=1}^{n} i^{-\alpha} \right)^{-1} \) is a normalization constant. Summing over all \( p_i \), the value of \( H(n) \) in order sense is given by:
\[
H(n) = \begin{cases} 
  \Theta(1), & \alpha > 1 \\
  \Theta\left(\frac{1}{\log n}\right), & \alpha = 1 \\
  \Theta(n^{\alpha-1}), & \alpha < 1
\end{cases}
\]

In this way, node 1 is both the source and destination of node 2, node 3 is both the source and destination of node 4, and so on.

2.2 Definitions

**Capacity**: Suppose packets arrive at node \( j \) as a Bernoulli process of rate \( \lambda_j \) packets/\( s \), i.e., a packet arrives during the current slot with probability \( \lambda_j \), and otherwise no packet arrives. Other stochastic arrival flows with the same average rate can be treated similarly, and the arrival model does not affect the region of rates the network can support [27]. For given \( \lambda_j \) rates, the network is stable if there is a scheduling algorithm guaranteeing that the queue of each node does not go to infinity. Thus, the per-node capacity of the network is the maximum rate \( \lambda \) that the network can stably support.

**Delay**: The delay of a packet is the time it takes from the source to reach its destination. The total network delay is the expectation of the average delay over all packets, all S-D pairs, and all random network configurations in the long term.

3 Capacity and Delay Analysis

In this section, we first investigate the fundamental structure of this network for transmission opportunities. Then we conduct performance analysis under the conventional 2-hop relay algorithm. By exploring popular cells’ potential in accelerating information delivery, we propose a 3-hop relay algorithm to improve the network performance.

3.1 Analysis of Transmission Opportunities

Consider a cell-partitioned network model as depicted in Figure 1. Nodes independently move according to location-popularity-based mobility model, where every timeslot each node jumps into the \( i \)-th most popular cell with probability \( p_i = \frac{H(n)}{i^\alpha} \) (\( 1 \leq i \leq n \)).

Let \( q_i \) denote the probability of finding at least two nodes within cell \( i \), and \( q''_i \) denote the probability of finding a S-D pair in cell \( i \). Due to the diversity of popularity, these two probabilities will vary according to different cells. Specifically, a popular cell is more likely to attract nodes to visit it and thus has at least two nodes or S-D pairs with a larger probability. Then, we obtain that
\[
q_i = 1 - (1 - p_i)^n - np_i(1 - p_i)^{n-1}.
\]
\[
q''_i = 1 - (1 - p_i^2)^x.
\]

Since \( q'_i \) represents the probability of finding at least two nodes within cell \( i \), the opposite event of it is there is no node (this occurs with a probability of \( (1 - p_i)^n \) under the condition that a particular node is in cell \( i \) with a probability of \( p_i \)) or only one node in cell \( i \) (and this happens with a probability of \( np_i(1 - p_i)^{n-1} \), where
n infers that the node in cell i can be any one among all nodes in the network). Hence, we have the expression of (3).

As for \( q'' \), it represents the probability of finding a S-D pair in cell i. Note that in our traffic pattern, the number of nodes n is divided into \( \frac{2}{n} \) groups and each group corresponds to two S-D pairs in the network. The probability that two nodes of any S-D pair are not in cell i is \( (1 - p_i^2) \). Since each group is independent of others, the probability that there is not any group (S-D pair) in cell i is \( (1 - p_i^2)^{\frac{2n}{n}} \), yielding (4).

Lemma 1: Consider a cell-partitioned MANET (with n nodes and n cells), in which cell \( i \) (1 \( \leq i \leq n \)) is assigned with a popularity \( p_i \). Let \( q' = \frac{1}{n} \sum_{i=1}^{n} q'_i \) be the average probability of finding at least two nodes within a cell, and \( q'' = \frac{1}{n} \sum_{i=1}^{n} q''_i \) be the average probability of finding a S-D pair in a cell. Then, we bound \( q' \) and \( q'' \) in order sense:

\[
q' = \begin{cases} 
\Theta(n^{\frac{1-n}{\log n}}), & \alpha > 1 \\
\Theta\left(\frac{\log \log n}{n}\right), & \alpha = 1 \\
\Theta(1), & \alpha < 1 
\end{cases} 
\]
\[
q'' = \begin{cases} 
\Theta(n^{\frac{1-\alpha}{\log n}}), & \alpha > 1 \\
\Theta\left(\frac{1}{\log n^{\sqrt{n}}}\right), & \alpha = 1 \\
\Theta\left(n^{\frac{-\alpha}{\log n}}\right), & \frac{1}{2} < \alpha < 1 \\
\Theta\left(\frac{1}{n}\right), & \alpha < \frac{1}{2} 
\end{cases} 
\]

Proof: We first divide \( p_i \) into two categories, e.g., \( p_i \in (0, \frac{1}{n}) \) and \( p_i \in (\frac{1}{n}, 1) \). It is easy to see that \( q'_i \) will monotonously decrease when \( p_i \) increases. Also note that when \( p_i = \frac{1}{n} \), \( q'_i = 1 - (1 - \frac{1}{n})^n - (1 - \frac{1}{n})^{n-1} = \Theta(1) \). Therefore, when \( p_i \) is greater than \( \frac{1}{n} \), \( q''_i \) is also a constant. Now we will calculate \( q'_i \) when \( p_i \in (0, \frac{1}{n}) \). In this case, we have

\[
q'_i = 1 - (1 - p_i)^n - np_i(1 - p_i)^{n-1} > 1 - 2(1 - p_i)^n. 
\]

Since \( q'_i \) is obviously larger than \( 1 - (1 - p_i)^n \), we have

\[
q'_i = \Theta(1 - (1 - p_i)^n) = \Theta\left(1 - (1 - p_i)^{\frac{1}{n}}np_i\right) = \Theta\left(1 - e^{-np_i}\right) = \Theta\left(1 - \sum_{k=0}^{\infty} (-1)^k \frac{(np_i)^k}{k!}\right) = \Theta(np_i). 
\]

where the fourth equation is due to lagrange polynomial approximation of \( e^{-np_i} \) and the fifth equation holds because \( np_i \) is smaller than 1.

Recall that \( p_i = \frac{H(n)}{i}, 1 \leq i \leq n \), then we have

\[
nq' = \sum_{p_i \in (0, \frac{1}{n})} q'_i + \sum_{p_i \in [1/n, 1)} q'_i. 
\]

To determine the exact values of two components above, we make a discussion on different values of \( \alpha \) since location popularity \( p_i \) is dependent on \( \alpha \).

**Case 1:** \( \alpha > 1, p_i = \Theta\left(\frac{1}{\log n}\right) \)

\[
\sum_{p_{i \in [1/n, 1)}} q'_i = \sum_{1 \leq i \leq n \atop \theta(1)} = \Theta\left(\frac{n}{\log n}\right). 
\]

**Case 2:** \( \alpha = 1, p_i = \Theta\left(\frac{1}{\log n}\log n\right) \)

\[
\sum_{p_{i \in [1/n, 1)}} q'_i = \sum_{1 \leq i \leq n \atop \theta(1)} = \Theta\left(\frac{n}{\log n}\right). 
\]

Therefore, when \( \alpha = 1, \ q' = \Theta\left(\frac{1}{\log n}\log n\right) \).

