QoS-splitting for RAT Selection in Heterogeneous Networks

ABSTRACT

Today, heterogeneous network is becoming more and more crowded with a huge increment of users and their diverse requirements. These increasing requirements turn to be a big challenge to traditional Quality of Service (QoS) based resource management. Fortunately, the emergence of mobile terminals with multiple radio-access technologies (RATs) and multi-homing functionalities ensures users to feasibly access to more than one networks. Case exists when a single network cannot meet the user's QoS requirement. To this end, we propose the concept of "QoS-splitting". When no single channel can satisfy the QoS requirement, user can connect to another network simultaneously to share his load. Our system model is originated from the QoS satisfaction game, but with split demand. To approach the Nash Equilibrium, we propose a satisfaction-guaranteed better reply (SBR) algorithm. The convergence property of SBR is proved by using quasi-potential function. Specifically, when given a complete interference graph, preference-based matching (PM) algorithm is proposed to obtain a more stable equilibrium. In this case, users can approach a unique pure Nash equilibrium. Simulation results illustrate the efficiency of SBR and PM algorithms. Moreover, it shows that more crowded the system is, more improvement in the number of satisfied users and social welfare will be achieved.

Keywords

QoS, RAT, Heterogeneous Network

1. INTRODUCTION

The current highly developed wireless technologies and the explosion of wireless users render the heterogeneity of the network an unavoidable trend. At the same time, more and more mobile terminals are equipped with multiple wireless access network interfaces in order to access the Internet through the heterogeneous networks (HetNets). There are typically two different types of the terminals with multiple interfaces:1) multi-mode mobile terminals; 2) multi-homed mobile terminals.

The widely used multi-mode terminal enables users to switch from one radio-access technology (RAT) to another, which is called vertical handover. This increases the flexibility of users and increases the total throughput obtained by each user. The *multi-homed terminal* means a terminal with multiple connections to multiple networks simultaneously and it enables the user to use multiple interfaces to share load at the same time.

Many researches have been focused on RAT selection in Het-Nets – how to select the most appropriate network to reach the requirement of Always Best Connected (ABC) [1] in either distributed manner or centralized manner. In previous work, no matter based on the centralized manner in which a central controller distributes the users or the distributed manner where users choose the network according to their own utility, only one RAT can be used at any given time. In this case, when a user's demand is too high to be met in any single network, he cannot be satisfied anymore.

In this paper, we propose the concept of *QoS-splitting* to further increase the capacity of the HetNets. When a user is dissatisfied, he can split his QoS demand and share part of his load by connecting to a second network. The QoS requirement for each single network is reduced and hence the user has a larger possibility to be satisfied. And we utilize a newly proposed game to model the distributed RAT selection problem–QoS satisfaction game [2], but with split demand. The main idea behind the QoS satisfaction game is that there are many players with diverse QoS demands and each of them wants to select a channel to use. When the channel data rate is high enough to satisfy his requirement, the user reaches its unit utility. And the data rate each user achieved depends on the congestion level–how many users are competing for the channel. Because of the split demand, the distributed algorithm proposed in [2] cannot work anymore. How to select the networks with appropriate ratio to maximize each user's utility is the main challenge when split demand is introduced in our system. We propose the satisfaction-guaranteed better reply (SBR) algorithm. We prove that there is a quasi-potential function with finite potential decrease for SBR, therefore the convergence property is guaranteed. Quasi-potential game and quasi-potential function are the concepts proposed in this paper acting as the extension of the classical potential game [3]. Later for a special case of complete interference graph, we propose another algorithm preference-based matching (PM), which can converge to a unique Nash equilibrium. In simula-

tion, we show that an obvious increase (several times better than) in the number of satisfied users and social welfare.

1.1 Related Work

Many researches have been conducted on the network selection or RAT selection in HetNets. Systematic tutorial on mathematical theories for modelling the network selection problem has been offered in a survey [4]. In [5], a central controller distribute the users to optimize some attributes. Network-assistant distributed algorithm has been proposed in [6], which is based on trial and error mechanism. A user-centric approach has been proposed in [7], in which the user makes decisions to select the best access networks without any signalling or coordination among the networks.

The game model is often congestion game [8] [9] in many analysis about multi-RATs selection in HetNets. Many papers have studied the convergence property of different sub-classes of this game. Recently, a more practical game model has been proposed in [2] – Quality of Service game. Unlike in the congestion game when users will go on selecting the resource with the least congestion level, users in the QoS satisfaction game will be satisfied and reluctant to deviate if the congestion level is smaller than certain threshold. The convergence is based on the demonstration of finite improvement property by utilizing potential game.

1.2 Contributions

Our main results and contributions can be summarized as follows:

- We propose the concept of QoS-splitting. By sharing certain ratio of the load with the second network, the user can be satisfied even in the situation when no single resource can meet his original demand;

- We use the QoS satisfaction game to model our system and propose the satisfaction-guaranteed better reply algorithm for general cases with frequency spatial reuse. The convergence property is proved by utilizing quasi-potential function.

- We propose the concept of quasi-potential game and prove it can converge under certain conditions.

- For a special case of complete interference graph, we propose the preference-based matching algorithm. This algorithm can reach a unique Nash equilibrium.

The rest of the paper is organized as follows. We present our system model in Section 2. In Section 3 we propose two distributed algorithms and investigate the convergence property. We then evaluate our algorithms' performance by simulations in Section 4. Finally we conclude our paper in Section 5.

2. SYSTEM MODEL

2.1 Network Model

We consider a heterogeneous network environment which has A channels of Wi-Fi, B channels for LTE and N users. For convenience, we will use "resource" in the following part to represent the collective set of Wi-Fi channels and LTE channels and we use resource A, resource B to represent the set of Wi-Fi channels and the set of LTE channels respectively. The set of resource A, resource B and users are denoted by $\mathbf{A} = \{0, 1, 2, ..., A\}, \mathbf{B} = \{0, 1, 2, ..., B\}$, and $\mathbf{N} = \{1, 2, ..., N\}$, respectively. $0 \in \mathbf{A}$ and $0 \in \mathbf{B}$ mean "virtual channel". When user connecting to virtual channel, they actually do not select any channel. And we use $\mathbf{R} = \mathbf{A} \cup \mathbf{B}$ to denote the set of resource, where $r \in \mathbf{R}$ can be either $a \in \mathbf{A}$ or $b \in \mathbf{B}$.

