Achieving Full View Coverage with Randomly-Deployed Heterogeneous Camera Sensors

Yibo Wu, Xinbing Wang
Department of Electronic Engineering
Shanghai Jiao Tong University
Shanghai, China
Email: {iceworld0324, xwang8}@sjtu.edu.cn

Abstract—A brand-new concept about the coverage problem of camera sensor networks, full view coverage, has been proposed recently to judge whether an object’s face is guaranteed to be captured. It is specially significant for camera networks since image shot at the frontal viewpoint considerably increases the possibility to recognize the object. In this paper, we investigate the necessary and sufficient conditions to achieve full view coverage under two random deployment scheme, uniform deployment and Poisson deployment. In uniform deployment, we define a centralized parameter – critical sensing area (CSA) – to evaluate the total requirements to reach asymptotic full view coverage for all heterogeneous sensors in the network. In Poisson deployment, we develop the probability for a point to achieve full view coverage. Our results reveal that under uniform deployment, the demands for sensors to achieve full view coverage merely lie in the area of the sensing region, regardless of its shape.

Keywords-full view coverage; heterogeneity; camera sensor

I. INTRODUCTION

Coverage is a fundamental concern in the research of wireless sensor networks. In recent years, the appearance and application of camera sensors has brought new vitality to this topic. The image or video provided by camera sensors considerably enriches the information retrieved from the monitored area, making sensor networks more practical and useful than traditional acoustic or thermal ones. Such networks have perspective applications including traffic monitoring, surveillance in estates, animal protection, and even online gaming [1]. However, different from traditional sensors with omnidirectional sensing ability, a camera sensor (camera or sensor for short) possesses an angle of view, beyond which it is not able to capture any information. This characteristic leads to new features of networks coverage problem, and thus new requirements for achieving full coverage.

Liu and Towsley’s work [3] offers a basic guideline for considerations on coverage problems of scalar sensor networks, such as mobility[10][18], deployment scheme [12][28], barrier formation [7][8], multiple coverage and connectivity [6][13][14][17]. These considerations are also valuable in studying camera sensor networks (CSNs), and have inspired a large quantity of innovative studies [4][19][20][21][22][24]. Among them, Wang and Cao’s work [4] absorbs us most as they put forward a novel concept in the judgment of coverage in CSNs, which is called full view coverage. An object is full view covered if its viewed direction is always closely enough to its facing direction, no matter what direction the object actually faces. The outstanding value of full view coverage lies in the guarantee of catching the face of an object. Studies in [5] proved that it’s more likely for computer recognition system to successfully recognize an object if its image is captured at or near the frontal viewpoint. Therefore, developing a CSN which is capable to full view cover the area of interest is of great significance, as the network not only ensures the detection of a target, but also owns a high probability to recognize it.

To construct such networks, it is essential to understand the conditions required for full view coverage, e.g. sensing radius, angle of view, deployment density, or the effective angle of full view coverage. In [4], Wang et al. derived a sufficient condition under random and uniform deployment, and a critical (i.e. both necessary and sufficient) condition under triangular lattice based deployment, both of which are on the density needed for full view coverage. Nevertheless, the critical condition of full view coverage under uniform or Poisson deployment is still an open problem. We believe that this problem is vital for the design and application for CSNs when random deployment is inevitable. For example, sensors have to be dropped by plane or artillery if the area of interest is hostile or hard to access, or deficiency on time, manpower or funds prevents careful arrangement of every single sensor.

In this paper, we concentrate on the critical condition of full view coverage under both uniform and Poisson deployment. We convert the coverage problem of an unit square into the coverage of a dense grid in it, which is a common method in the analysis of area coverage [6]. Follow this route, we derive both the necessary and sufficient conditions of full view coverage under uniform deployment, and then conjecture that according to our results, the critical condition of full view coverage seems to be nonexistent. Namely, when the sensing parameter (including sensing radius and angle of
view) falls in a certain interval, whether the area of interest is full view covered cannot be predetermined, but depends on the actual deployment of the network. Plus, we provide the probability that an arbitrary point meets the necessary and sufficient conditions under Poisson process, and explain how the differences between two deployment schemes multiply difficulties in further exploration.

Heterogeneity is another consideration in many network problems including coverage [18][25]. In practice, cameras with different sensing radii and angles of view commonly appear in the same network due to the underlying reasons. Cameras may come from different manufacturers and thus have different sensing parameters. Or sometimes, both high-end and low-end cameras are chosen to simultaneously meet quality requirements and funds limitation. Plus, cameras sensing capability will decline as time passes by or under the obstruction of terrains, resulting in difference between cameras which are deployed in different times or places. As a result, we also consider heterogeneous sensors due to its inherence.

To deal with heterogeneous cameras with unequal sensing parameters, we divide cameras into different groups according to their sensing radii and angles of view. Further, our study shows that under random and uniform deployment, the sensing ability of a camera is proportional to its sensing area, so the critical sensing area (CSA) will be defined later to help analysis. This index takes different sensing parameters of all the cameras into consideration, and therefore represents the overall requirements for cameras in CSNs to achieve full view coverage. Moreover, it aims to judge the asymptotic coverage, which means that we consider the coverage problem when the total number of cameras approaches to infinity.

Our main contribution is highlighted here.

- We define CSA to describe the requirements of full view coverage. This parameter is centralized and thus represents the whole demand for heterogeneous sensors.
- We derive the geometric necessary condition of full view coverage. When \( n \) sensors are uniformly deployed in an unit square, we obtain that the CSA to reach the necessary condition of full view coverage is:

\[
s_{N,c}(n) = -\frac{\pi}{2n \log(1 - \frac{\pi}{n \log n})}.
\]

- Similarly, we provide the geometric sufficient condition of full view coverage, and present that the CSA to reach the sufficient condition under uniform deployment is:

\[
s_{S,c}(n) = -\frac{2\pi}{n \log(1 - \frac{2\pi}{n \log n})}.
\]

- We derive the probability that an arbitrary point in the operation region reaches the necessary or sufficient condition of full view coverage under 2-dimensional Poisson process, and explain the difficulty in continuing the study of this topic.
- By evaluating our results, we provide some intuitive characteristics of CSNs. We declare that under uniform deployment, the sensing ability of a camera is proportional to its sensing area, and we conjecture that the critical condition of full view coverage may not exist.

The remainder of the paper is organized as follows. Basic models and definitions are described in Section II. In Section III, we study the CSA to meet the necessary condition of full view coverage under uniform deployment. In Section IV, we study the corresponding CSA to achieve the sufficient condition. We provide the probability of full view coverage under Poisson deployment in Section V. Section VI presents some evaluations on our work and comparisons with related studies. Conclusion is provided in Section VIII.