**Case 3:** \( \alpha < 1, p_i = \Theta\left(\frac{n^{\alpha-1}}{\log n}\right) \)

In this case, for \( i \in [1/n, 1] \), \( np_i \) is in the interval of \([1, n^\alpha]\). Therefore,

\[
nq' = \sum_{p_{i \in [1/n, 1)}} q'_i + \sum_{i \in [1/n, 1]} q'_i = 0 + \sum_{i \in [1/n, 1]} nq_i = \Theta(n). 
\]

Combining all the three cases, we obtain the values of \( q' \).

Next we calculate the expected value of \( q'_i \), where \( q''_i \) is given in (4). Recall that

\[
q''_i = 1 - (1 - p_i^2)^{\frac{2n}{n}}. 
\]

Similar to previous analysis, \( q''_i \) can also be represented by two parts, as follows.

\[
q'' = \begin{cases} 
\Theta(1), & p_i \in \left[\frac{1}{\sqrt{n}}, 1\right) \\
\Theta(np_i^2), & p_i \in (0, \frac{1}{\sqrt{n}}) 
\end{cases} 
\]

Therefore,

\[
nq'' = \sum_{p_{i \in (0, \frac{1}{\sqrt{n}})}} q''_i + \sum_{p_{i \in [1/n, 1)}} q''_i. 
\]
We calculate this equation according to different values of popularity exponent.

**Case1:** $\alpha > 1$, $p_i = \Theta(\frac{1}{\sqrt{n}})$

$$nq'' = \sum_{p_i \in (0, \frac{1}{\sqrt{n}})} \Theta(np_i^2) + \sum_{p_i \in [\frac{1}{\sqrt{n}}, 1)} \Theta(1)$$

$$= \sum_{i \in (\frac{1}{\sqrt{n}}, n]} \Theta(n^{\frac{\alpha}{2\alpha}}) + \sum_{i \in [1, \frac{1}{\sqrt{n}}]} \Theta(1) \cdots (17)$$

$$= \Theta(n^{\frac{\alpha}{2\alpha}}).$$

**Case2:** $\alpha = 1$, $p_i = \Theta(\frac{1}{\log n})$

$$nq'' = \sum_{i \in (\frac{1}{\sqrt{n}}, n]} \Theta(\frac{n}{i^2 \log n}) + \sum_{i \in [1, \frac{1}{\sqrt{n}}]} \Theta(1)$$

$$= \Theta \left( \frac{\sqrt{n}}{\log n} \right) \cdots (18)$$

**Case3:** $\alpha < 1$, $p_i = \Theta(n^{\frac{\alpha-1}{\alpha}})$

To proceed, we further divide this case into three subcases, e.g., $\alpha \in [0, \frac{1}{2})$, $\alpha = \frac{1}{2}$ and $\alpha \in (\frac{1}{2}, 1)$. When $\alpha \in (\frac{1}{2}, 1)$, we have

$$nq'' = \sum_{i \in (\frac{2}{\alpha - 1}, \frac{1}{\sqrt{n}})} \Theta(np_i^2) + \sum_{i \in [1, \frac{1}{\sqrt{n}}]} \Theta(1) \cdots (19)$$

$$= \Theta \left( n^{\frac{\alpha}{2\alpha - 1}} \right).$$

When $\alpha = \frac{1}{2}$, we have $nq'' = \Theta(\log n)$.

When $\alpha \in [0, \frac{1}{2})$, for any $i$, $p_i$ is smaller than $\Theta(\frac{1}{\sqrt{n}})$. Therefore,

$$nq'' = \sum_{p_i \in (0, \frac{1}{\sqrt{n}})} \Theta(np_i^2)$$

$$= \sum_{i \in [1, n]} \Theta \left( \frac{n^{2\alpha - 1}}{i^{2\alpha}} \right) = \Theta(1) \cdots (20)$$

Combining all the proof above, we complete this lemma. \hfill \Box

### 3.2 Capacity and Delay Under 2-Hop Relay Algorithm

In this subsection, we employ conventional 2-hop relay algorithm [5], [13] to investigate the throughput and delay. Here we briefly describe the algorithm. A packet is first sent from the source to an available node, which will act as a relay for the packet. Then the packet is carried by the relay until it encounters the destination and delivers the packet.

Packets are transmitted and routed through the network according to the scheduling algorithm. The algorithm decides which packet to transmit at each timeslot without violating the physical constraints of the cell-partitioned network. Since nodes will act as relays, there will be multiple packets in the buffer of each node. If the buffer is not large enough, packets may be dropped, which will also affect the network performance. In the following analysis, we will calculate how large the buffer needed to be to ensure stability. A scheduling algorithm is stable if buffers of all the nodes do not overflow.

**Theorem 1**: Consider a cell-partitioned network (with $n$ nodes and $n$ cells) under the 2-hop relay algorithm, and assume that each node moves into cell $i$ with probability $p_i$ every timeslot and its buffer size is $n_b$. Then the capacity of the network is:

$$\mu = \left\{ \begin{array}{ll}
\Theta \left( \frac{n^{1-\alpha}}{\alpha} \right), & \alpha > 1 \\
\Theta \left( \frac{\log \log n}{\log n} \right), & \alpha = 1 \\
\Theta(1), & \alpha < 1
\end{array} \right. \cdots (21)$$

where the average delay is $\Theta(n)$ for any $\alpha$ and

$$n_b = \left\{ \begin{array}{ll}
\Theta \left( \frac{n^{1-\alpha}}{\alpha} \right), & \alpha > 1 \\
\Theta \left( \frac{\log \log n}{\log n} \right), & \alpha = 1 \\
\Theta(n), & \alpha < 1
\end{array} \right. \cdots (22)$$

The network is stable if all nodes communicate at rate $\lambda < \mu$. The proof of the above theorem involves proving that $\lambda \leq \mu$ is necessary for network stability. Here we provide the main ideas behind the proof.

**Proof**: In [27] it is shown that the capacity region depends only on the steady state node distribution when nodes change cells in a reshuffled and independent manner. Suppose all nodes send at the same data rate (i.e., $\lambda_j = \lambda$ for all $j$), then the sum rate of new packets entering the network is $n\lambda$. If $S$ and $D$ are in the same cell, their packets are transmitted once. Since the average number of cells containing a S-D pair is $nq''$ during a given timeslot, the rate of single-hop transfers between sources and destinations is at most $nq''$. While the average number of cells that can support a packet transfer (i.e., there are at least two nodes in it) is $nq''$ in a given timeslot, the maximum rate of other transfers taking two or more hops is thus $\frac{nq'' + nq''}{2}$. Therefore, $n\lambda \leq nq'' + \frac{nq''}{2}$, yielding the necessary condition. Combined with (5) and (6) in Lemma 1, we can get the expression (21).