Users are static and heterogeneous, which means they have different throughput by connecting to different resources due to their location-differences. We introduce the interference graph to show the impacts of distances on BSs, APs and users. $G = (\mathbf{N}, \boldsymbol{\varepsilon})$ is an undirected and unweighted graph where $\mathbf{N}$ is the set of users and $\boldsymbol{\varepsilon}$ is the set of edges. For any pair of users $m, n \in \mathbf{N}, \exists\{m, n\} \in \boldsymbol{\varepsilon}$ if and only if user m and user n are close enough to cause congestion or interference when connecting to the same channel. Followed by this concept, we introduce the "neighboring" set $\mathbf{Ne}_n = \{m : \{m, n\} \in \boldsymbol{\varepsilon}\} \cup \{n\}$. In other words, user n's neighboring set contains users that will interfere with n if they choose the same channel.

We assume that each user has two connections and can access to at most two resources at any given time, which is different from that in a traditional single-RAT model. $\mathbf{x} = \{x_1, x_2, ..., x_N\} \in \mathbf{A}^N$ and $\mathbf{y} = \{y_1, y_2, ..., y_N\} \in \mathbf{B}^N$ are the state of channel selection of resource A and resource B respectively, where $x_n \in \mathbf{A}$ and $y_n \in \mathbf{B}$. Similarly, for the convenience in the following statement, we use $z_n \in \mathbf{A} \cup \mathbf{B}$ to denote that user n chooses one of the resources, either a or b.

2.2 Throughput Model

The throughput a user can achieve varies with the networks, the location, the transmission rate and the congestion level of the channel (i.e. how many users are connecting to the same resource).

Different medium access control (MAC) protocols of the different access networks in HetNets shape different throughput curves. Here we use $Q_n(r)$ to represent the QoS of user n when connecting to resource r. Specifically, we have

$$Q_n(r) = B_n^r \cdot f(\mathbf{\Lambda}_n^r), \tag{1}$$

where B_n^r is the mean channel throughput of user n on resource r, $f(\cdot)$ is the "influence function" which demonstrates the influence of other users who select the same channel. $\mathbf{\Lambda}_n^r = \{n \in \mathbf{Ne}_n : z_n = r\}$ is the set of users who will interfere with user n.

In our model, different users may have different B_n^r on the same resource and each user also may have different B_n^r on different resources. This is more practical in analyzing the real world issue where there may be diversity in user's transmission technologies and the environmental effects. We use the Shannon Capacity to get B_n^r

$$B_n^r = W_r \log_2 \left(1 + \frac{\zeta_n \cdot g_n^r}{\omega_n^r}\right), \tag{2}$$

where W_r is the bandwidth of channel r, ζ_n is the transmission power adopted by user n, g_n^r is the user-specific channel

Table 1: Main Notations

N	User set
A	Set of resource A
B	Set of resource B
x_n	User n's selection on resource A
y_n	User n's selection on resource B
q_n	Ratio on resource A of user n
s	Set of strategies
T	Set of thresholds
G	Interference graph
$\mathbf{Ne}_n$	User n's neighbouring set
$\mathbf{\Lambda}_n^r$	Set of users choosing resource r and will interfere with n
I_n^r	Congestion level for user n on resource r
$Q_n(r)$	Throughput function

gain and ω_n^r is the power of noise.

The "influence function" varies according to the access protocols. Here we mainly introduce two access mechanisms which can be applied to Wi-Fi network and LTE network, respectively.

- *Persistence-probability-based Random Access Mechanism*

  Under this mechanism, each user n will contend for the channel r with probability $p_n \in (0,1)$. If multiple users are contending for the same channel, a collision will occur and nobody can transmit. The only situation that a user can transmit is when nobody else will contend for this channel. We can compute the long-run expected throughput of each user n choosing resource r as follows [10]:

$$Q_n(r) = B_n^r \cdot p_n \cdot \prod_{i \in \mathbf{\Lambda}_n^r \setminus \{n\}} (1 - p_i). \qquad (3)$$

- *Time-fair TDMA MAC*

  Unlike in the last mechanism, where the QoS depends on the specific combination of other users who share the resource r, in this mechanism, the QoS of user n on resource r only depends on the total number of users that selects resource r. Here the time accessing to the wireless medium is fairly-shared among all the users such that each user has the same time span. Therefore, the throughput of a user n connected to a resource r is given by [7]:

$$Q_n(r) = \frac{B_n^r}{|\mathbf{\Lambda}_n^r|}. \qquad (4)$$

2.3 QoS Satisfaction Game

To model our system, we introduce the framework of QoS satisfaction game [2], which is a modified version of the classical congestion game. In the QoS satisfaction game model, there is a demand D_n^r for every user. Each user will try to select the resource with less user sharing to make himself satisfied (i.e. the user's throughput is larger than or equal to his demand). When the user is satisfied, he will be reluctant to change his strategy. Since the throughput function is always a monotonic curve along with the congestion level, we can find the congestion level where the corresponding throughput equals to the demand. This particular congestion level is called the "Threshold" T_n^r.

The QoS satisfaction game is modeled as the tuple $(\mathbf{N}, \mathbf{R}, \mathbf{T})$. $\mathbf{N} = \{1, 2, ..., N\}$ is the set of users, $\mathbf{R} = \{1, 2, ...R\}$ is the set of resources and $\mathbf{T} = \{\mathbf{T}_n^r : n \in \mathbf{N}, r \in \mathbf{R}\}$ is the set of thresholds. The set of strategy profiles is $\mathbf{s} = \{s_1, s_2, s_3, ..., s_N\}$, where each user n chooses a strategy $s_n \in \mathbf{R}$. Since the user is either satisfied or dissatisfied, we have the utility function as follows:

$$U_n(s) = \begin{cases} 1, & \text{if } T_n^{s_n} \geq I^{s_n}, \\ 0, & \text{if } T_n^{s_n} < I^{s_n}, \end{cases} \qquad (5)$$

where $I^r = |\{n \in \mathbf{N} : s_n = r\}|$ is the congestion level of resource r.

We then introduce the concept of QoS-splitting so that more user can be satisfied.

DEFINITION 1 (**QoS-splitting**). *A user is performing* QoS-splitting *when he shares his load by connecting to a second network to split the QoS demand into two smaller ones.*

We can easily see that a lower QoS demand means a larger threshold. In other words, the user can tolerant more "neighbor" users by lowering his QoS demand. We use q_n to denote the ratio of user n choosing resource A and $\mathbf{q} = \{q_1, q_2, ..., q_N\}$. Specifically, we divide the ratio into 11 levels $q_n \in \{0, 0.1, 0.2, 0.3, ..., 0.9, 1\}$. Then our threshold becomes ratio-dependent. We use $T_n^r(q_n)$ to define user n's threshold when he chooses resource r with ratio q_n and this can be computed by the throughput function $Q_n(\cdot)$ mentioned in section 2.2. Specifically for $q_n = 0$, we define $T_n^r(q_n) \equiv N$.