II. NOTATIONS AND MODEL

In this section, we present the sensing and deployment model of camera sensor used in our work, describe the definition and meaning of full view coverage, and define critical sensing area to evaluate the conditions for achieving full view coverage.

A. Sensing and Deployment Model

A camera sensor \( S \) can sense perfectly in a sector of radius \( r \) and angle \( \phi \), but will not sense outside the sector. Without confusion, \( S \) also denotes the location of the sensor. The angular bisector of \( \phi \) is recognized as orientation of \( S \), denoted by \( \vec{f} \). This model is commonly used in literature [20][21][22], called binary sector model. Further, since the quality of information provided by a camera is sensitive to its viewpoint, there are another two essential directions to be considered. The direction which a point \( P \) faces towards is called its facing direction. The vector \( \vec{PS} \) is called the object’s viewed direction, which reflects the viewpoint of sensor \( S \). Figure 1 illustrates these directions which will be useful in subsequent discussion.

Figure 1. For sensor \( S \) and point \( P \), the orientation, viewed direction and facing direction are depicted respectively.

We consider heterogeneous sensors similar to [18]. To describe sensors of different qualities, we partition sensors to \( u \) groups \( G_1, G_2, \ldots, G_u \), where \( u \) is a constant. As the total
number of sensors is \( n \), each group \( G_y(y = 1, 2, \cdots, u) \) has \( n_y = c_y n \) sensors, where \( c_y \) is a constant invariant to \( n \). Clearly, \( c_y \) satisfies \( 0 < c_y < 1 \) and \( \sum_{y=1}^{u} c_y = 1 \). All sensors in group \( G_y \) own identical sensing radius \( r_y \) and angle \( \phi_y \), but either \( r_y \neq r_z \) or \( \phi_y \neq \phi_z \) will hold if \( y \neq z, y, z = 1, 2, \cdots, u \). We mainly study the asymptotic coverage here, implying that \( n \) is a variable approaching to infinity, whereas \( r_y \) and \( \phi_y \) are dependent variables of \( n \), sometimes denoted by \( r_y(n) \) and \( \phi_y(n) \). When the total number of sensors \( n \) changes, the requirements for \( r_y(n) \) and \( \phi_y(n) \) should change along with \( n \).

In our work, sensors are deployed according to two different schemes, uniform deployment and Poisson deployment. The Uniform deployment scheme means that total \( n \) sensors are deployed in the operational region randomly, uniformly and independently. The Poisson deployment scheme means that \( n \) sensors are deployed according to 2-dimensional Poisson point process. Both methods are widely recognized as proper estimations for randomly distributed sensors. The operational region is an unit square, which is supposed to be a torus so that we can ignore the boundary effect. Wherever a sensor actually locates, its orientation \( \vec{f} \) faces towards all possible directions with equal probability, and \( \vec{f} \) stays the same once a sensor is deployed. This means that the orientation of the camera cannot steer during operation.

### B. Description of Full View Coverage

Full view coverage appears as a new concept specially considered for camera sensor networks. Wang et al. firstly defined it in [4]. We modify its description by adding a term safe direction.

**Definition 1.** For a point \( P \), its facing direction \( \vec{d} \) is safe if there is a sensor \( S \), such that \( P \) is covered by \( S \) and \( \angle(\vec{d}, PS) \leq \theta \). Then \( P \) is full view covered if all possible \( \vec{d} \) is safe. Here, \( \theta \in (0, \pi] \) is a predefined constant parameter called effective angle.

![Diagram](image_url)

Figure 2. Point A is more likely to be full view covered than point B. When point B faces up, there is no camera closely catching its frontal image.

From the definition, we know that when the object faces to a safe direction \( \vec{d} \), there is an upper bound \( \theta \) between the viewed direction \( \vec{PS} \) and the facing direction \( \vec{d} \). Then full view coverage guarantees an upper limit between the viewed direction and the facing direction of an object. This means that there is always a camera viewing near the frontal point of the object, which further ensures the object’s face to be captured. Traditional scalar sensor networks do not care the face of targets, since they usually detect objects through the energy they emitted, but the characteristic of full view coverage is specially important for camera networks. The frontal image will effectively raise the probability to recognize the object, whether it is an endangered wild animal, a human being, or an armored vehicle. As a trade off, full view coverage of an area requires much more sensors compared with traditional \( 1/k \)-coverage. Therefore, full view coverage is more suitable for high quality and high expense service.

### C. Definition of Critical Sensing Area

According to binary sector model, a sensor in group \( G_y \) possesses a sensing area \( s_y = \phi_y r_y^2/2 \). It will be presented later that compared with \( r_y \) and \( \phi_y \), \( s_y \) plays a more decisive role in the sensing process. For all sensors in the network, denote \( s_c = \sum_{u=1}^{u} c_y s_y \) as the weighted summation of all their sensing areas. Then the critical sensing area of camera sensor networks, a key parameter to evaluate the conditions for full view coverage, is defined here.

**Definition 2.** \( s_c(n) \) is the critical sensing area(CSA) for an event \( H \) if

\[
\lim_{n \to \infty} P(H) = 1, \text{if } s_c \geq cs_c(n) \text{ for any } c > 1; \\
\lim_{n \to \infty} P(H) < 1, \text{if } s_c < cs_c(n) \text{ for any } 0 < c < 1.
\]

According to the definition, no matter what value of \( s_y \) is for a special group \( G_y \), when the order of the weighted sum \( s_c \) exceeds the order of CSA, event \( H \) is sure to happen asymptotically. On the other hand, \( H \) may not happen if the order of \( s_c \) is lower than that of CSA. Then CSA is a centralized parameter to judge whether an event will happen in heterogeneous camera sensor networks as \( n \) approaches to infinity. It provides an unified standard and an overall judgement for sensors of different sensing radii and angles of view. If we denote event \( H \) as the full view coverage of the operational region, then the CSA is able to express the requirements for CSNs to achieve full view coverage.

### III. Necessary Condition Under Uniform Deployment

We believe that it’s considerably difficult to develop a critical condition for full view area coverage, if it really exists. Therefore, we first focus on necessary condition only. We find a CSA to achieve the necessary condition for full view coverage under uniform deployment.
Theorem 1. In CSNs, if $n$ sensors are randomly and uniformly deployed in an unit square, the CSA to reach the necessary condition for full view coverage of effective angle $\theta$ is

$$s_{N,c}(n) = - \frac{\pi}{2} \log \left( 1 - \frac{1}{n \log n} \right). \quad (1)$$

Subscript $N$ in $s_{N,c}(n)$ means it’s special for necessary condition.