For a given packet, there are two possible routings: one is the direct “S-D” single-hop path and the other is the “S-R-D” 2-hop path. Only a $O\left(\frac{1}{\sqrt{n}}\right)^2$ fraction of packets are transmitted by “S-D” single-hop path, which implies that most of the packets are transmitted to their destinations via relay nodes. Thus, the delay is mainly composed of two parts: (i) S-R delay, which is the average waiting time for a source to be scheduled for transmissions. Since there are $np_i$ nodes in cell $i$, a node has an opportunity to transmit with probability $\frac{1}{np_i}$. The waiting time in cell $i$ is geometrically distributed with mean $np_i$. With probability $p_i$ a source is in cell $i$, the average S-R delay is $\sum_{i=1}^{n} np_i^2$; (ii) R-D delay, which is the time it takes for the relay node to reach the destination. The probability of this event is

2. “S-D” single-hop path happens when both $S$ and $D$ are in the same cell. And the average probability of finding a S-D pair in a cell is $q''$ (6), which reflects the fraction of packets reaching their destinations in a single hop.
\[ \sum_{i=1}^{n} (p_i^2 \cdot \frac{1}{np_i}) \], which is given by the probability some relay and destination are scheduled for transmissions in cell \( i \) multiplied by the conditional probability they meet in cell \( i \). The former probability is \( \frac{1}{np_i} \), while the latter one is \( p_i^2 \). Thus this event occurs every \( \Theta(n) \) timeslots given that \( \sum_{i=1}^{n} p_i = 1 \). Combined with the fact that S-R delay is \( O(n) \), the average delay is \( \Theta(n) \) for any \( \alpha \).

Since each node can serve as a relay for the other \( n-1 \) nodes equally, it can be modeled as a node having \( n-1 \) parallel subqueues buffering packets from different sources. Suppose subqueue \( j \) (\( j = 1, 2, \ldots, n-1 \)) stores packets from node \( j \), it can be regarded as a Bernoulli/Bernoulli queue with input rate \( \lambda_j = \frac{1}{n-1} \) (for node \( j \)), the other \( n-1 \) nodes act as relays for it equally) and service rate \( \mu_j' = \frac{1}{n} \) (because each packet needs \( \Theta(n) \) timeslots to be successfully transmitted). The resulting expected number of occupancy packets at subqueue \( j \) is \( \tilde{L}_j = \frac{\lambda_j'}{1-\lambda_j'} \), where \( \lambda_j' \triangleq \frac{\lambda_j}{\mu_j} \). Thus the buffer size of each node is equivalent to the sum queue length of all subqueues, i.e., \( n_b = \sum_{j=1}^{n-1} \tilde{L}_j \), which completes the proof.

\[ 3.3 \text{ Capacity and Delay Under 3-Hop Relay Algorithm} \]

In previous subsection we find that 2-hop relay algorithm incurs an excessive delay of \( \Theta(n) \). This is due to the fact that the algorithm only admits a one-to-one transmission every timeslot in a popular cell even though there are a number of nodes in it. Inspired by this, we propose a modified relay algorithm to improve the network performance, which utilizes the potential transmission opportunities in popular cells and restricts packets to 3-hop paths. We show that the algorithm can achieve the maximal capacity with a bounded average delay \( \Theta(\sqrt{n}) \).

A cell is defined to be a popular cell if it contains at least \( E[n_p] \) nodes. And the number of popular cells is \( E[n_p] \) in a network. Since the popular cell has a high node density, we deploy an AP in the center and each node within the cell can reach the AP in a single-hop fashion. The nodes in the same popular cell take turns to broadcast packets. Once an opportunity arises for a relay node (called initial relay) to connect with the AP, all the packets in its buffer will be instantly broadcasted to all the nodes within the cell. These nodes will then act as accelerated relays to forward the packet to the destination. Note that these multiple accelerated relays are vitally important, as they boost the chances of running across the destination significantly.

3-Hop Relay Algorithm: During a timeslot, for each cell with at least two nodes:

1) If there exists a S-D pair in the cell, randomly select such a pair uniformly over all possible pairs within the cell. If the source has a new packet intended for the destination, transmit it and then delete it from its buffer. Else remain idle.

2) If there is no S-D pair within the cell, randomly designate a node in the cell as sender. Then for such a cell:

i) Broadcast Transmission: If it is a popular cell and the designated sender has packets, transmit all the packets in its buffer to all other nodes in the cell with the assistance of AP. If a packet is received by its destination successfully, delete the packet from the buffers of all nodes holding it. Else remain idle.

ii) If it is not a popular cell, independently choose another node as receiver among the remaining nodes in the cell. With equal probability, randomly choose from the two options:

- Source-to-Relay Transmission: If the sender has a new packet which has never been transmitted before, relay the packet to the designated receiver. Else remain idle.
- Relay-to-Destination Transmission: If the sender has a packet destined for the designated receiver, transmit. Once the destination has received it, the packet will be dropped from the buffers of all nodes holding it. Else remain idle.

Note that the algorithm schedules the single-hop S-D transmissions whenever possible, while S-R, R-D and broadcast transmissions happen independently.

![Fig. 2: A decoupled view of the network as seen by the packets transmitted from a single source to the destination.](image-url)

The algorithm has a nice decoupling feature between sessions, as depicted in Figure 2. There are three roles for individual nodes in the network: a source, intermediate relays and a destination. And relays can act as either initial relays or accelerated relays. Transmissions of packets from other sources are mapped as random ON/OFF service opportunities. Service at the first stage (source) is Bernoulli with rate \( \mu \) while the Bernoulli rate is \( \frac{2E[n_p]}{n} \) at the second stage (initial relay). At the third stage (accelerated relay), it is Bernoulli with rate \( \frac{2E[n_p]}{n-2} \).

Theorem 2: Consider the same assumptions for the network as Theorem 1. If the exogenous input flow to node \( j \) is a Bernoulli flow of rate \( \lambda_j \) (where \( \lambda_j < \mu \) for network stability) and its buffer size is \( n_b \), then under the 3-hop relay algorithm, the average delay \( W_j \) for the
traffic of node $j$ satisfies:

$$E[W_j] = \left\{ \begin{array}{ll}
\Theta(n^{\alpha/\alpha+\alpha}), & \alpha > 1 \\
\Theta(\sqrt{n\log n}), & \alpha = 1 \\
\Theta(n^{1/\alpha+1}), & \alpha < 1
\end{array} \right. \quad (23)$$

where $\mu = \frac{q''+2q'''}{3}$ and

$$n_b = \left\{ \begin{array}{ll}
\Theta\left(n^{\alpha/\alpha+\alpha}\right), & \alpha > 1 \\
\Theta\left(\sqrt{n\log n} \cdot \log \log n\right), & \alpha = 1 \\
\Theta\left(n^{1/\alpha+1}\right), & \alpha < 1
\end{array} \right. \quad (24)$$

Proof: Since every timeslot packets arrive at source $j$ with probability $\lambda_j$ and are scheduled for transmissions with probability $\mu$, source $j$ can be represented as a Bernoulli/Bernoulli queue with input rate $\lambda_j$ and service rate $\mu$.