As we have mentioned before, users have the network they most prefer. So when they connect to some network they do not like, the "cost" should be added in our analysis. The more ratio of resource B is, the higher the cost will be. The cost reaches its peak value when $q_n = 0$ (i.e. user n only access to resource B). However, no matter how large the cost is, the utility of a satisfied user should be larger than the dissatisfied one's. Considering the above properties, we define our utility for a satisfied user to be $(1 + q_n) \times 0.5$.
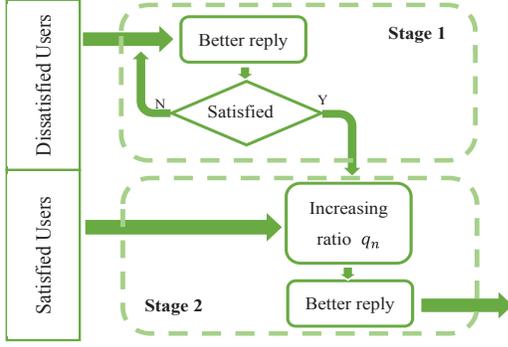
Figure 1: Update Scheme.

Our system can be modeled by a tuple: $\{\mathbf{N}, \mathbf{A}, \mathbf{B}, \mathbf{T}, G\}$. Unlike the original QoS satisfaction game which is single-resource selection, users are allowed to use at most two RATs in our system. So we define our strategy profile $\mathbf{s}$ to be $\mathbf{s} = \{s_1, s_2, ..., s_N\}$ where $s_n = (x_n, y_n, q_n)$. What's more, the user is said to be satisfied if and only if all the resource he selects can satisfy his demands at the same time. Thus we have our utility function as follows:

$$U_n(\mathbf{s}) = \begin{cases} (1 + q_n) \times 0.5, & \text{if satisfied,} \\ 0, & \text{otherwise,} \end{cases} \quad (6)$$

where satisfied means $T_n^{x_n}(q_n) \geq I_n^{y_n} \wedge T_n^{y_n}(1 - q_n) \geq I_n^{y_n}$. Here, we add subscript n to the congestion level because our game model is a spatial one. Interference graph is included in our analysis, so the congestion level should be user-specific, $I_n^r = |\mathbf{\Lambda}_n^r|$.

The goal of each user is to maximize their own utility U_n.

All the main notations have been listed in Table 1.

3. GAME SOLUTIONS AND ALGORITHMS
In this section we propose two different solutions to our game model. They all take the distributed manner to reach a stable state where nobody will tend to change his strategy.

3.1 Satisfaction-guaranteed Better Reply
For each user, their goal is to maximize their own utilities. The user wants to increase his own utility under the following two situations:

   i. The user is dissatisfied and wants to change his strategy to be satisfied;

   ii. The user is satisfied but he wants to minimize the cost contributed by resource B.

Thus, the update scheme of every single user can be divided into two stages just as Fig. 1 illustrates. In the first stage, users perform better reply update asynchronously (i.e. one by one at each time slot). Their goal in the first stage is to make themselves satisfied regardless of the cost. When a user is satisfied, he may still want to update in order to minimize the cost given by resource B. From the utility function,

we can see that the minimum utility increase is 0.5 when a dissatisfied user becomes satisfied, but the maximum utility increase is 0.5 for a satisfied user performing update to increasing his ratio. This implies that being satisfied is much more important than purely reducing the cost. Thus, we utilize the *satisfaction-guaranteed better reply* scheme. The key idea is that users can increase the ratio in resource A only when he has been satisfied.

Algorithm 1 Satisfaction-guaranteed BR (SBR)

1: **while** 1 **do**
2:   **for** user $n = 1, N$ **do**
3:     **while** user n is dissatisfied **do**
4:       Change his strategy while keeping the ratio the same;
5:       **if** user n is satisfied $\vee$ the ratio cannot be adjusted **then**
6:         Break;
7:       **else**
8:         Adjust the ratio;
9:       **end if**
10:     **end while**
11:   **end for**
12:   **if** Nobody changes his strategy **then**
13:     Break;
14:   **end if**
15:   **while** user n is satisfied $\wedge$ ratio $q_n < 1$ **do**
16:     Increasing the ratio by 0.1 $q_n = q_n + 0.1$;
17:     Compute the set of channels that make the user n satisfied $\mathbf{\Upsilon}_n(\mathbf{s})$;
18:     **if** $\mathbf{\Upsilon}_n(\mathbf{s}) \neq \emptyset$ **then**
19:       Continue;
20:     **else**
21:       Stick to the previous channel;
22:       Break;
23:     **end if**
24:   **end while**
25: **end while**

3.1.1 Dissatisfied Stage
In this stage, each dissatisfied user will try their best to make themselves satisfied. In other words, every dissatisfied user will take turns to perform better reply to increase their utilities. At the beginning of each time slot, user n will try to find a channel that meets his demand at the current ratio. If no such channel exists, the user will try to change his ratio, lowering his QoS demand in resource A ,increasing his threshold in resource A until he is satisfied (if the resource B is dissatisfying, he will lowering the demand in resource B to increase his threshold in resource B). If the user cannot be satisfied at any ratio in either resource A or resource B, he will stick to his old strategy.

3.1.2 Cost-minimizing Stage
Because connecting to resource B will add certain cost to the user's utility, each user wants to increase his ratio in resource A to minimize the cost under the satisfaction-guaranteed circumstance. For each satisfied user who connects to two resources at the same time, they will first try to increase their ratio q_n by 0.1. Due to the increase in the ratio, their thresholds for resource A will decrease accordingly. So

their current channel may be inappropriate (i.e. $I_n^r > T_n^r$). Then they will try to find a channel in resource A that can meet their demand. If there exists such channels, they will transfer to one of them and preparing for the next turn of ratio-increasing. If no channel can satisfy the user with new thresholds, the user will give up updating and stick to his old strategy.

THEOREM 1. *The Satisfaction-guaranteed better reply can converge to Nash equilibria from any initial stages.*

The proof is given in Appendix.

As a solution to this problem, we propose an algorithm shown in Algorithm 1.

3.2 Preference-based Matching

In this subsection, we consider the special case of complete interference graph and propose another algorithm that can converge to a unique Nash equilibrium. So the congestion level I_n^r is degenerated into I^r.

As we have already mentioned, users and channels have different goals in our system. This is quite similar to College Admissions Games [11] [12] except for the fact that we do not have a quota for each channel. In College Admissions Game, each student has a preference list which ranks the colleges and colleges also have their own preference list over students. It is of interest to study how we assign students to colleges while satisfying, as much as possible, all preferences.