Figure 3. $\sqrt{m} \times \sqrt{m}$ dense grid in an unit square

A. Geometric Analysis

We convert the coverage of the unit square to the coverage of all points of a $\sqrt{m} \times \sqrt{m}$ dense grid $M$. Kumar et al. proved in [6] that in binary disc sensing model, if $m \geq n \log n$, then conditions realizing the coverage of the dense grid is sufficient to guarantee the coverage of the whole square area. While in binary sector model, we suppose that as long as $\lim_{n \to \infty} \phi(n) > 0$, $m = n \log n$ is also sufficient large for full view coverage. Namely, conditions to achieve full view coverage of $M$ will also ensure full view coverage of the unit square. On the other hand, the coverage of $M$ is obviously necessary for coverage of the whole area, so the coverage of the unit square is equivalent to the coverage of $M$. We can focus our attention on the latter in the following analysis.

Here we develop the necessary geometric condition for full view coverage of an arbitrary point $P$ in the dense grid. As the left of Figure 4 shows, $C(P, r)$ is a circle centered at $P$ with radius $r$. Without lost of generality, denote the dashed radius $r_P$ as the start line. Rotate the start line for angle $2\theta$ anticlockwise, we get a sector $T_1$. Similarly there are sectors $T_2, T_3, \cdots, T_{k_N}$, where $k_N = \lceil \frac{\pi}{2\theta} \rceil$. Between sector $T_{k_N}$ and $T_1$, there may be one more sector $T_\alpha$ with central angle $\alpha \in (0, 2\theta)$. Denote the angular bisector of $T_\alpha$ as $d_\alpha$. We create sector $T_{k_N+1}$ such that its angular bisector is also $d_\alpha$ and its circular angle is $2\theta$, as showed on the right of Figure 4. Therefore, the necessary condition for full view coverage is such that, there is at least one sensor falling within $T_j$ and covering $P$ for all $j = 1, 2, \cdots, k_{N+1}$.

![Figure 4](image-url)

In the following explanation of geometric necessary condition, we refer to sensors who successfully cover point $P$ when we mention sensors. Given no sensor locates in one sector $T_j, j = 1, 2, \cdots, k_N$, we choose the angular bisector of $T_j$ as the facing direction $\vec{d}$. Then for any sensor $S$ outside $T_j$, $\angle(\vec{d}, \vec{PS}) > \theta$ must hold. So $\vec{d}$ is unsafe, resulting in failure of full view coverage. Thus, every $T_j, j = 1, 2, \cdots, k_N$ requires a sensor. If sector $T_\alpha$ exists, i.e. $2\pi = k_N \times 2\theta + \alpha, \alpha \in (0, 2\theta)$, a sensor is also needed in sector $T_{k_N+1}$. If not, $d_\alpha$ must be unsafe due to the same reason. Hence, this necessary condition indicates that at least $\lceil \frac{\pi}{2\theta} \rceil$ sensors are needed to achieve full view coverage of a point.

The above analysis reveals an intrinsic feature of full view coverage, that is, the angular bisector of a sector with angle $2\theta$ is unsafe if the sector has no sensor. This implies that sector $T_{k_N+1}$ needs not be located exactly as Figure 4 shows. It can rotate to some extent, without influence on the requirements we derived, as long as its angular bisector is still within $T_\alpha$.

Let $F_{N,P}$ be the event that an arbitrary point $P$ in dense grid $M$ does NOT fulfill the necessary condition of full view coverage. Since

$$P(\{\text{Sensor } S \text{ in group } G_y \text{ falls in } T_j\}) \times P(\{S \text{ has proper orientation}\}) = \frac{2\theta}{2\pi} \cdot \pi y^2 \cdot \frac{\phi_y}{2\pi} = \frac{\theta s_y}{\pi}$$

We have

$$P(F_{N,P}) = 1 - \left[ 1 - \prod_{y=1}^{u} \left( 1 - \frac{\theta s_y}{\pi} \right) \right]^{\left\lceil \frac{\pi}{2\theta} \right\rceil} \quad (2)$$

The probabilities that at least one sensor falls into different sectors actually have some correlation between each other, on considering that one sensor cannot fall into other sectors if it is already in one. However, this impact is negligible.
as \( n \to \infty \), so we suppose that those probabilities are independent.

Denote the event that all points in dense grid \( \mathbb{M} \) meet the necessary condition of full view coverage with effective angle \( \theta \) as \( \mathcal{H}_N \). Ignoring the boundary effect, \( \mathbb{P}(\mathcal{F}_{N,P}) \) for all \( P \in \mathbb{M} \) is identical. Here we explore the upper and lower bound of \( \mathbb{P}(\mathcal{H}_N) \), using the main idea illustrated in Figure 5. Suppose each circle represents the event that one point fails to achieve the necessary condition of full view coverage (i.e., \( \mathcal{F}_{N,P} \)). Then the event \( \mathcal{H}_N \) is denoted by the shaded area. Referring to Bonferroni inequalities, we have the upper bound

\[
\mathbb{P}(\mathcal{H}_N) \leq \sum_{P \in \mathbb{M}} \mathbb{P}(\mathcal{F}_{N,P}) \tag{3}
\]

and the lower bound

\[
\mathbb{P}(\mathcal{H}_N) \geq \sum_{P \in \mathbb{M}} \mathbb{P}(\mathcal{F}_{N,P}) - \sum_{P_i, P_j \in \mathbb{M}} \mathbb{P}(\mathcal{F}_{N,P_i} \cup \mathcal{F}_{N,P_j}) \tag{4}
\]

Figure 5. If one single circle denotes the event \( \mathcal{F}_{N,P} \), the event \( \mathcal{H}_N \) is represented by the whole shaded area.

B. Proof of Theorem 1—Necessary Part

We now begin the proof of Theorem 1, following the basic idea in [18].

**Proposition 1.** In CSNs, if \( n \) sensors are randomly and uniformly deployed in an unit square, and they possess \( s_c = -\frac{\pi}{\theta n} \log \left( 1 - \sqrt[4]{1 - \frac{e^{-\xi}}{n \log n}} \right) \), where \( \xi \) is a positive constant, then

\[
\lim \inf_{n \to +\infty} \mathbb{P}(\mathcal{H}_N) \geq e^{-\xi} - e^{-2\xi}
\]

**Proof:** To ease the complexity of the proof, we provide three lemmas first.

**Lemma 1.** Given a variable \( x \) satisfies \( 0 < x < \frac{1}{2} \), then \( \log(1 - x) \in \left( -(x + \frac{5}{6}x^2), -(x + \frac{1}{2}x^2) \right) \).