We first show that $\mu = \Theta(q''+q''')$ still holds under this algorithm. Let $r_1$ denote the probability that the source is scheduled to transmit directly to the destination, and $r_2$ denote the probability that it is scheduled to transmit to one of its relays. Thus, we have $\mu = r_1 + r_2$. Since the proposed relay algorithm schedules transmissions into and out of relay nodes with equal probability, $r_2$ also equals the probability that relay nodes are scheduled to broadcast in a popular cell or transmit to the destination. Hence, the total rate of transmission opportunities over the network is $n(r_1 + 3r_2)$ every timeslot. Meanwhile, a transmission opportunity occurs in cell $i$ with probability $q''_i$, we thus have:

$$\sum_{i=1}^{n} q''_i = n(r_1 + 3r_2). \quad (25)$$

Recall that $q''_i$ is the probability that cell $i$ contains a S-D pair. Since the algorithm schedules the S-D transmissions whenever possible, then $r_1$ satisfies:

$$\sum_{i=1}^{n} q''_i = nr_1. \quad (26)$$

It follows from (25) and (26) that $r_1 = q''$, $r_2 = \frac{q''+q'''}{3}$. Thus, the total rate of transmissions out of the source node is given by

$$\mu = r_1 + r_2 = \frac{q''+2q'''}{3}. \quad (27)$$

Next, we derive the average delay $E[W_j]$ for the packets from source $j$. There are four possible routings from a source to its destination: one is the 3-hop path along “source-initial relay-accelerated relay-destination”; one is the 2-hop path along “source-initial relay-destination” or “source-accelerated relay-destination”, and another is the single-hop path from source to destination. Since only $O\left(\frac{1}{\sqrt{n}}\right)$ of the packets reach their destinations in a single-hop or 2-hop manner, their transmissions do not affect the delay in order sense and thus can be omitted.

As for the 3-hop routing, the packet delay is composed of three parts: the waiting time at source, initial relay, and accelerated relay, respectively. Since source $j$ can be regarded as a Bernoulli/Bernoulli queue with input rate $\lambda_j$ and service rate $\mu$, the resulting occupancy at source $j$ is $L_a = \rho \frac{\lambda_j}{1-\rho}$, where $\rho = \frac{\lambda_j}{\mu}$. From Little’s theorem, the average waiting time in the source is given by $E[W_s] = \frac{L_a}{\lambda_j} = \frac{1-\rho}{\rho \mu}$. Moreover, since the queue is reversible, the output process is also a Bernoulli stream of rate $\lambda_j$. Packets from this output process will be transmitted to the initial relay, and we will calculate its input rate and service rate in the following step.

A given packet from source $j$ is delivered to the initial relay with probability $\frac{r_2}{\mu}$ (because in an ordinary cell, the packet is intended for a relay node with probability $\frac{r_2}{\mu}$, and $n-2$ relay nodes share the opportunity equally). Hence, the input rate of the relay is $\lambda_{fr} = \frac{r_2}{\mu} \lambda_j$. On the other hand, with probability $\mu_{fr} = \frac{r_2}{\mu} E[n_a]$ the relay is scheduled for a broadcast transmission in a popular cell. This probability is constituted by: (a) a broadcast opportunity arises in the initial relay with probability $\frac{r_2}{\mu}$, (b) the relay node moves to a popular cell with probability $E[n_a]$, and (c) it is scheduled for a broadcast transmission with probability $\frac{1}{E[n_a]}$. Hence, the average waiting time in the initial relay node is $E[W_{fr}] = \frac{\lambda_{fr}}{\mu_{fr} - \lambda_{fr}}$. From Little’s theorem, the average waiting time at the initial relay node is $E[W_{fr}] = \frac{1}{\mu_{fr} - \lambda_{fr}}$.

After the broadcast in the second hop, there are $E[n_a]$ nodes holding the duplicates of a given packet. Then the packet will wait at these accelerated relays until one of them comes across the destination. Similarly, any accelerated relay can be treated as a continuous-time M/M/1 queue with input rate $\lambda_{ar}$ and service rate $\mu_{ar}$. A given packet from the initial relay node is transmitted to an accelerated relay with probability $\frac{E[n_a]}{n-2}$ (because the packet is broadcast to $E[n_a]$ accelerated relays in a popular cell and each of the $n-2$ relay nodes are equally likely). Therefore, every timeslot, an accelerated relay independently receives a packet with probability $\lambda_{ar} = \frac{\lambda_{fr} E[n_a]}{n-2}$. Meanwhile, one of the $E[n_a]$ accelerated relay nodes is scheduled for a potential packet transmission to the des-

3. A transmission opportunity arises when a node is scheduled to transmit to another node. Such an event occurs with probability $\mu$ every timeslot, independent of whether or not there is a packet waiting in the queue, corresponding to a service process with Bernoulli nature. Thus, the time average service rate of a node equals the transmission probability.

4. If no technique is employed, an initial relay node will receive multiple packets in ordinary cells before it gets a broadcast opportunity in popular cells. Then the queue will grow to infinity as time increases. With the assistance of AP, packets in the buffer can be served instantly once an opportunity arises for the relay to connect with AP. In this way, the transmission stability at the second stage can be guaranteed.
tination with probability \( \mu_{ar} = \frac{r_2 E[n_a]}{n} \). The probability is determined by the following factors: (a) an "accelerated relay-to-destination" opportunity arises in an accelerated relay with probability \( r_2 \), (b) the probability that some accelerated relay carrying the packet communicates with the destination is \( \sum_{i=1}^{n} \left( p_i^2 \cdot \frac{1}{np} \right) \) given that \( \sum_{i=1}^{n} p_i = 1 \), and (c) each of the \( E[n_a] \) accelerated relays gets the opportunity with equal probability. Due to the nature of the M/M/1 queue, the resulting occupancy at any accelerated relay is \( \lambda_{relay} = \frac{\lambda}{\mu_{ar} - \lambda_{ar}} \). From Little’s theorem, the average waiting time in the accelerated relay-to-destination opportunity arises in an accelerated relay is determined by the following factors: (a) an "accelerated relay redundancy" opportunity with equal probability. Due to the nature of the M/M/1 queue, the resulting occupancy at any accelerated relay is \( \lambda_{relay} = \frac{\lambda}{\mu_{ar} - \lambda_{ar}} \). From Little’s theorem, the average waiting time in the accelerated relay node is thus \( E[W_{ar}] = \frac{\lambda}{\mu_{ar} - \lambda_{ar}} \).

Therefore, the total network delay is

\[
E[W_j] = E[W_s] + E[W_{fr}] + E[W_{ar}]
\]

and hence,

\[
E[W_j] = 1 - \frac{\lambda_j}{\mu} + \frac{1}{\mu} \frac{E[n_a]}{n} \left( \frac{1}{\mu E[n_p] - \lambda_j} + \frac{1}{\mu E[n_a]} \right)
\]

\[
= 1 - \frac{\lambda_j}{\mu} + \Theta \left( \frac{\mu(n-2)}{\mu E[n_p] - \lambda_j} + \Theta \left( \frac{\mu(n-2)^2}{\mu E[n_a]} \right) \right)
\]

\[
= \Theta \left( \frac{n}{E[n_p]} \right) + \Theta \left( \frac{n}{E[n_a]} \right).
\]

From the definitions of \( E[n_a] \) and \( E[n_p] \), we have

\[
E[n_a] = \frac{nH(n)}{E^n[n_p]}.
\]

It follows that

\[
E[W_j] = \Theta \left( \frac{n}{E[n_p]} + \frac{E^n[n_p]}{nH(n)} \right)
\]

\[
\geq \Theta \left( \frac{nE[n_p]}{(1 + \alpha)H(n)} \right),
\]

where the equality holds when \( E[n_p] = E[n_a] = \Theta \left( (nH(n))^{\frac{1}{1+\alpha}} \right) \). Connecting (2) and (31), we obtain the average delay in order sense.