In our system, each user can make their preference list based on their thresholds of different channels. This is reasonable because what the user wants is to be satisfied. In other words, the user does not care about exactly how crowded this channel is, as far as his threshold is large enough. Thus, choosing a channel with higher threshold means the user can tolerate a more severe congestion level. For channels, their goal is to provide high QoS to as many users as possible. Compared with connecting to a user with low threshold, connecting to the one with high threshold allows the channel to transmit data for more users. Therefore, users' threshold is the criterion for making the channel's list.

At the initial stage, all users will propose to the channel that ranks the top. According to the users' thresholds, each channel ranks the users that propose to them, such as $T_{n_1}^r, T_{n_2}^r, \dots$. The channel approves the proposal of top I^r users, where $T_{I^r} \geq I^r > T_{I^r+1}$, and rejects the others.

In the following stages, these rejected users will go on to propose to their second preferred channels, third preferred channels and so on until they are accepted.

When they have been rejected by the last preferred channel on their list, they begin QoS-splitting, decreasing their ratio q_n by 0.1 and propose to the most preferred channel again.

Users who are thrown out of their previous channel due to the new comers will join in the propose process in the next time slot.

Users will stop changing their strategies either because they have been satisfied or they cannot be satisfied no matter what he tries(i.e. neither changing ratio nor deviating channel is helpful).

The algorithm is shown in Algorithm 2.

Algorithm 2 Preference-based Matching (PM)

1: **for** user $n = 1, N$ **do**
2:     Make the preference list
3: **end for**
4: **while** 1 **do**
5:     Rejected users propose to the preferred channel sequentially according to their preference list;
6:     **if** User has been rejected by the least preferred channel **then**
7:         **if** The ratio can be further adjusted **then**
8:             QoS-splitting, adjust the ratio of the dissatisfying resource by 0.1;
9:         **else**
10:            Remain dissatisfied;
11:         **end if**
12:     **end if**
13:     Channels rank the user propose to them according to their thresholds;
14:     Channels accept users in the sequence of the preference list and accept as many users as possible while guaranteeing that every accepted user is satisfied.
15:     **if** Nobody has changed their strategy **then**
16:         Break;
17:     **end if**
18: **end while**

THEOREM 2. *The Preference-based Matching method can reach a unique pure Nash equilibrium.*

PROOF. First, we prove that accepting one user can at most kick off one user. Considering our "accepting criterion", we must have $T_{\min}^r \geq I^r$ or the lowest-threshold one will not be accepted, where $T_{\min}^r \equiv \min\{T_{n_1}^r, T_{n_2}^r, \dots, T_{n_{I^r}}^r\}$. When a new user is accepted, $I'^r = I^r + 1$. If $T_{\min}^r \geq I'^r$, then it is quite good that no user will be thrown out. If $T_{\min}^r < I'^r$, then the lowest-threshold user will be thrown out. After he has been thrown out, $I''^r = I'^r - 1 = I^r$, the congestion level returns to its previous value, and the remaining users will not be thrown out because $\forall T \in \{T_n^r : z_n = r\}, T \geq I^r$.

Each time a new comer is accepted, the average threshold of the channel r is increased, no matter whether the lowest-threshold user is thrown out or not. This means the channel can accept more users. More users can be accepted without decreasing the total number of satisfied users in our system.

With QoS-splitting, the threshold of user n on resource A can be increased. By doing another turn of such propose-approve process, the average threshold of the channel is raised again.

What's more, in our scheme, we decrease the ratio in resource A only under the condition that no channel can accept this user and we decrease it step by step. This is be-

cause users want to minimize their cost if they are satisfied, and they are reluctant to decrease their ratio unless they are dissatisfied.

Because the average threshold of each channel is increasing, the number of users thrown out or rejected must decrease. If the number of users is finite, there must be a state where nobody can change their strategies – the satisfied one cannot increase his ratio and the dissatisfied one cannot become satisfied anymore.

About the uniqueness in theorem 3, because at each time slot, there's only one possible behaviour of the user. When the user is dissatisfied, he will propose to the next preferred channel or change his ratio. There is only one improvement path for each user. So the final Nash equilibrium is unique. □

4. SIMULATION RESULTS

We consider a heterogeneous network with $A = 4$ and $B = 4$ channels. Since user's demand is diverse, their thresholds are a random variables that follows the uniform distribution from 0 to T_{max}, where T_{max} is the maximum threshold in our system. In a network with resource A, we adopt the persistence-probability-based random access mechanism (see section 2.2) for the medium access control. And the contention probability of each user is set to be equal $p_n = p$. In a network with resource B, we adopt the time-fair TDMA mechanism (see in section 2.2). For convenience, we use $SN(\cdot), SW(\cdot)$ to denote the number of satisfied users and the social welfare (the sum of all players' utilities, $\sum_{n=1}^{N} U_n$).

We first implement with $N = 50$ users, contention probability $p = 0.1$ and the $T_{max} = 5$ for both resource A and resource B. In SBR, users are randomly scattered over a region of $100 \times 100\text{m}^2$, and users will interfere with each other if the distance between them is equal to or less than 50m. In other words, we have an incomplete interference graph for SBR. While for PM, we use the complete interference graph. The dynamics of the number of satisfied users and the social welfare are shown in Fig. 2. This illustrates that the proposed SBR and PM can converge to a pure Nash equilibrium.

Next, we are going to check the impact of the number of users on the final results and compare our algorithms with the traditional one without QoS-splitting. The total user number $N \in \{10, 20, 30, 40, 50, 60, 70, 80, 90, 100\}$. We implement in a complete interference graph with the maximum threshold $T_{max} = 10$. The results are shown in Fig. 3.

As we can see from Fig. 3, our algorithms are better than the traditional one without QoS-splitting, in terms of the social welfare and the number of satisfied users. When N is small, $SN = SW$ and they increase as N rises. This is because the resource is not crowded and all users can be satisfied. As N grows larger, the system steps into the "saturated" state when SN cannot increase anymore. For SBR, $SN(SBR)$ and $SW(SBR)$ even begin to decrease due to the over-crowding system, but for PM, $SN(PM)$ and $SW(PM)$ can keep almost unchanged no matter how crowded the system is. We also evaluate the time steps each algorithm takes to reach Nash equilibrium. Both SBR and PM's time steps
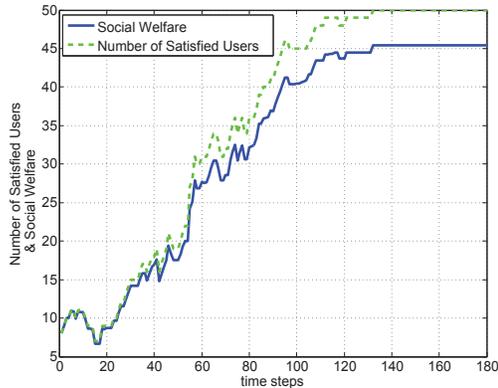
(a) Satisfaction-guaranteed Better Reply
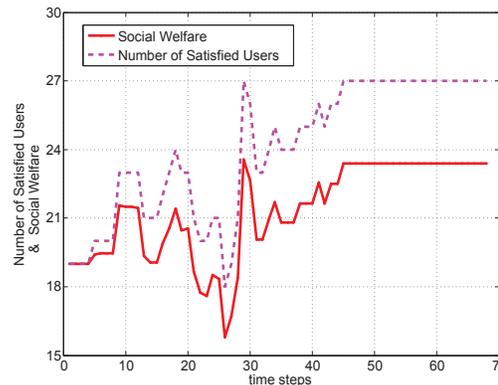
(b) Preference-based Matching