**Proof:** The proof is similar to part of the proof of Lemma 4.1 in [18].

**Lemma 2.** Given a variable \( x = x(n) \) satisfies \( 0 < x(n) < \frac{1}{2} \), and a variable \( y = y(n) > 0 \), then \( (1 - x)^y \sim e^{-xy} \) if \( x^2y \) approaches to zero as \( n \to +\infty \).

**Proof:** According to Lemma 1, \( x \) satisfies

\[
\log(1 - x) \in \left( -(x + \frac{5}{6}x^2), -(x + \frac{1}{2}x^2) \right).\]

Times \( y \) on both sides and take exponent, then

\[e^{-xy} - \frac{5}{6}x^2y < (1 - x)^y < e^{-xy} - \frac{1}{2}x^2y.\]

Use \( x^2y \to 0 \), and we have

\[(1 - x)^y \sim e^{-xy}.\]

**Lemma 3.** If \( s_c = -\frac{\pi}{\theta n} \log \left( 1 - \sqrt[4]{1 - \frac{e^{-\xi}}{n \log n}} \right) \), then \( s_c, s_y \) and \( s_y^2n(y = 1, 2, \ldots, u) \) all approach to zero as \( n \to \infty \).

**Proof:** When \( n \) goes to infinity,

\[
s_c = -\frac{\pi}{\theta n} \log \left( 1 - \sqrt[4]{1 - \frac{e^{-\xi}}{n \log n}} \right)
\leq -\frac{\pi}{\theta n} \log \left( \frac{e^{-\xi}}{n \log n} \right)
\sim -\frac{1}{n} \log \left( \frac{e^{-\xi}}{n \log n} \right)
= \frac{\log n + \log \log n + \xi}{n} \to 0
\]

As \( s_c = \sum_{y=1}^u c_y s_y \), and \( s_c = \Theta \left( \frac{\log n + \log \log n}{n} \right) \), we know that \( s_y = \Theta \left( \frac{\log n + \log \log n}{n} \right) \) and \( s_y^2n = \Theta \left( \frac{(\log n + \log \log n)^2}{n} \right) \). Therefore, for all \( y = 1, 2, \ldots, u \), \( s_y \to 0 \) and \( s_y^2n \to 0 \). Then the proof is done.

We bound the two terms on the right side of (4) respectively and begin with the first term. Take logarithm of \( \prod_{y=1}^u \left( 1 - \frac{\theta s_y}{\pi} \right)^{n_y} \) and we have

\[
\log \left[ \prod_{y=1}^u \left( 1 - \frac{\theta s_y}{\pi} \right)^{n_y} \right]
= n \sum_{y=1}^u c_y \log \left( 1 - \frac{\theta s_y}{\pi} \right)
\geq -n \sum_{y=1}^u c_y \left[ \frac{\theta s_y}{\pi} + \frac{5}{6} \left( \frac{\theta s_y}{\pi} \right)^2 \right] \tag{5}
\]

Step (5) is derived by applying Lemma 1 with the fact that \( \frac{\theta s_y}{\pi} < \frac{1}{2} \) when \( n \) approaches to infinity. Then,
\[
\log \left[ \prod_{y=1}^{u} \left( 1 - \frac{\theta s_y}{\pi} \right)^{n_y} \right] \\
\geq -\frac{\theta n}{\pi} s_c - 5 \frac{\theta n}{6 \pi} \sum_{y=1}^{u} c_y(s_y)^2 \\
\geq -\frac{\theta n}{\pi} s_c - 5 \frac{\theta n}{6 \pi} (s_c)^2 \tag{6}
\]

Step (6) is due to \( s_i < c_j \) for \( i, j = 1, 2, \ldots, u \) when \( n \) is sufficiently large, according to Lemma 3.

Since \( \frac{5}{6} \theta n (s_c)^2 \to 0 \), for any \( \epsilon > 0 \),

\[
\log \left[ \prod_{y=1}^{u} \left( 1 - \frac{\theta s_y}{\pi} \right)^{n_y} \right] \geq -\frac{\theta n}{\pi} s_c - \epsilon,
\]

for all \( n > N_\epsilon \). Let \( \beta = e^{-\epsilon} \) and take exponent of both sides, we get

\[
\prod_{y=1}^{u} \left( 1 - \frac{\theta s_y}{\pi} \right)^{n_y} \geq \beta e^{-\frac{\theta n}{\pi} s_c} \tag{7}
\]

Use (7) in the first term

\[
\sum_{P \in \mathcal{M}} P(F_{N,P}) \\
= m \left\{ 1 - \left[ 1 - \prod_{y=1}^{u} \left( 1 - \frac{\theta s_y}{\pi} \right)^{n_y} \right]^{\frac{1}{2}} \right\} \\
\geq m \left\{ 1 - \left[ 1 - \beta e^{-\frac{\theta n}{\pi} s_c} \right]^{\frac{1}{2}} \right\} \\
= m \left\{ 1 - \beta \left( 1 - \left[ 1 - \beta \right]^{\frac{1}{2}} \right) \right\} \\
= m \left\{ 1 - \beta \left( 1 - \frac{1}{2} \right) \right\} \\
= m \left\{ 1 - \beta \right\} \\
\]

Since the above inequality holds for any \( \beta < 1 \), we have

\[
\sum_{P \in \mathcal{M}} P(F_{N,P}) \geq e^{-\xi} \tag{8}
\]

For the second term, Lemma 3 proves that when \( n \to \infty \), all sensing areas \( s_y \) goes to zero, which implies that sensors who cover point \( P \) cannot further cover another point \( P_j \). The probabilities that \( P \) or \( P_j \) is full view covered depends on different sets of sensors. Hence, \( P(F_{N,P}) \) and \( P(F_{N,P_j}) \) are independent. We have

\[
\sum_{P \neq P_j} P(F_{N,P} \cup F_{N,P_j}) \\
= \sum_{P \neq P_j} P(F_{N,P})P(F_{N,P_j}) \\
= m^2 \left\{ 1 - \left[ 1 - \prod_{y=1}^{u} \left( 1 - \frac{\theta s_y}{\pi} \right)^{n_y} \right]^{\frac{1}{2}} \right\}^2 \tag{9}
\]

Use Lemma 2 in (9),

\[
\sum_{P \neq P_j} P(F_{N,P} \cup F_{N,P_j}) \\
\sim m^2 \left\{ 1 - \left[ 1 - e^{-\frac{\theta n}{\pi} \sum_{y=1}^{u} n_y s_y} \right]^{\frac{1}{2}} \right\}^2 \\
= m^2 \left\{ 1 - \left[ 1 - \frac{1}{\sqrt{1 - e^{-\xi}} n \log n} \right]^{\frac{1}{2}} \right\}^2 \\
= e^{-2\xi} \tag{10}
\]

Combining (8) and (10), the result of Proposition 1 follows.