To guarantee the network stability, the arrival rate should be less than the service rate at any stage of each transmission. Then we have

\[
\begin{align*}
\frac{\mu - \lambda_j}{\mu} & > 0 \\
\frac{r_2 E[n_a]}{n} - \frac{\lambda_j}{\mu E[n_p]} & > 0 \\
\frac{r_2 E[n_a] - \lambda_j}{\mu E[n_a]} & > 0
\end{align*}
\]

yielding the stability condition \( \lambda_j < \mu \).

Moreover, from the perspective of the whole network, the sum of new packets generated by popular cells should be less than the sum of packets served by ordinary cells in any time slot. Once a packet is delivered to its destination, each relay holding the duplicate will delete the corresponding packet from the buffer, leading to \( E[n_a] \) served packets. Thus,

\[
E[n_a]E[n_p]n \leq (nq' - E[n_p])E[n_a].
\]

Therein the factor \( nq' - E[n_p] \) denotes the number of ordinary cells which can support a packet transmission. It follows that \( n_q = \Theta \left( \frac{nq'}{E[n_p]} \right) \). Combined with (5) in Lemma 1, we get the value of \( n_q \).

### 3.4 Minimum Delay Analysis Under Two Schemes

In the previous subsection, we obtain the achievable delay by constructing two schemes without relay redundancy. One question arises: what is the minimum delay a network can guarantee without relay redundancy?

In location-based scenarios, delay can be further improved by allowing multi-user reception in popular cells, which are defined as transmission redundancy. Here relay redundancy refers to the redundancy put forward in previous works, which could happen in any cell having at least two nodes. Transmission redundancy occurs in cells which have more than \( E[n_a] \) nodes. In transmission redundancy, a packet can be received by all other nodes in the same cell in a timeslot through broadcasting. The crucial difference is that, once transmission redundancy of a packet incurs, it will generate multiple duplicate-carrying relays for the packet. And to obtain the same number of duplicates, each packet is required to be retransmitted multiple times to different relays by relay redundancy since relay redundancy generates at most one duplicate each time. Note that rich node resource in popular cells enables transmission redundancy, which thus may not work in uniform scenarios.

Now we explore the minimum delay of the network without relay redundancy. Specifically, we consider an ideal situation where the network is empty and a source sends a single packet to its destination. Firstly, we adopt a scheme without considering relay (i.e., non-redundancy scheme) since relaying packets cannot help improve delay. Secondly, we explore the minimum delay of the network when implementing transmission redundancy (named redundancy scheme for briefedness).

**Lemma 2:** Let \( p' \) be the average probability that any two nodes move into the same cell, then the order sense of \( p' \) is

\[
p' = \begin{cases} 
\Theta \left( \frac{1}{\log n} \right), & \alpha > 1 \\
\Theta \left( \frac{\log n}{n^{\alpha-2}} \right), & 1/2 < \alpha < 1 \end{cases}
\]

**Proof:** Denote \( p'_i \) as the probability that node 1 encounters node 2 in cell \( i \) in a timeslot. We have that \( p'_i = p_i^2 = \frac{H^2(n)}{\mu^2} \). Thus, the probability that these two nodes are in the same cell is \( p' = \sum p'_i \). Then we have

\[
p' = \sum_{i=1}^{n} \frac{H^2(n)}{i^{2\alpha}} = \int_{1}^{n} \frac{H^2(n)}{i^{2\alpha}} di.
\]

Combined with (2), we get the value of \( p' \). □

**Theorem 3:** Algorithms which do not use any redundancy cannot achieve an average delay of less than \( E[W_{min}'] \). Algorithms permitting transmission redundancy cannot achieve an average delay of less than \( E[W_{min}''] \).
$E[W'_{\text{min}}]$ and $E[W''_{\text{min}}]$ are given by

$$E[W'_{\text{min}}] = \begin{cases} \Theta(1), & \alpha > 1 \\ \Theta(\log^2 \frac{1}{n}), & \alpha = 1 \\ \Theta\left(\frac{1}{n^{\alpha(1-\alpha)}}\right), & \frac{1}{2} < \alpha < 1 \end{cases} \quad (36)$$

$$E[W''_{\text{min}}] = \begin{cases} \Theta(1), & \alpha > 1 \\ \Theta\left(\log^2 \frac{1}{n}\right), & \alpha = 1 \\ \Theta\left(\frac{n^{2(1-\alpha)}}{\alpha}\right), & \frac{1}{2} < \alpha < 1 \end{cases} \quad (37)$$

**Proof:** For the first situation, the time required for S-D transmission depends only on the encountering probability of S and D. Denote $W'_{\text{min}}$ as the minimum amount of time it takes the source to reach the destination. Suppose that the source meets the destination at the $h$th time slot, then the probability of this event is

$$P\{W'_{\text{min}} = h\} = p'(1 - p')^{h-1} \quad (38)$$

where $(1 - p')^{h-1}$ implies that these two nodes have not met each other in the former $h - 1$ time slots. Hence, the expectation of $W'_{\text{min}}$ is

$$E[W'_{\text{min}}] = \sum_{h=1}^{+\infty} h p'(1 - p')^{h-1} = p' \left(\sum_{h=1}^{+\infty} (1 - p')^h\right) = \frac{1}{p'} \quad (39)$$

wherein $\left(\sum_{h=1}^{+\infty} x^h\right)' = \frac{1}{(1-x)^2}$ for any $|x| < 1$ is employed.

From Lemma 2, we can obtain $E[W''_{\text{min}}]$.

Then we propose a simple 2-hop routing scheme to allow transmission redundancy. As an initial step, the source broadcasts the packet to all the nodes within the cell when it moves into a popular cell. These duplicate-carrying nodes will act as relays until one of them reaches the destination. Note that the actual number of relay nodes depends on the number of nodes in the same popular cell as the source (at the timeslot when the source entered a popular cell).