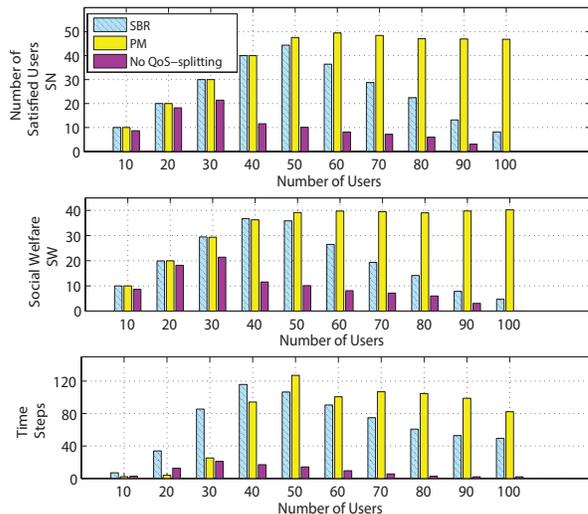Figure 2: The dynamics of the number of satisfied users and social welfare.

Figure 3: Performance Comparison of SBR, PM and the one without QoS-splitting.

increase along with the total user number N before "saturated" state and decrease afterwards. However, before the "saturated" state, PM is much efficient than SBR when they
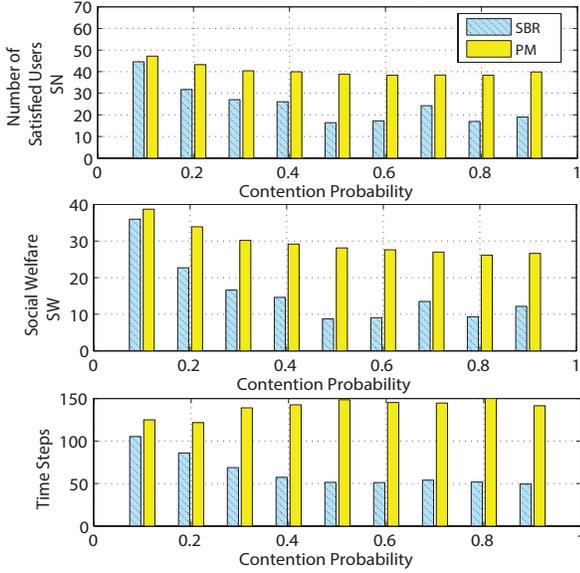
Figure 4: The impact of contention probability p_n on final results.

can reach the same SN and SW. In the "saturated" state, PM takes longer to converge. In our simulation, when being in the complete interference graph, PM outperforms SBR. This is because via SBR, the system can reach more than one Nash equilibrium. Some equilibria can improve the number of satisfied users and social welfare a lot while some can only improve a little. So generally, the final state reached by PM is more stable than SBR. However, no matter in what situation, the proposed algorithms are much better than the traditional method.

We also do simulation with the variant contention probability p ranges from 0.1 to 0.9 to see the impact it has on our final results. The number of satisfied users, social welfare and the time steps are illustrated in Fig. 4. According to our function for persistence-probability-based random access mechanism (3), higher p results in lower threshold increase when ratio of load on this network decrease by 0.1. As shown in Fig. 4, the SN and SW declines slightly as p increases. But the PM is more stable than SBR in terms of the impact of contention probability.

5. CONCLUSION

In this paper, a novel concept, QoS-splitting, to achieve higher capacity in HetNets is proposed. We first formulate the system model based on the knowledge in wireless communications and game theory, then we design a distributed algorithm via better reply. To prove the convergence property, we propose the concept of quasi-potential game, which can be utilized to prove the FIP. Later, we consider a special case of complete interference graph and propose another algorithm that can guide the users to reach a unique Nash equilibrium. The simulation results illustrate that our scheme improves the number of satisfied users and the social welfare compared with the traditional one without QoS-splitting.

6. REFERENCES

[1] E. Gustafsson and A. Jonsson, "Always best connected," *IEEE Wireless Commun.*, vol. 10, no. 1, pp. 49-55, February 2003.

[2] R. Southwell, X. Chen and J. Huang, "Quality of service games for spectrum sharing" *Proceedings of IEEE INFOCOM,* Turin, Italy, April 2013.

[3] A. Monderer and L. S. Shapley, "Potential Games," *Games and Economic Behavior,* vol. 14, no. 44, pp. 124-143, 1996.

[4] L. Wang and G. S. Kuo, "Mathematical modeling for network selection in heterogeneous wireless networks - a tutorial," Accepted for publication in *IEEE Communications Survey and Tutorials*, 2012.

[5] Q. Ye, B. Rong, Y. Chen, M. Al-Shalash, C. Caramanis, and J. G. Andrews, "User association for load balancing in heterogeneous cellular networks," in *arXiv:1205.2833v1,* March 2012.

[6] P. Coucheney, C. Touati, and B. Gaujal, "Fair and efficient user-network association algorithm for multi-technology wireless networks," in *Proceedings of IEEE INFOCOM,* Rio de Janeiro, Brazil, April 2009.

[7] E. Aryafar, A. Keshavarz-Haddad, M. Wang and M. Chiang, "RAT Selection Games in HetNets," in *Proceedings of IEEE INFOCOM*, Turin, Italy, April 2013.

[8] R. Rosenthal, "A class of games possessing pure-strategy Nash equilibria," *International Journal of Game Theory,* vol. 2, pp. 65-67, 1972.

[9] I. Michtaich, "Congestion games with player-specific payoff functions," *Games and Economic Behavior,* vol. 13, no. 1, pp. 111-124, 1996.

[10] X. Chen, J. Huang, "Distributed spectrum access with spatial reuse" *arXiv:1209.5130v1,* September 2012

[11] D. Gale and L. Shapley, "College admissions and the stability of marriage," *American Mathematical Monthly,* vol. 69, pp. 9-15, 1962.