From Proposition 1 we know that if \( s_c = -\frac{2}{\delta n} \log \left( 1 - \frac{\sqrt{q}}{n \log n} \right) \), the failure probability of guaranteeing the necessary condition of full view coverage is bounded away from zero, implying that \( s_c \geq s_{N,c}(n) \) is necessary for reaching such condition.

C. Proof of Theorem 1–Sufficient Part

**Proposition 2.** In CSNs, if \( n \) sensors are randomly and uniformly deployed in an unit square, and \( s_c = q s_{N,c}(n) = -\frac{2}{\delta n} \log \left( 1 - \frac{\sqrt{q}}{n \log n} \right) \), where \( q > 1 \), then

\[
\liminf_{n \to +\infty} P(H_N) = 0 
\]

*Proof:* Use the bound in (3), we have

\[
P(H_N) \leq \sum_{P \in \mathcal{M}} P(F_{N,P}) \\
= m \left\{ 1 - \left[ 1 - \prod_{y=1}^{u} \left( 1 - \frac{\theta s_y}{\pi} \right)^{n_y} \right]^{\frac{1}{2}} \right\} \\
\sim m \left\{ 1 - \left[ 1 - e^{-\frac{\theta n}{\pi} \sum_{y=1}^{u} n_y s_y} \right]^{\frac{1}{2}} \right\} \\
= m \left\{ 1 - \left[ 1 - \frac{1}{\sqrt{1 - e^{-\xi}} n \log n} \right]^{\frac{1}{2}} \right\} \\
\]

Since for constant \( k \geq 1 \) and \( q > 1 \), the inequality

\[
\left( 1 - \frac{1}{m^q} \right)^q \leq 1 - \frac{1}{1 - \frac{1}{m^q}} \tag{11}
\]

holds when \( m \) is large enough. Therefore,

\[
P(H_N) \leq m \left\{ 1 - \left[ \frac{1}{\sqrt{1 - e^{-\xi}} n \log n} \right]^{\frac{1}{2}} \right\} \\
= \frac{1}{m^q-1} \to 0 \tag{12}
\]

Then the proof is completed. ■
From Proposition 2, if \( s_c = q s_{N,c}(n) \), the probability that the dense grid cannot achieve the necessary condition of full view coverage approaches zero as \( n \to \infty \), which means that \( s_{N,c}(n) \) is sufficient to reach such condition. Consequently, Theorem 1 is proved by the combination of Proposition 1 and Proposition 2.

IV. SUFFICIENT CONDITION UNDER UNIFORM DEPLOYMENT

Now we turn to the sufficient condition of full view coverage. To begin with, we provide the sufficient condition to full view cover a point \( P \) through geometric analysis.

Rotate the start line \( \vec{r} \) (also shown as the dashed line in Figure 6) anticlockwise with angle \( \theta \) (note that this angle is \( 2\theta \) in the necessary condition in Section III) to get sector \( T'_1, T'_2, \ldots, T'_{k_s} \), where \( k_s = \lceil \frac{2\pi}{\theta} \rceil \). Then between \( T'_{k_s} \) and \( T'_1 \) there may be one more sector \( T'_\alpha \) with central angle \( \alpha \in (0, \theta) \). Create \( T'_{k_s+1} \), such that its angular bisector is the same as \( T'_\alpha \) and its circular angle is \( \theta \). The sufficient condition for point \( P \) to be full view covered is such that, at least one sensor locates in \( T'_j \) and cover \( P \) for all \( j = 1, 2, \ldots, k_s + 1 \).

The proof is simple. Wherever the facing direction \( \vec{d} \) is towards, it must be within at least one of the sectors created above (perhaps in two sectors simultaneously). Due to the assumption, there is at least one sensor \( S \) falling in that sector and covering point \( P \), and \( \angle(\vec{d}, PS) \leq \theta \) holds. Then \( \vec{d} \) is safe, and full view coverage of \( P \) is guaranteed. Sector \( T'_{k_s+1} \) performs all the same as other sectors here, so a sensor is also needed in \( T'_{k_s+1} \). According to this sufficient condition, if properly deployed, \( \lceil \frac{2\pi}{\theta} \rceil \) sensors are enough to achieve full view coverage of a point.

![Figure 6](image)

Denote the event that point \( P \) does NOT fulfill the sufficient condition of full view coverage as \( \mathcal{F}_{S,P} \), and the event that the dense grid reaches the sufficient condition of full view coverage as \( \mathcal{H}_S \). Then we have

\[
\mathbb{P}(\mathcal{F}_{S,P}) = 1 - \left( 1 - \prod_{y=1}^{u} \left( 1 - \frac{\theta S_y}{2\pi} \right)^{n_y} \right)^{\lceil \frac{2\pi}{\theta} \rceil} \tag{13}
\]

According to the ideas in Figure 5, the upper and lower bound of \( \mathbb{P}(\overline{\mathcal{H}_S}) \) is

\[
\mathbb{P}(\overline{\mathcal{H}_S}) \leq \sum_{P \in M} \mathbb{P}(\mathcal{F}_{S,P}) \tag{14}
\]

\[
\mathbb{P}(\overline{\mathcal{H}_S}) \geq \sum_{P \in M} \mathbb{P}(\mathcal{F}_{S,P}) - \sum_{P_i, P_j \in M} \mathbb{P}(\mathcal{F}_{S,P_i} \cup \mathcal{F}_{S,P_j}) \tag{15}
\]

We present the CSA to achieve the sufficient condition of full view coverage under uniform deployment.

**Theorem 2.** In CSNs, if \( n \) sensors are randomly and uniformly deployed in an unit square, the CSA to reach the sufficient condition for full view coverage of effective angle \( \theta \) is

\[
s_{S,c}(n) = -\frac{2\pi}{\theta n} \log \left( 1 - \left[ \frac{2\pi}{\theta n} \sqrt{1 - \frac{1}{n \log n}} \right] \right). \tag{16}
\]

Subscript \( S \) in \( s_{S,c}(n) \) means it’s special for sufficient condition.

**Proof:** The proof also consists of necessary part and sufficient part, using similar skills in the proof of Theorem 1. For the necessary part, we have Proposition 3.

**Proposition 3.** In CSNs, if \( n \) sensors are randomly and uniformly deployed in an unit square, and \( s_c = \frac{2\pi}{\theta n} \log \left( 1 - \left[ \frac{2\pi}{\theta n} \sqrt{1 - \frac{1}{n \log n}} \right] \right) \), where \( \xi \) is a positive constant, then

\[
\lim_{n \to +\infty} \mathbb{P}(\overline{\mathcal{H}_S}) \geq e^{-\xi} - e^{-2\xi} \geq 0.
\]

In the sufficient part, we use Proposition 4.

