Capacity Scaling in Mobile Wireless Ad Hoc Network with Infrastructure Support

Wentao Huang¹  Xinbing Wang¹  Qian Zhang²

¹Department of Electronic Engineering
Shanghai Jiao Tong University

²Dept. of CS and Engi
HK Univ of Science & Technology

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Outline

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   - Methods to Increase Capacity
   - The General (Unanswered) Problem

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   - Mobility Model

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   - Weak Mobility
   - Trivial Mobility
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Capacity of wireless ad hoc network not scalable: in a static ad hoc wireless network with $n$ nodes, the per-node throughput is limited as $O\left(\frac{1}{\sqrt{n}}\right)$.

Interference is the main reason behind.

In mobile ad hoc wireless networks with $n$ nodes, the per-node throughput remains constant $\Theta(1)^{[2]}$.

Full mobility is assumed: The stationary distribution of each node is uniform over the whole network.

A Store-carry-forward communication scheme exploit node mobility to carry traffic across network.

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Partial Mobility

- Even if node mobility does not cover the whole network area, it still helps.
- If mobility range is limited to $f(n)$ portion of the network diameter, per-node capacity is $\Theta(f(n))$.[3]

In static ad hoc network with $n$ wireless nodes and $k$ base stations, the per-node capacity is $\Theta(k/n)^{[4]}$.

Assume base stations have unlimited bandwidth and $\Theta(n) > k > \Theta(\sqrt{n})$.

Main Question

What is the capacity if the network features both mobility and infrastructure?
How to achieve capacity?
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Assumptions

- $n$ wireless nodes moving over closed connected region
- Independent, stationary and ergodic mobility processes
- Uniform permutation traffic: each node is origin and destination of a single traffic flow with rate $\lambda(n)$
- Omni-directional antennas and a single wireless channel with bandwidth $W$ bps
- $k = n^K$ static inter-connected base stations
- Bandwidth between base stations are $c(n)$ bps, and $kc = n^\phi$
Definition

Let $d_{ij}$ denotes the distance between node $i$ and node $j$, and $R_T$ the common transmission range, then a transmission from $i$ to $j$ at rate $W$ is successful if:

$$\begin{cases} 
    d_{ij} < R_T \\
    d_{kj} > (1 + \Delta)R_T
\end{cases}$$

for any other $k$ transmitting simultaneously.
Asymptotic Capacity

Definition

Asymptotic per-node capacity \( \lambda(n) \) of the network is said to be \( \Theta(g(n)) \) if there exist two positive constants \( c \) and \( c' \) such that:

\[
\begin{align*}
\lim_{n \to \infty} \Pr \{ \lambda(n) = cg(n) \text{ is feasible} \} &= 1 \\
\lim_{n \to \infty} \Pr \{ \lambda(n) = c'g(n) \text{ is feasible} \} &< 1
\end{align*}
\]
Home-point Based Mobility

Every node has a “home-point” and tends to move around it.
Home-point Based Mobility

The shape of the spatial stationary distribution of each node’s presence is an arbitrary, non-increasing function $s(d)$ of the distance from the home-point.
Distribution of Home-points

Clustering home-points characterizes the heterogeneous node density.

Uniform model

Clustered model
Scaling the Network Size

- Network size grows as more nodes are joining.

- Assume that $L = f(n) \sim n^\alpha \quad \alpha \in [0, 1/2]$

constant size, increasing density
Scaling the Network Size

- Network size grows as more nodes are joining.

- Assume that \( L = f(n) \sim n^\alpha \) \( \alpha \in [0, 1/2] \)

\[ L \]

\[ \uparrow \]

*increasing size, constant density*
Scaling the Network Size

- Network size grows as more nodes are joining.

\[ L = f(n) \approx n^\alpha \quad \alpha \in [0, 1/2] \]

- Node mobility does not depend on network size.
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Main Results

If the network is uniformly dense, then per-node capacity is $\Theta(1/f(n)) + \Theta(\min(k^2c/n, k/n))$.

Recall that $f = n^\alpha$, $k = n^K$ and $kc = n^\varphi$. 
Uniformly Dense Networks

**Definition**

*Local density* of nodes at point $X$ is defined as:

$$
\rho(X) = \sum_{i=1}^{n+k} E[1_{Z_i \in B(X, 1/\sqrt{n})} | \mathcal{F}_{n+k}]
$$

where $B(X, 1/\sqrt{n})$ is the disk centered at $X$ with radius $1/\sqrt{n}$, and $\mathcal{F}_{n+k}$ are the locations of all wireless nodes and BSs.