[12] A. Roth and M. A. O. Sotomayor, *Two-Sided Matching: A Study in Game-Theoretic Modeling and Analysis.* New York, USA: Cambridge University Press, March 1992.

APPENDIX

Before proving theorem 1, we first define the *update set* $\mathbf{\Xi}_n$ and introduce the definition of *quasi-potential game*.

DEFINITION 2 (**Update Set**). *In the game* $\Gamma = (\mathbf{N}, \mathbf{s})$ *with payoff function* $U_n(\mathbf{s})$, *the update pair* (s_n, s'_n) *is the strategy pair that will increase user* n*'s payoff by deviating from* s_n *to* s'_n. *update set* $\mathbf{\Xi}_n$ *is the set of update pairs,* $\mathbf{\Xi}_n = \{(s_n, s'_n) : U_n(s'_n, \mathbf{s}_{-n}) - U_n(s_n, \mathbf{s}_{-n}) > 0, s_n, s'_n \in \mathbf{s}_n\}$.

DEFINITION 3 (**Quasi-Potential Game**). *For a game* $\mathbf{\Gamma} = (\mathbf{N}, \mathbf{s})$ *with payoff function* $U_n(\mathbf{s}) : n \in \mathbf{N}$, *where* $\mathbf{N}$ *is the set of users and* $\mathbf{s} = \{s_1, s_2, ..., s_N\}$ *is the set of strategy profiles. The update set for every user* n *is* $\mathbf{\Xi}_n$. *If* $\exists \mathbf{\Theta}_n \subset \mathbf{\Xi}_n$ *and* $\exists P : \mathbf{s} \to \Re$ *that for* $\forall n \in \mathbf{N}$ *and* $\forall \mathbf{s}_{-n}$ *that has:*

$$\begin{cases} P(s'_n, \mathbf{s}_{-n}) - P(s_n, \mathbf{s}_{-n}) > 0, & \text{for } \forall(s_n, s'_n) \in \mathbf{\Xi}_n \backslash \mathbf{\Theta}_n, \\ P(s'_n, \mathbf{s}_{-n}) - P(s_n, \mathbf{s}_{-n}) \leq, 0 & \text{for } \forall(s_n, s'_n) \in \mathbf{\Theta}_n, \\ |\mathbf{\Theta}_n| < \infty, \end{cases}$$

then we say $\mathbf{\Gamma}$ *is a* quasi-potential game *and* P *is the* quasi-potential function. *The set* $\mathbf{\Theta}_n$ *is the* decrease set.

LEMMA 1. *For any finite quasi-potential games, it possesses the finite improvement property if the quasi-potential function has bounds and the total decrease δ contributed by $(s_n, s'_n) \in \mathbf{\Theta}_n$ is finite.*

PROOF. Because the decrease is finite and the potential function has bounds, the peak value of P is sure to reach after certain steps of updates. In other words, the potential gap $P(s)_{\max} - P(s)_{\min}$ in the original game is modified to be $P(s)_{\max} - P(s)_{\min} + \delta$, then we only consider the potential-increase updates, and because potential gap is finite, the peak value is sure to be reached within finite steps. $\quad\square$

LEMMA 2. *Satisfaction-guaranteed better reply is a quasi-potential game.*

PROOF. We define a function $P(\mathbf{s}) = \sum_{n \in \mathbf{N}} F_n(\mathbf{s})$ which maps each strategy profile $\mathbf{s}$ to an real number. Here we have

$$
F_n(\mathbf{s}) = \begin{cases} F_{an}(\mathbf{s}), & \text{if } q_n = 1, \\[2mm] \dfrac{F_{bn}(\mathbf{s})}{N^2} - C(0.1), & \text{if } q_n = 0, \\[2mm] F_{an} + \dfrac{F_{bn}(\mathbf{s})}{N^2} - C(q_n), & \text{if } 0 < q_n < 1, \end{cases} \quad (7)
$$

for each users $n \in \mathbf{N}$, and $C(q_n) = 2\frac{\ln q_n}{\ln(1-p_n)}$ is the cost function, $F_{an}(\mathbf{s}) = 2T_n^{x_n}(q_n) - I_n^{x_n}$, $F_{bn}(\mathbf{s}) = 2T_n^{y_n}(1-q_n) - I_n^{y_n}$.

Next, we show that the function P is a quasi-potential function for our game model. By checking the utility function (6), we can see that a user will do better reply update in the following cases:

  i. ratio q_n does not change:
    (a) $S(a) = 0, S(b) = 0 \Longrightarrow S(a) = 1, S(b) = 1$;
    (b) $S(a) = 1, S(b) = 0 \Longrightarrow S(a) = 1, S(b) = 1$;
    (c) $S(a) = 0, S(b) = 1 \Longrightarrow S(a) = 1, S(b) = 1$;
  ii. ratio q_n decreases by 0.1:
    (a) $S(a) = 0, S(b) = 0 \Longrightarrow S(a) = 1, S(b) = 1$;
    (b) $S(a) = 0, S(b) = 1 \Longrightarrow S(a) = 1, S(b) = 1$;
 iii. ratio q_N increases by 0.1:
    (a) $S(a) = 0, S(b) = 0 \Longrightarrow S(a) = 1, S(b) = 1$;
    (b) $S(a) = 1, S(b) = 0 \Longrightarrow S(a) = 1, S(b) = 1$;
    (c) $S(a) = 1, S(b) = 1 \Longrightarrow S(a) = 1, S(b) = 1$;

where $S(a)$ and $S(b)$ represents the satisfaction degree of resource A and resource B respectively. ($S(\cdot) = 1$ denotes this resource is satisfactory and $S(\cdot) = 0$ means it is not satisfactory).

We use $\mathbf{s} = (s_n, \mathbf{s}_{-n})$ and $\mathbf{s}' = (s'_n, \mathbf{s}_{-n})$ to denote the strategy profile before and after updating.

1) For the group of updates that does not have ratio changing:

Because there is not ratio-changing, the update process in resource A and resource B are independent. And due to their similarity, we only analyze the update process in resource A here.

For a user n performing a BR update, we have:

$$
F_n(\mathbf{s}') - F_n(\mathbf{s}) = (2T_n^{x'_n} - I_n^{x'_n}(\mathbf{s}')) - (2T_n^{x_n} - I_n^{x_n}(\mathbf{s})). \quad (8)
$$

For other users, we only care about these in set $\mathbf{\Lambda}_n^{x_n}$ and $\mathbf{\Lambda}_n^{x'_n}$, because there's no influence on users who is not n's neighbour or who does not choose the channel x_n or x'_n.