**Proposition 4.** In CSNs, if \( n \) sensors are randomly and uniformly deployed in an unit square, and \( s_c = q s_{N,c}(n) \) = \( -\frac{2\pi}{\theta n} \log \left( 1 - \left[ \frac{2\pi}{\theta n} \sqrt{1 - \frac{1}{n \log n}} \right] \right) \), where \( q > 1 \), then

\[
\lim_{n \to +\infty} \mathbb{P}(\overline{\mathcal{H}_S}) = 0.
\]

The result will follow after both propositions above are proved.

V. PROBABILITY OF FULL VIEW COVERAGE UNDER POISSON DEPLOYMENT

In this section, we consider full view coverage under 2-dimensional Poisson point process, which is commonly used in literature to model random deployment of sensors. The same as uniform deployment, the probability that sensors locate in a region is related to both the deployment density and the area of the region. However, Poisson process behaves fairly different from uniform deployment. In 2-dimensional
Poisson process with density \( \lambda \), the probability that exactly \( k \) sensors fall in a region of area \( A \) is

\[
P(k) = \frac{(\lambda A)^k e^{-(\lambda A)}}{k!}.
\]

We develop the probability that a point in the region achieves the necessary condition of full view coverage here. Such probability owns similar geometric meaning to \( \mathbb{P}(\mathcal{F}_{N,P}) \) in Section III.

Theorem 3. If \( n \) sensors are deployed according to 2-dimensional Poisson process in an unit square, then the probability that an arbitrary point meets the necessary condition of full view coverage is

\[
P_N = \left[ 1 - \prod_{y=1}^{u} (1 - Q_{N,y}) \right]^{\left\lceil \frac{n}{s} \right\rceil},
\]

where \( Q_{N,y} \) satisfies

\[
Q_{N,y} = \sum_{k=1}^{n_y} \frac{(\theta_{n_y} r_y^2)^k e^{-(\theta_{n_y} r_y^2)}}{k!} \left[ 1 - \left( 1 - \frac{\phi_y}{2\pi} \right)^k \right].
\]

Proof: Remind the necessary condition of full view coverage depicted in Figure 4. Each sector \( T_j \) needs at least one sensor in it and this sensor should cover point \( P \). In Poisson process, the probability that such event happens is different from equation (2).

As total \( n \) sensors are placed in the unit square, the density \( \lambda = n \). Hence all the sensors in the network are distributed according to Poisson process of density \( n \). For each group \( G_y \), sensors in \( G_y \) are also Poisson distributed, but the density is \( n_y \). Since the area of sector \( T_j \) is \( \theta r^2 \), the probability that \( k \) sensors in \( G_y \) falls within sector \( T_j \) is

\[
P(k) = \frac{(\theta r^2)^k e^{-(\theta r^2)}}{k!}
\]

(17)

Then those sensors who are in sector \( T_j \) must have proper orientations. To be specific, at least one of them should have proper orientation in order to cover point \( P \). For total \( k \) sensors in \( T_j \), the probability that at least one covers point \( P \) is

\[
1 - \left( 1 - \frac{\phi_y}{2\pi} \right)^k
\]

(18)

We have mentioned in Section II that where a camera locates has no impact on its orientation. Hence, equation (17) and (18) is independent. We have the probability that in group \( G_y \), at least one sensor falls in sector \( T_j \) and covers \( P \) is

\[
\sum_{k=1}^{n_y} \frac{(\theta r^2)^k e^{-(\theta r^2)}}{k!} \left[ 1 - \left( 1 - \frac{\phi_y}{2\pi} \right)^k \right] = Q_{N,y}.
\]

We denote it as \( Q_{N,y} \). Since sensors in different groups are independent from each other, we can naturally express \( P_N \) as

\[
P_N = \left[ 1 - \prod_{y=1}^{u} (1 - Q_{N,y}) \right]^{\left\lceil \frac{n}{s} \right\rceil}.
\]

Then the result follows.

Similarly, we derive the probability for sufficient condition as below.

Theorem 4. If \( n \) sensors are deployed according to 2-dimensional Poisson process in an unit square, then the probability that an arbitrary point meets the sufficient condition of full view coverage is

\[
P_S = \left[ 1 - \prod_{y=1}^{u} (1 - Q_{S,y}) \right]^{\left\lceil \frac{n}{s} \right\rceil},
\]

where \( Q_{S,y} \) satisfies

\[
Q_{S,y} = \sum_{k=1}^{n_y} \frac{(\theta y^2)^k e^{-(\theta y^2)}}{k!} \left[ 1 - \left( 1 - \frac{\phi_y}{2\pi} \right)^k \right].
\]

Proof: The proof is similar to the proof of Theorem 3 and hence is omitted.

Using the two theorem above, we can calculate the probability that whether a point meets the necessary or sufficient condition of full view coverage, given the parameters of the network under Poisson process. However, \( P_N \) and \( P_S \) own more practical meanings. As presented in many works [3][18], when edge effect is neglected, the probability that an arbitrary point is covered equals the expectation of the fraction of area which is covered. Thus, \( P_N \) and \( P_S \) represent the expected value of how much part of the operational region achieves the necessary or sufficient condition of full view coverage. Similarly, \( \mathbb{P}(\mathcal{F}_{N,P}) \) and \( \mathbb{P}(\mathcal{F}_{S,P}) \) possesses the same meaning under uniform deployment.

The research on Poisson point process model is limited to this step. We don’t further develop any centralized requirements like CSA for it because Poisson process behaves much different from uniform deployment. From \( P_N \) and \( P_S \) we present, we discover that the sensing ability of a sensor is no longer directly related to its sensing area. The mathematical expression of \( P_N \) and \( P_S \) undergoes a complicated interaction between parameters \( r_y, \phi_y, n_y \) and \( \theta \). Therefore, it’s nearly impossible to derive any centralized parameter to evaluate network performance under Poisson deployment. Future study on this topic needs more effective mathematical tools to handle such complex expressions.

VI. Performance Evaluation

Next, we investigate the CSAs we derive in Section III and Section IV from several different aspects, in order to provide some intuitive guidance for future study and design of camera sensor networks.
A. Decisive Role of Sensing Area

In the geometric analysis of uniform deployment, there are two ways to understand the probability that a sensor $S$ covers a point $P$. First, as sensors are uniformly deployed in the region, the probability that $S$ falls in circle $C(P, r)$ (a circle centered at $P$ with radius $r$) is proportional to $C(P, r)$’s area, $\pi r^2$. If $S$ further owns a proper orientation, that is, $\angle(S\overrightarrow{P}, \overrightarrow{f}) \leq \frac{\pi}{2}$, then $P$ is covered by $S$. The overall probability is $\pi r^2 \cdot \frac{\phi}{2\pi} = \frac{1}{2} \phi r^2$, rightly the sensing area of $S$. Second, sensors are uniformly deployed seen by a fixed point, so reversely, points are also uniformly deployed seen by a fixed sensor. Then a point will be sensed as long as it locates within the sensing area of $S$, the probability of which is proportional to $\frac{1}{2} \phi r^2$.