To prove the result (37), again consider delivering a single packet from the source to its destination. By employing the 2-hop routing scheme mentioned above, we have $E[W''_{\text{min}}] = W''_1 + W''_2$. Here $W''_1$ represents the expected number of timeslots required for the source to move into a popular cell, and $W''_2$ represents the expected time it takes for the destination to come in contact with a duplicate-carrying node. The probability that the source is within a popular cell is $\Theta\left(\frac{\sum_i E[n_i] n_i}{E[n]_{\alpha-1}}\right)$ where the sum of nodes in all popular cells is $n \sum_i E[n_i] n_i$, and hence the average time required to reach a popular cell is equal to the inverse of this quantity. Since with probability $\Theta(1)$ the source is scheduled for broadcast transmission (only the source has the packet to transmit), we have

$$W''_1 = \Theta\left(\frac{1}{\sum_i E[n_i] n_i} \right). \quad (40)$$

To calculate $W''_2$, denote $P(W''_2)$ as the probability that one of the $n_{2} = \sum_i E[n_i] n_i w_i$ nodes transmits to the destination in a cell during a timeslot. Therein $w_i = p_i \sum_i E[n_i] n_i$ represents the probability that a node is from cell $i$. Because with average probability $p'$ any two nodes encounter in a cell, we have

$$P(W''_2) = \Theta\left(1 - (1 - p')^n_{2}\right) = \Theta\left(1 - (1 - p')^n_{2}\right) \quad (41)$$

$$= \Theta\left(1 - e^{-n_{2}p'}\right).$$

If $n_{2}p'$ is less than 1, $P(W''_2) = \Theta(n_{2}p')$. And if $n_{2}p'$ is equal to or greater than 1, $P(W''_2) = \Theta(1)$. Hence, $W''_2$ is in order sense equivalent to $\max\left(\frac{1}{n_{2}p'}, 1\right)$. Summing $W''_1$ and $W''_2$, we can obtain the minimum delay of the packet

$$E[W''_{\text{min}}] = \Theta\left(\frac{1}{\sum_i E[n_i] n_i} \right) + \Theta\left(\max\left(\frac{\sum_i E[n_i] n_i}{np' \sum_i E[n_i] n_i}, 1\right)\right) \quad (42)$$

where

$$\sum_{i=1}^{\sum_i E[n_i]} p_i = \begin{cases} \Theta(H(n)(E[n_i])^{-\alpha}), & \alpha < 1 \\ \Theta\left(H(n)\log(E[n_i])\right), & \alpha = 1 \\ \Theta(H(n)), & \alpha > 1 \end{cases} \quad (43)$$

and

$$\sum_{i=1}^{\sum_i E[n_i]} p_i^2 = \begin{cases} \Theta(H^2(n)(E[n_i])^{-2\alpha}), & \alpha < \frac{1}{2} \\ \Theta(H^2(n)(E[n_i])^{-2\alpha}), & \alpha = \frac{1}{2} \\ \Theta(H^2(n)), & \alpha > \frac{1}{2} \end{cases} \quad (44)$$

Plug (34) and (2) into (42), we make a derivation of $E[W_{\text{min}}]$ on different values of $\alpha$, which can be summarized as follows:

**Case1:** When $\alpha < \frac{1}{2}$, $n_{2}p' = \frac{n_{2}^{\alpha-1}}{E[n]_{\alpha-1}} < 1$

$$E[W''_{\text{min}}] = \Theta\left(\left(\frac{n}{E[n]_{\alpha-1}}\right)^{-\alpha}\right) + \Theta\left(\left(\frac{E[n_i]}{n_{2}^{\alpha-1}}\right)^{\alpha}\right) \quad (45)$$

where the equality holds when $E[n_i] = \Theta(1)$.

**Case2:** When $\alpha = \frac{1}{2}$, $n_{2}p' = \frac{\sqrt{\log(E[n])\log n}}{\sqrt{n}E[n]} < 1$

$$E[W''_{\text{min}}] = \Theta\left(\sqrt{\frac{n}{E[n]}}\right) + \Theta\left(\frac{\sqrt{nE[n]}}{\log(E[n])\log n}\right) \quad (46)$$

where the equality holds when $E[n_i] = \Theta(1)$.

**Case3:** When $\frac{1}{2} < \alpha < 1$, $n_{2}p' = \frac{n_{2}^{\alpha-1}}{E[n_i]^{\alpha-1}}$. If $n_{2}^{\alpha-1} < \frac{1}{E[n_i]}$
nodes in popular cells

optimal values can provide guidelines on how to assign
under which these inequalities are tight, we can solve the

E of redundancy scheme is affected by three factors: the

directly since the expected number of nodes in any cell is

degraded. In view of this, we employ relay redundancy
to reduce delay and investigate the delay-capacity trade-
off under 3-hop relay algorithm.

Remarks. From Theorem 3, we can make the following
fundamental observations. First, the minimum delay of
non-redundancy scheme is determined only by the
average encountering opportunity of two nodes, which
increases as \( \alpha \). Second, there is a gap between \( E[W'_{\text{min}}] \)
and the average delay (\( \Theta(n) \)) achieved by the 2-hop relay
algorithm. Note that such a gap does not exist in
uniform scenarios. The main reason is, two destined
nodes meeting in the same cell may wait a long time
to be scheduled for transmissions because the number
of nodes in some cells scales as \( n \) due to location popu-
larly. While in uniform scenarios they can communicate
directly since the expected number of nodes in any cell is
of constant order, independent of \( n \). Third, redundancy
scheme outperforms non-redundancy scheme for any \( \alpha \)
in terms of the minimum delay. And the minimum delay of
redundancy scheme is affected by three factors: the
number of popular cells \( E[n_p] \), the average number of
nodes in popular cells \( n_2 \) and the average encountering
opportunity of two nodes \( n_2' \). By studying the conditions
under which these inequalities are tight, we can solve the
optimal choices for these key parameters. Note that these
optimal values can provide guidelines on how to assign
node resource for transmission redundancy. Hence, it is
desirable to use transmission redundancy in popular
cells to forward the packet from multiple relays to the
destination.

3.5 Delay and Capacity Tradeoffs

In previous subsections, we assume that there is only a
single initial relay node receiving packets from the
source. Intuitively, if more initial relays obtain the packet,
the packet will be spread more quickly and the delay
may be further improved. However, the capacity will be
degraded. In view of this, we employ relay redundancy
to reduce delay and investigate the delay-capacity trade-
off under 3-hop relay algorithm.

Suppose there are \( m \) nodes initially holding a replica
of a given packet. Then these nodes are scheduled as fol-

\[
E[W_{\text{min}}] = \Theta \left( \frac{n}{m E[n_p]} \right) \quad \text{for } n \leq m E[n_p],
\]

with corresponding capacity of \( \Theta \left( \frac{n q''}{m} \right) \).

Proof: For a given packet, there will be \( k E[n_p] \)
accelerated nodes carrying the duplicate after \( k \) nodes
spread the packet in distinct popular cells. Note that
a special case cannot be ignored here, that is, when
\( m < k E[n_p], \) network performance cannot benefit from
the accelerated nodes. In this case, 3-hop algorithm
degenerates into 2-hop algorithm with no acceleration
and all the transmissions are along 2-hop path starting
out with \( m \) duplicates. Since all nodes send at the same
data rate \( \lambda \) and a packet has \( m \) replicas, then the sum
rate of new packets entering the network is \( n m \lambda \). Hence,
\( n m \lambda \leq n q'' + \frac{n (q'' - q')}{2} \) for \( m > k E[n_p] \). Otherwise,
when \( m \leq k E[n_p], \) the transmissions will benefit from \( k E[n_p] \)
accelerated nodes. In this case, all the transmissions are
along 3-hop path with acceleration. Similarly, we have
\( n m \lambda \leq n q'' + \frac{n (q'' - q')}{3} \) for \( m \leq k E[n_p] \). Based on
the above analysis, we have \( \lambda < \Theta \left( \frac{q''}{m} \right) \).