For those choosing channel x_n (except user n), each of them has a potential increase by 1 due to the congestion level decrease.

$$
\sum_{m \in \mathbf{\Lambda}_n^{x_n}} F_m(\mathbf{s}') - F_m(\mathbf{s}) = I_n^{x_n} - 1. \quad (9)
$$

For those choosing channel x'_n (before updating), each of them has a potential decrease by 1 due to the congestion level increase.

$$
\sum_{m \in \mathbf{\Lambda}_n^{x'_n}} F_m(\mathbf{s}') - F_m(\mathbf{s}) = -I_n^{x'_n} + 1. \quad (10)
$$

We use $\Delta(\mathbf{s} \to \mathbf{s}')$ to show the potential change for our quasi-potential function, $\Delta(\mathbf{s} \to \mathbf{s}') = P(\mathbf{s}') - P(\mathbf{s})$. The total potential change is

$$
\Delta(\mathbf{s}' \to \mathbf{s}) = F_n(\mathbf{s}') - F_n(\mathbf{s}) + \sum_{m \in \mathbf{\Lambda}_n^{x_n} \cup \mathbf{\Lambda}_n^{x'_n}} F_m(\mathbf{s}') - F_m(\mathbf{s}).
$$
$$(11)$$

Substituting equation (8), (9), (10) into equation (11) gives:

$$
\Delta(\mathbf{s} \to \mathbf{s}') = 2(T_n^{x'_n} - I_n^{x'_n}(\mathbf{s}')) - 2(T_n^{x_n} - I_n^{x_n}(\mathbf{s})). \quad (12)
$$

Since it is a better reply for user n to deviate from channel x_n to x'_n, we must have inequalities:

$$
T_n^{x'_n} \geq I_n^{x'_n}(\mathbf{s}'), \quad (13)
$$
$$
T_n^{x_n} \leq I_n^{x_n}(\mathbf{s}) - 1. \quad (14)
$$

Thus we have:

$$
\Delta(\mathbf{s} \to \mathbf{s}') \geq 2. \quad (15)
$$

2) For the group of ratio changing:

When updating process contains ratio-changing, the time slot t consists of the following two parts:

  i. *Ratio Changing*: the user n changes his ratio q_n accordingly, $\mathbf{s} \Rightarrow \mathbf{s}''$,where $\mathbf{s} = ((x_n, y_n, q_n), \mathbf{s}_{-n})$, $\mathbf{s}'' = ((x_n, y_n, q'_n), \mathbf{s}_{-n})$;
  ii. *Better Reply*: check whether he is satisfied after ratio changing, if not, performing update $\mathbf{s}'' \Rightarrow \mathbf{s}'$, where $\mathbf{s}' = ((x'_n, y'_n, q'_n), \mathbf{s}_{-n})$.

Here because the potential value changes if and only if there are successful updates, we have $q'_n \neq q_n, x'_n \in \mathbf{A}, y'_n \in \mathbf{B}$.

Before analyzing the ratio-changing phase, we want to explain the property that for $\forall q_n, q'_n \neq 0$

$$T_n^{x_n}(q'_n) - T_n^{x_n}(q_n) + \frac{\ln q_n}{\ln q'_n} = 0. \quad (16)$$

This is because we use function (3) to compute the ratio-dependent threshold. Applying the same rule for q_n in our quasi-potential function, we can use the change in term $\frac{\ln q_n}{\ln(1-p_n)}$ to counteract the effect of threshold change in resource A.

In phase 1, the user n will not deviate from his current channels, so the only change in potential value is the change in the thresholds of user n. Since $q_n = 1, 0$ are special cases, so we divide the ratio change into 5 different cases:

a) $q_n = 0.1, q'_n = 0$

$$\Delta(\mathbf{s} \to \mathbf{s}'') = -2(T_n^{x_n}(q_n) - I_n^{x_n}) \\ + \frac{2}{N^2}(T_n^{y_n}(1 - q'_n) - T_n^{y_n}(1 - q_n)), \quad (17)$$

b) $q_n = 0, q'_n = 0.1$

$$\Delta(\mathbf{s} \to \mathbf{s}'') = 2(T_n^{x_n}(q'_n) - I_n^{x_n}) \\ + \frac{2}{N^2}(T_n^{y_n}(1 - q'_n) - T_n^{y_n}(1 - q_n)), \quad (18)$$

c) $q_n = 1, q'_n = 0.9$

$$\Delta(\mathbf{s} \to \mathbf{s}'') = \frac{2}{N^2}(T_n^{y_n}(1 - q'_n) - I_n^{y_n}) \\ + 2(T_n^{x_n}(q'_n) - T_n^{x_n}(q_n)) - 2\frac{\ln q'_n}{\ln(1 - p_n)}, \quad (19)$$

d) $q_n = 0.9, q'_n = 1$

$$\Delta(\mathbf{s} \to \mathbf{s}'') = 2(T_n^{x_n}(q'_n) - T_n^{x_n}(q_n)) \\ + 2\frac{\ln q_n}{\ln(1 - p_n)} - \frac{2}{N^2}(T_n^{y_n}(1 - q_n) - I_n^{y_n}), \quad (20)$$

e) $0.1 < q_n < 0.9, 0.1 < q'_n < 0.9$

$$\Delta(\mathbf{s} \to \mathbf{s}'') = 2(T_n^{x_n}(q'_n) - T_n^{x_n}(q_n)) + 2\frac{\ln q_n}{\ln q'_n} \\ + \frac{2}{N^2}(T_n^{y_n}(1 - q'_n) - T_n^{y_n}(1 - q_n)). \quad (21)$$

We are going to analyze this case by case. First we want to give the bounds of threshold change in both resource A and resource B. According to equation (3) and (4) we can compute the $T_n^r(q_n)$ in terms of $T_n^r(q_n)|_{q_n=1}$. We use $\Delta T_n^{x_n}$ and $\Delta T_n^{y_n}$ to show the change in threshold when ratio changes by 0.1. Because we have $0 < T_n^r(1) \leq N$, we can see that

$$1 \leq \quad \Delta T_n^{x_n} \quad \leq 6, \quad (22)$$

$$0 \leq \frac{T_n^{y_n}(1)}{9N^2} \leq \quad \frac{\Delta T_n^{y_n}}{N^2} \quad \leq \frac{5T_n^{y_n}(1)}{N^2} \leq \frac{5}{N}. \quad (23)$$

a) For case $q_n = 0.1, q'_n = 0$, the only reason for this update is that user in resource A is not satisfied, that is $T_n^{x_n}(q_n) - I_n^{x_n} \leq 1$. Ratio-decreasing will result in the threshold B's decrease, however, from inequalities (22), we know the maximum change in threshold B is $5/N$, which is smaller than