The above two understandings reveal an inherent feature of uniform deployment scheme. In terms of sensing ability, it is the area of the sensing region really matters, rather than the shape of it. In our derivation of CSAs for necessary and sufficient conditions, $r$ and $\phi$ show their impact merely on sensing area $s$, but not directly on CSAs, which also proves the decisive role of sensing area. Cameras with different $r$ and $\phi$ but own the same $s = \phi r^2 / 2$ will perform all the same in the network. Further, we conjecture that sensors with irregular sensing regions also satisfy this characteristic. This provides important hints on the judgement of the quality of camera sensors.

B. Impact of Parameter $n$ and $\theta$

Here we analyze the influence of $n$ and $\theta$ on the CSAs we have obtained. To be concise, we use $s_c(n)$ to denote either $s_{N,c}(n)$ or $s_{S,c}(n)$.

When $n$ is fixed, $s_c(n)$ becomes larger as $\theta$ decreases. This means that given a constant number of sensors, we need sensors of larger sensing area when a better view of an object’s face is demanded. It’s obvious since larger sensing region renders more sensors to cover a certain object, and thus they have more chances to catch its face. More specifically, if $n$ is very large, the influence of the $\theta$ on the radical expression $\sqrt{1 - \frac{1}{n \log n}}$ is comparatively small, while the $\theta$ in the coefficient $-\frac{\pi}{2n}$ is the primary variable. Then we have $s_c(n) \propto \frac{1}{\theta}$. This shows a inverse proportion between the effective angle $\theta$ and the required sensing area $s_c(n)$. Since $\theta$ determines the quality of full view coverage, this relationship between performance and requirements may be vital to network designers. Figure 7 presents how the CSAs for necessary and sufficient conditions decrease as $\theta$ varies from 0.1$\pi$ to 0.5$\pi$. We set $n = 1000$ in the plotting. The decreasing trend of CSAs is quite similar to inverse proportional function, which matches our analysis above.

On the other hand, when $\theta$ is fixed, $s_c(n) \to 0$ as $n$ goes to infinity according to Lemma 3. This corresponds with the instinct that with certain effective angle $\theta$, only smaller sensing area is needed to achieve full view coverage if the total number of sensors increases. Figure 8 illustrates the relationship between the CSAs and the number of cameras $n$, when $\theta$ is set to $\frac{\pi}{4}$. In the figure, the requirement for sensing area is extremely large when $n = 100$ (about 0.5 in sufficient condition, half the area of the unit square). It’s thus evident that full view coverage is unpractical if the number of cameras is insufficient, since under this situation the required sensing area is not tolerable. Further, the decline of CSAs slows down after $n$ exceeds 1000, this is because when cameras in the region are dense enough to provide full view coverage, more cameras cannot offer an effective improvement in the network performance. This reveals the importance of deciding the proper number of cameras to be deployed according to their sensing ability.

C. The Gap Between Necessary and Sufficient Conditions

A comparison of $s_{N,c}(n)$ and $s_{S,c}(n)$ reveals that with the same $\theta$, $s_{N,c}(n) < s_{S,c}(n)$ always holds. Approximately, $s_{S,c}(n)$ is two times of $s_{N,c}(n)$, which is mainly due to the difference of their coefficient $-\frac{\pi}{2n}$ and $-\frac{\pi}{n}$. In Figure 7 and Figure 8, the gaps between two lines of CSA also proves
this difference. This implies that there may not exist any CSA to rightly achieve the full view asymptotic coverage of an unit square in CSNs.

Geometrically, the necessary condition we derive in Section III is surely not sufficient, since the point can be full view covered only if sensors are distributed as evenly as possible. Once two adjacent sensors are too far away from each other, there will be a hole direction where no sensor is located and the object can face towards to avoid full view coverage. On the left of Figure 9, the closest angle between viewed direction and facing direction \( \beta = \angle(\vec{P} \vec{S}, \vec{d}) > \theta \), then \( P \) is not full view covered, though it satisfies the necessary condition. On the other side, the sufficient condition is more than necessary, because some sensors might be redundant if they stay close enough. On the right of Figure 9, sensor \( S \) can be removed without breaking the full view coverage of point \( P \), since \( \beta = \angle S_i P S_j \) satisfies \( \beta \leq 2\theta \).

As a result, when the weighted summation of all sensing areas is below \( s_{N,c}(n) \), the area cannot be full view covered; when it is above \( s_{S,c}(n) \), full view coverage will surely be guaranteed. But when the weighted sum is between \( s_{N,c}(n) \) and \( s_{S,c}(n) \), whether the area is full view covered is a random event, depending on the actual deployment of sensors. The exact critical condition for full view coverage is left to future study.

VII. COMPARISON WITH RELATED WORK

We compare our results with several related works [4][6][13][18] in the coming section, intending to make a clear picture about the status of our study.

A. Comparison with 1-Coverage

Full view coverage is surely more demanding than traditional 1-coverage. We try to establish their relationships in the following discussion. Considering the situation \( \theta = \pi \), from Figure 4 we discover that the number of sectors reduces to only one (\( T_1 \)). Then the necessary condition for full view coverage of point \( P \) degenerates to 1-coverage of \( P \). The corresponding CSA to achieve it is

\[
\begin{align*}
s_{N,c}(n) &= -\pi \frac{1}{\theta n} \log \left( 1 - \frac{1}{\theta n} \right) \\
&= -\frac{1}{n} \log \left( 1 - 1 + \frac{1}{n \log n} \right) \\
&= \frac{\log n + \log \log n}{n} 
\end{align*}
\]

In [18], Wang et al. studied the critical ESR for 1-coverage under disk sensing model and uniform deployment, \( R_*(n) = \sqrt{\frac{\log n + \log \log n}{\pi n}} \) (THEOREM 4.1). If we recognize disk sensor as a special kind of camera sensor with angle \( \phi = 2\pi \) (e.g. an omnidirectional camera consists of several cameras bundled together, or a camera quickly rotates around), we can convert critical ESR to critical sensing area, as they both use weighted summation to evaluate heterogeneous sensors. The CSA for 1-coverage is \( \pi R_*(n)^2 = \frac{\log n + \log \log n}{n} \), which exactly matches our result in (19).