Next we analyze the average delay in two cases
mentioned above. If \( m \leq k E[n_p], \) the delay is mainly
composed of two parts: (i) second-hop delay, which is
the time it takes for \( k \) initial relay nodes to reach
distinct popular cells and are scheduled for broadcast
transmissions. The probability of this event is \( \Theta \left( \frac{k}{n} \cdot \frac{E[n_p]}{E[n_p]} \cdot \frac{1}{m E[n_p]} \right) \), which implies that this event occurs
every \( \Theta \left( \frac{kn}{m E[n_p]} \right) \) timeslots; (ii) third-hop delay, which is
the time it takes for one of the \( k E[n_p] \) accelerated relay
nodes to reach the destination. The probability of this
event is \( \Theta \left( \frac{k E[n_p]}{n} \right) \), which means that this event occurs
For these cases, $p_i = \frac{H(n)}{n^2}, 1 \leq i \leq n.$

every $\Theta\left(\frac{n}{kE[n_a]}\right)$ timeslots. The average delay of a packet is then given by

$$E[W_j] = \Theta\left(\frac{kn}{mE[n_p]}\right) + \Theta\left(\frac{n}{kE[n_a]}\right)$$

$$\geq \Theta\left(\frac{n}{\sqrt{mE[n_a]}}\right)$$

(52)

where the equality holds when $k = \sqrt{m}$ under the condition that $E[n_a] = E[n_p] = \Theta\left(\frac{nH(n)}{\sqrt{n}}\right)$.

If $m > kE[n_a]$, the average delay is the time required for one of $m$ nodes to reach the destination. From Theorem 1, we know that a relay node delivers packets to the destination with probability $\frac{1}{n}$. Since there are $m$ relay nodes, the probability increases to $\frac{m}{n}$, yielding the average delay $\Theta\left(\frac{m}{n}\right)$.

From Theorem 4, we can observe that both capacity and delay decrease as $m$ increases. It implies that by changing the number of replicas of a packet, we can regulate the tradeoffs between capacity and delay.

4 DISCUSSION

4.1 Impacts of Location Popularity on Performance

In this subsection, we compare the performance of 3-hop relay algorithm (3-hop scheme for briefness) with that of 2-hop relay algorithm (2-hop scheme for briefness), both with no relay redundancy. We further discuss the impacts of location popularity on their performance. Before that, we first illustrate how $\alpha$ affects the location popularity in Figure 3. It can be seen that when $\alpha$ is small, nodes tend to visit the entire network area uniformly, leading to a more homogeneous distribution. As $\alpha$ increases, nodes tend to gather in a few hot locations, resulting in a more diversified distribution.

Impact of $\alpha$ on capacity: As depicted in Figure 4(a), both 3-hop scheme and 2-hop scheme can achieve a capacity of $\Theta(1)$ for any $\alpha \leq 1$. However, the capacity decays as $\Theta(n^{1-\alpha})$ for $\alpha > 1$. The intuitive explanation is as follows. To obtain a higher throughput, we should enable more cells to be active for transmissions. Since nodes are more dense in popular cells and more sparse in ordinary cells as $\alpha$ increases, the majority of cells become idle and cannot be efficiently utilized for transmissions, leading to a poor utilization on network space and inferior capacity performance.

Impact of $\alpha$ on delay: From Figure 4(a), we can observe that the delay of 2-hop scheme is independent of $\alpha$ while the delay of 3-hop scheme varies with $\alpha$ dramatically. Specifically, when $\alpha = 1$, 3-hop scheme achieves the minimal delay of $\Theta(\sqrt{n})$. And the delay becomes worse when $\alpha$ either decreases in region $\alpha < 1$ or increases in region $\alpha > 1$. We present an intuitive insight for this phenomenon. In a 3-hop transmission, the average packet delay is $\Theta\left(\frac{n}{E[n_p]}\right) + \Theta\left(\frac{1}{E[n_a]}\right)$, which is composed of the second-hop delay $\Theta\left(\frac{n}{E[n_p]}\right)$ and the third-hop delay $\Theta\left(\frac{1}{E[n_a]}\right)$. Figure 5 shows the restrictive relation between $E[n_a]$ and $E[n_p]$. For $\alpha > 1$, the delay bottleneck is in the second hop. Due to the fact that a few locations (small $E[n_p]$) occupy a large number of nodes (large $E[n_a]$)
every timeslot, delay performance is limited by a small number of popular cells in the second hop. For $\alpha < 1$, the delay bottleneck is in the third hop. Since nodes tend to be uniformly distributed in the network, $E[n_a]$ is small and $E[n_{p}]$ is large. Therefore, delay performance is limited by a small number of relays in the third hop. Thus, when $E[n_a] = E[n_{p}]$, there is a better tradeoff between the second-hop and third-hop transmissions, and the maximal $E[n_a]$ and $E[n_{p}]$ can be reached when $\alpha = 1$ as shown in Figure 5. As a result, the optimal delay performance can be achieved in this case.

Besides, 3-hop scheme outperforms 2-hop scheme in terms of delay for any $\alpha$, as shown in Figure 4(a). It is worth mentioning that such an improvement is not at the cost of capacity. The main reason for this improvement is that the acceleration mechanism in 3-hop scheme largely enhances the transmission ability of popular cells. Note that we only need to place APs in a few popular cells, on the purpose of ensuring network stability. The fraction of popular cells is at most $O(1/\sqrt{n})$ for the network, indicating a small deployment cost.

Impact of $\alpha$ on buffer size: Buffer is a big concern in DTN. It is shown that 2-hop scheme is no longer optimal in location heterogeneity scenarios, without the consideration of buffer. However, does it mean that 3-hop scheme requires more buffer to achieve better performance?

As illustrated in Figure 4(b), we can find that buffer = capacity $\times$ delay holds in such an inhomogeneous scenario. Compared with 2-hop scheme, 3-hop scheme needs much smaller buffer size, which indicates that 3-hop scheme brings less backlog than 2-hop scheme.

In 2-hop scheme, the buffer size keeps unchanged in region $\alpha \in [0, 1]$ while decreases as $\alpha$ in region $\alpha \in (1, \infty)$, which has the same tendency as transmission opportunities. This is essentially due to the fact that the average transmission time remains the same.

In 3-hop scheme, buffer size decreases as $\alpha$. Here we also present an intuitive explanation. In region $\alpha \in [0, 1]$, ordinary cells contains a majority of nodes, ensuring constant transmission opportunities. When $\alpha$ increases in this region, more nodes gathered in popular cells enable quicker acceleration. Thus the time required for a transmission becomes shorter. The factors above result in less average backlog and thus the needed buffer size minish-
es. When $\alpha$ increases in region $\alpha \in (1, \infty)$, an increasing number of cells will become idle and thus transmission opportunities sharply decreases. Meanwhile, since a large number of nodes crowd into popular cells, for each node the waiting time for acceleration becomes longer. Nevertheless, as transmission opportunities decay at a greater rate than the growth of waiting time, the backlog in buffer continues to fall.