1 since we assume the number of users N is large enough. Therefore, in this case, $\Delta(\mathbf{s} \to \mathbf{s}'') > 0$.

b) For case $q_n = 0, q'_n = 0.1$, there must be $T_n^{x_n}(q_n) - I_n^{x_n} \geq 0$, otherwise, it is not a successful update and is out of the scope of our discussion. Besides, ratio-increasing can cause the increase in threshold B. Thus, we have $\Delta(\mathbf{s} \to \mathbf{s}'') > 0$.

c) For case $q_n = 1, q'_n = 0.9$, there must be $T_n^{y_n}(1 - q'_n) - I_n^{y_n} \geq 0$ so that this update is successful. In addition, due to equation (16), we have $(T_n^{x_n}(q'_n) - T_n^{x_n}(q_n)) - \frac{\ln q_n}{\ln(1 - p_n)} = 0$, so the potential change in this case is still a non-negative value $\Delta(\mathbf{s} \to \mathbf{s}'') \geq 0$.

d) For case $q_n = 0.9, q'_n = 1$, we have $(T_n^{x_n}(q'_n) - T_n^{x_n}(q_n)) - \frac{\ln q_n}{\ln(1 - p_n)} = 0$. But $T_n^{y_n}(1 - q_n) - I_n^{y_n}$ may be greater than 0. Since $T_n^{y_n}(1 - q_n)|_{q_n=0.9} = 10T_n^{y_n}(1) \leq 10N$ and $I_n^{y_n} \geq 0$, for the worst case when $T_n^{y_n}(1 - q_n) - I_n^{y_n} = 10N$, the biggest potential change is $-\frac{20}{N}$.

e) For other cases, we divide our demonstration into two parts – the ratio-increase part and the ratio-decrease part.

For a ratio-increase

$$\begin{cases} T_n^{x_n}(q'_n) - T_n^{x_n}(q_n) + \frac{\ln q_n}{\ln q'_n} = 0, \\ T_n^{y_n}(1 - q'_n) - T_n^{y_n}(1 - q_n) \geq 0. \end{cases}$$

Thus we have $\Delta(\mathbf{s} \to \mathbf{s}'') \geq 0$ under this case;

For a ratio-decrease

$$\begin{cases} T_n^{x_n}(q'_n) - T_n^{x_n}(q_n) + \frac{\ln q_n}{\ln q'_n} = 0, \\ T_n^{y_n}(1 - q'_n) - T_n^{y_n}(1 - q_n) \leq 0. \end{cases}$$

Then in the *better reply* phase, users will first check whether they are satisfied in both resource A and resource B. If they have already been satisfied without channel-altering, they stick to their current channels, and the potential value does not decrease. If they become dissatisfied on their current channels, they will perform better reply with the ratio unchanged. We have already proved that there's at least 2 in potential increase.

After analyzing all the update scenario, we have proved that $P(\mathbf{s})$ is a quasi-potential function and the game model is a quasi-potential game. And the decreasing set Θ_n is as the following shows: $\{((x_n, y_n, q_n), (x_n, y_n, q_n - 0.1)) : 0.1 < q_n < 1\} \cup \{(x_n, y_n, 0.9), (x_n, y_n, 1))\}$. $\square$

LEMMA 3. *The potential decrease contributed by decreasing set Θ_n is finite in the satisfaction-guaranteed better reply.*

PROOF. The decrease set Θ_n contains 10 decrease pairs | nine of them are pure ratio decrease (without channel altering) and one is ratio increasing from 0.9 to 1.

We first consider the ratio-decrease cases. The only situation when a user can decrease his ratio for more than 9 times is the user sometimes will increase his ratio in updating. So

we assume that the user first decrease his ratio by 0.1 and increase his ratio by 0.1 next turn while $x_n = x'_n, y_n = y'_n$. The only reason that the user will decrease his ratio in the first turn is because his resource A becomes dissatisfying

$$I_n^{x_n}(s) > T_n^{x_n}(q_n). \tag{24}$$

And if he can increase his ratio next turn while still stick to the channel x_n, there must be

$$I_n^{x_n}(s') \leq T_n^{x_n}(q_n). \tag{25}$$

From inequalities (24) and (25) we can get

$$I_n^{x_n}(s) > I_n^{x_n}(s'). \tag{26}$$

This implies that there must be at least one user deviating from channel x_n, which means the potential increases at least 2.

Combining the above "decrease-deviate-increase" process together, we get at least 2 in potential increase. Thus the worst case for the potential decrease is that a user can decrease his ratio continuously for 8 times and never change his channels. The maximum decrease is

$$N \cdot \frac{2\sum_{q_n=0.1}^{0.8}(T_n^{y_n}(0.9 - q_n) - T_n^{y_n}(1 - q_n))}{N^2}$$
$$= \frac{2(T_n^{y_n}(0.1) - T_n^{y_n}(0.9))}{N} \leq \frac{20T_{\max}}{N} \leq 20, \tag{27}$$

where $T_{\max} = \max_{n \in \mathbf{N}} T_n^r(1) \leq N$ is the maximum value of thresholds.

The decrease caused by $(x_n, y_n, 0.9), (x_n, y_n, 1)$ is also finite since if a user wants to perform it for a second time, there must be an update $(x_n, y_n, 1), (x_n, y_n, 0.9)$. This update will counteract the decrease caused by the first $0.9 \Rightarrow 1$ update. Thus the total decrease caused by this is at most 2. $\quad\square$

Then we begin our proof for theorem 1.

PROOF. In lemma 2 we have proved the SBR is a quasi-potential game and in lemma 3, we have proved the potential decrease is finite. We are going to demonstrate that P is bounded above and below.

For any strategy profile we have

$$0 \quad \leq I_n^r \quad \leq N, \tag{28}$$
$$0 \quad \leq T_n^r(1) \quad \leq T_{\max} \leq N, \tag{29}$$
$$0 \quad \leq T_n^r(q_n) \quad \leq T_n^r(0.1) = 10T_n^r(1). \tag{30}$$

Thus we have

$$-N \leq (2T_n^r - I_n^r) \leq 20N. \tag{31}$$

The maximum potential is when every user's ratio $q_n = 1$, for $\forall n \in \mathbf{N}$. Hence, we have $-N^2 \leq P(s) \leq 20N^2$. Combined with the fact that there might be at most 20 decrease in potential (inequality (27)) we have

$$P(s)_{\max} - P(s)_{\min} \leq 21N^2 + N. \tag{32}$$

Then according to the lemma 1, since SBR is a finite quasi-potential game with finite potential decrease and bounded potential function, it possesses the finite improvement property. Thus our SBR can converge to Nash equilibria. $\quad\square$