B. Comparison with \( k \)-coverage

In practical, 1-coverage appears to lack robustness as sensors often fail due to unexpected events. As a result, much attention has been paid to \( k \)-coverage to improve service quality and achieve high fault tolerance. Intuitively, full view coverage with effective angle \( \theta \) implies \( k \)-coverage where \( k = \lceil \frac{\theta}{2\pi} \rceil \). This can be easily understood when referring to the necessary condition we have created. To some extent, full view coverage is even more demanding than \( k \)-coverage, since full view coverage also requires a relatively even distribution of sensors around the object. But in \( k \)-coverage there is no limitation in the relative positions of sensors.

In [13], Bai et al. investigated the optimal deployment density and deployment pattern on 2-coverage of uniformly deployed disk sensors. They defined the deployment density as the ratio of the area of the sensing disks to the area of Voronoi polygons generated by the sensor, where Voronoi polygon is a widely used approach to evaluate congruent deployment patterns. Since we don’t apply Voronoi polygon in analysis, this difference impedes further match between our results and theirs.

Kumar et al. explored the requirements to achieve \( k \)-coverage for uniformly deployed disk sensors in [6]. They used \( n \) as the total number of sensors deployed in an unit square, and \( p \) as the probability that an arbitrary sensor turns off and sleeps to prolong its lifetime. Therefore, \( np \) denotes the number of sensors which are currently awake. As they stated (THEOREM 5.1), if

\[
c(n) = 1 + \frac{u(np) + k \log \log (np)}{\log (np)} \tag{20}
\]

for some \( u(np) \), where \( c(n) = \frac{np + \pi r^2}{\log(np)} \) and \( u(np) = o(\log \log (np)) \), then the unit square region is asymptotically
$k$-covered as $n \to \infty$. To translate their results into our framework, suppose $p = 1$ and substitute $\frac{n \pi r^2}{\log(np)}$ with $c(n)$ on the left side of (20),

$$\frac{n \pi r^2}{\log n} = 1 + \frac{[u(n) + k \log \log n]}{\log n}$$

Denote $s_K(n) = \pi r^2$ as the area of the sensing disk, then we have

$$s_K(n) = \frac{\log n + k \log \log n + u(n)}{n} \quad (21)$$

As $u(n) = o(\log \log n)$, we can ignore it when only consider the order of (21). So this work provided a sufficient condition for a region to be $k$-covered, that is, the sensing area of each homogeneous sensor should satisfy $s_K(n) = \Theta \left( \frac{\log n + k \log \log n}{n} \right)$. Given $k = \left\lceil \frac{\pi}{\theta} \right\rceil$, our aim is to prove $s_{N,c}(n) \geq s_K(n)$, which means that the necessary condition of full view coverage is more demanding than the sufficient condition of $k$-coverage. Recall the inequality (11),

$$\left(1 - \frac{1}{\sqrt{1 - \frac{1}{m}}} \right)^q \leq 1 - \sqrt{1 - \frac{1}{m^q}}.$$

Replace $q$ with $k$ ($k \geq 1$),

$$\left(1 - \frac{1}{\sqrt{1 - \frac{1}{m}}} \right)^k \leq 1 - \sqrt{1 - \frac{1}{m^k}} \leq \frac{1}{m^k}$$

Thus,

$$s_{N,c}(n) = \frac{\pi}{\theta n} \log \left( 1 - \frac{\pi}{\sqrt{n \log n}} \right) \approx \frac{k}{n} \log \left( 1 - \sqrt{1 - \frac{1}{m}} \right) = \frac{\log m^k}{n} \geq \frac{k \log n + k \log \log n}{n} \geq s_K(n)$$

Till now, we have rigorously proved the higher demanding of full view coverage compared with $k$-coverage. Namely, in random and uniform deployment, a certain condition which achieves $k$-coverage of the region cannot guarantee full view coverage with effective angle $\theta$, where $k = \left\lceil \frac{\pi}{\theta} \right\rceil$.

### C. Comparison with Wang and Cao’s Work

In literature, the most related work of this paper is [4], where Wang and Cao explored an estimation on the lower bound of probability that an unit square is full view covered, given certain $r, \phi$ and $\theta$. Reversely, we can calculate the required $r, \phi$ and $\theta$ if full view coverage is needed. This can be seen as a sufficient condition for full view coverage, which highly overlaps Section IV of our study. Nevertheless, their research and ours differ both in methods of derivation and evaluation.

We both consider random and uniform deployment in an unit square and utilize dense grid to help analysis of area coverage, but Wang uses triangular lattices with exact computation on the side length of lattices. In Lemma 4.5, they presented that the edge length of lattices $l$ should satisfy $l \leq \min \{ 2 \Delta r, \Delta \phi \min \} / \sqrt{3 \cot \Delta \theta}$, given certain $\Delta r, \Delta \phi$ and $\Delta \theta$. Then $(r, \phi, \theta)$ which achieves full view coverage of the grid points will guarantee full view coverage of the whole region with condition $(r + \Delta r, \phi + \Delta \phi, \theta + \Delta \theta)$. On the other hand, we use square lattices and directly suppose total grid points $m = n \log n$ (i.e.edge length $\frac{1}{\sqrt{m}} = \frac{1}{\sqrt{n \log n}}$) is enough according to [6].

In developing the probability of full view coverage for a single point, Wang and Cao took the dependence of probabilities that sensors fall into different sectors into consideration, while we ignore it since we consider asymptotic coverage and when $n \to \infty$, such correlation becomes little. By this approximation, we obtain a much simpler result than Theorem 4.7 in [4], making it possible for further completion of CSAs. Critical sensing area is highly evaluated in our opinions, because it directly provides requirements for camera parameters. Designers and engineers can assess the demand for the quality of cameras on the basis of it.

In general, Wang and Cao’s work is more rigorous and complete, while ours seems to provide simpler results and more direct guidance to CSNs design.

### VIII. Conclusion

Coverage property of camera sensor networks is a fundamental issue, among which full view coverage draws our attention due to its emphasis on capturing the objects’ face. Finding the critical condition to achieve full view coverage is a problem that still demands future research. In this paper, we provide both the necessary and sufficient conditions to obtain full view coverage in an unit square under both uniform and Poisson deployment. Also, we take heterogeneous sensors into consideration and provide a centralized parameter, critical sensing area (CSA), to evaluate the conditions of full view coverage. Our results offer some valuable insights in the analysis and design for camera sensor networks, especially for high quality service. Our future work will be set on continuing pursuit for critical condition of full view coverage, or extending our results in...
probabilistic sensing models. Besides, the critical condition to reach barrier full view coverage will be an absorbing topic as well.

REFERENCES