**Definition**

A network is said to be *uniformly dense* if for any $X \in \mathcal{O}$, there exist two positive constants $h$ and $H$, such that

$$
h < \rho^n(X) < H \quad \text{w.h.p.}
$$
Even though the home-points may be clustering, the local node density is possible to be (almost) uniformly dense.
Theorem

If \( f(n) \sqrt{\gamma(n)} = o(1) \), where \( \gamma(n) = \frac{\log m}{m} \), and \( k = O(n) \), then the network is uniformly dense.
The optimal transmission range is $R^*_T = \Theta(1/\sqrt{n})$. 

Based on a simple scheduling policy with optimal range $R^*_T$, for any pair of nodes $(i,j)$, the link capacity between them is

$$
\mu(i,j) = \Theta \left( \Pr \left\{ d_{ij} \leq \frac{1}{\sqrt{n}} \right\} \right)
$$
Link capacities can be evaluated in terms of contact probabilities: 
\[ \mu(i, j) = \Theta \left( \Pr \left\{ d_{ij} \leq \frac{1}{\sqrt{n}} \right\} \right) \]

We can construct a random geometric graph:
- vertices stand for home-points of the nodes
- edges are weighted by link capacity \( \mu_{ij} \)

Network capacity is obtained by solving the maximum concurrent flow problem over the constructed graph.
Upper Bound of Capacity

Upper bound of capacity: network cut
Upper Bound of Capacity
Upper Bound of Capacity

We need to compute the total link capacity crossing the cut
- Red edge: wireless node to wireless node
- Yellow edge: wireless node to BS
- Black edge: BS to BS

Proof’s Idea: Tessellate the network into squares. Upper and lower bound the number of BSs within each square. Upper and lower bound the distance from a particular node’s home-point to this square. Sum over all squares and turn it into integral.
Mobility dominant state: capacity \( \lambda = \Theta(1/f(n)) \).
Lower (Achievable) Bound of Capacity

Infrastructure dominant state: capacity $\lambda = \Theta \left( \min \left\{ \frac{k^2 c(n)}{n}, \frac{k}{n} \right\} \right)$. 

- Infrastructure dominant state: capacity $\lambda = \Theta \left( \min \left\{ \frac{k^2 c(n)}{n}, \frac{k}{n} \right\} \right)$. 

Source

Cell Area: $\Theta(1)$

Destination
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Weak Mobility

- Occurs when node mobility is not strong enough to offset clustering.
- But within a cluster, mobility still plays a significant role to improve capacity.
- Therefore we use infrastructure to deliver inter-cluster traffic, and use mobility to deliver intra-cluster traffic.
Definition

Let $\sqrt{\tilde{\gamma}(n)} = r \sqrt{\frac{\log(n/m)}{n/m}}$. If $f(n) \sqrt{\tilde{\gamma}(n)} = o(1)$, $f(n) \sqrt{\gamma(n)} = \omega(1)$, then the network falls into weak mobility regime.

Theorem

*Under weak mobility, per-node capacity is $\lambda = \Theta \left( \min \left( \frac{k^2 c}{n}, \frac{k}{n} \right) \right)$. The optimal transmission range is $R_T = r(n) \sqrt{\frac{m(n)}{n}}$.***
Node mobility is so weak that even within a cluster the node density is not uniformly dense.

Mobility will not have any effect on capacity, either inter-network traffic or intra-network traffic.

Communication schemes of static networks can be readily applied.
Definition

If \( f(n) \sqrt{\gamma(n)} = \omega(\log \frac{n}{m}) \), then the network falls under trivial mobility regime.

Theorem

*Under trivial mobility, per-node capacity is* \( \lambda = \Theta \left( \min \left( \frac{k^2c}{n}, \frac{k}{n} \right) \right) \).

*The optimal transmission range is* \( R_T = r(n) \sqrt{\frac{m(n)}{k}} \).
Summary

- Mobile ad hoc network with infrastructure support
- Node mobility divided into strong, weak and trivial cases
- Network capacity is determined for each case, and optimal communication schemes are proposed.
Thanks for listening.