4.2 Impacts of Location Popularity on the Minimum Delay

From Figure 6, we can observe that the minimum delay of two schemes decreases as $\alpha$. Intuitively, the reason is as follows: As $\alpha$ increases, more nodes move to popular cells. The encountering probability of two nodes thus increases. More duplicates will also generate in popular cells under redundancy scheme. The factors above will improve the transmission efficiency of a single S-D pair. Furthermore, redundancy scheme outperforms non-redundancy scheme for any $\alpha$, which indicates that employing transmission redundancy can help reduce delay in location-based scenarios. It is worth to note that permitting transmission redundancy does not work in uniform scenarios since homogeneous node distribution substantially weakens the effect of acceleration.

Impacts of $\alpha$ on the Minimum Delay of Non-Redundancy Scheme: The minimum delay of non-redundancy scheme is determined only by one factor: the encountering opportunity of two nodes presented in Figure 7(a).

When $\alpha = 0$, our results can recover Neely’s scaling laws [13]: Algorithms which do not use redundancy cannot achieve an average delay of less than $\Theta(n)$. When $0 < \alpha \leq \frac{1}{2}$, the minimum delay is $\Theta(n)$ due to the fact that location diversity is moderate and nodes’ encountering probability remains $\Theta(\frac{1}{n})$ in this region. When $\frac{1}{2} < \alpha < 1$, the minimum delay decays as $\alpha$, since location diversity helps increase nodes’ encountering opportunity in this region. When $\alpha = 1$, it is possible to achieve the best delay $\Theta(1)$, which is essentially due to the fact that the encountering probability of any two nodes is $\Theta(1)$ in this case. And such $\Theta(1)$ probability also holds in region $\alpha \in (1, \infty)$.

Impacts of $\alpha$ on the Minimum Delay of Redundancy Scheme: Under the redundancy scheme, in order to achieve the minimum delay, $n_2$ (the average number of nodes in popular cells), $E[n_p]$ (the number of popular cells) and $\rho$ (the average encountering probability of two nodes) should all vary with the popularity exponent $\alpha$, as shown in Figure 7.

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{fig6.png}
\caption{Minimum delay of two schemes for different values of $\alpha$: non-redundancy scheme and redundancy scheme.}
\end{figure}

When $\alpha = 0$, redundancy scheme achieves the same minimum delay as non-redundancy scheme, verifying that transmission redundancy cannot help reduce delay in uniform scenarios. When $0 < \alpha \leq \frac{1}{2}$, $n_2$ increases as $\alpha$
while $E[n_p]$ remains unchanged. Hence, the probability of moving to some popular cell increases (a better first-hop delay) and more duplicates are generated. Even though the encountering probability remains $\Theta(\frac{1}{n})$, the time to complete a R-D transmission becomes shorter. When $\frac{1}{2} < \alpha < \frac{3}{4}$, $E[n_p]$ increases as $\alpha$ while $n_2$ stays the same. Thus, the total number of nodes in popular cells becomes larger (a better first-hop delay). Since $p'$ continues to increase in this region, the second-hop delay is also improved. Note that when $\frac{3}{4} \leq \alpha \leq 1$, we can obtain the best possible delay $\Theta(1)$ by setting $E[n_p] = \Theta(n)$, which implies that all the cells are popular cells (a $\Theta(1)$ first-hop delay). Since the sum encountering opportunity of $n_2$ relays and the destination is $\Theta(1)$, the second-hop delay is $\Theta(1)$. When $\alpha > 1$, a $\Theta(1)$ delay can be achieved for any $E[n_p]$, due to the fact that the average number of relays reaches $\Theta(n)$ and the encountering opportunity of $R$ and $D$ is $\Theta(1)$.

![Fig. 7: Optimal values for three factors (i.e., the encountering probability of two nodes, the average number of nodes in popular cells, the number of popular cells) corresponding to $\alpha$.](image)

### 4.3 Delay-Capacity Tradeoffs

In this subsection, we discuss how location popularity affects the delay-capacity tradeoff, which is established when relay redundancy $m$ varies from 1 to $n$.

![Fig. 8: Tradeoffs for different regions of $\alpha$. We set $\alpha = 0.5$, 1, 1.5 to represent these three regions, respectively.](image)

As illustrated in Figure 8, an optimal tradeoff can be achieved for $\alpha = 1$. When $\alpha$ decreases in region $\alpha \in [0, 1]$, the tradeoff curves move rightwards, indicating a worse tradeoff. As for $\alpha \in (1, \infty)$, the tradeoff also declines as $\alpha$ increases. The reason is as follows. In region $\alpha \in [0, 1]$, due to a moderate network diversity, there is only a small fraction of nodes in popular cells. While the capacity is mainly contributed by the transmissions in ordinary cells, a constant capacity can thus be guaranteed. Meanwhile, as $\alpha$ decreases, location diversity becomes more smooth, bringing about more uniform node visitations. The effect of acceleration occurred in popular cells thus be weakened, and for a given packet, less duplicates are generated. Thus, the time to complete a transmission becomes longer. In region $\alpha \in (1, \infty)$, as $\alpha$ increases, location diversity becomes more sharp. Then a small fraction of popular cells will occupy the majority of nodes, resulting in both degraded capacity and delay. Only when $\alpha = 1$, transmissions between ordinary cells and popular cells are balanced, and thus an optimal tradeoff between capacity and delay can be achieved.

By letting $\alpha = 0$, our results can recover Neely's scaling laws [13]: $\Theta(1/m)$ per-node capacity with an average delay of $\Theta(n/m)$ under i.i.d. mobility model. Moreover, Figure 8 shows that compared with [13], we can achieve a better tradeoff when $\alpha \leq 1$, i.e., $\frac{\text{delay}}{\text{capacity}} \geq \Theta(\sqrt{n})$. It further demonstrates that by exploiting location popularity, the network performance of MANETs can be significantly improved.

Besides, it is notable that region 1 and region 2 refer to the situation that the network cannot stably support the 3-hop store-carry-accelerate-forward paradigm. Thus it degenerates into the 2-hop store-carry-forward paradigm in this case. For $0 < \alpha < 1$, it achieves the same behavior as that of [13] in region 1. And for $\alpha > 1$, the performance deteriorates further in region 2 since acceleration does not work and a large number of nodes in popular cells cannot be utilized.

### 5 Conclusion and Future Work

In this paper, we investigate the impacts of location popularity on scaling laws of MANETs. First, we adopt traditional 2-hop store-carry-forward paradigm to analyze the delay and capacity, and find that the performance is worse than that of uniform scenarios. By proposing a 3-hop store-carry-accelerate-forward paradigm utilizing potential transmissions in popular cells, we show that the delay can be reduced up to a factor of $\Theta(\sqrt{n})$ without sacrificing the capacity for any $\alpha$. It indicates that exploiting location popularity can largely improve the performance in MANETs. Besides, our tradeoff under 3-hop relay algorithm is better than that of uniform scenarios when $\alpha \leq 1$. And our results can cover the uniform scenario by setting $\alpha = 0$. Moreover, an optimal performance can be achieved when $\alpha = 1$, with an almost constant capacity and $\Theta(\sqrt{n})$ delay. Therefore, our analysis may provide insight on the design and deployment of large-scale location-based networks.

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