On the Capacity of k-MPR Wireless Networks

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***Abstract*—The capacity of wireless ad hoc networks is mainly restricted by the number of concurrent transmissions. Recent studies found that multi-packet reception (MPR) can increase the number of concurrent transmissions and improve network capacity. This paper studies the capacity of 2-D wireless networks wherein each node can decode at most k simultaneous transmissions within its receiving range. We call such networks k-MPR wireless networks. For comparison, we call traditional networks 1-MPR wireless networks. Suppose that the number of nodes in a wireless network is n and each node can transmit at W bits/sec. For arbitrary k-MPR wireless networks, we show that when $k = O(n)$, the capacity gain over 1-MPR networks is $\Theta(\sqrt{k})$. When $k = \Omega(n)$, the capacity is $\Theta(Wn)$ bit-meters/sec and the network is scalable. For random k-MPR wireless networks, we show that when $k = O(\sqrt{\log n})$, the capacity upper bound and lower bound match and the capacity gain over 1-MPR networks is $\Theta(k)$. When $k = \Omega(\sqrt{\log n})$, even the lower bound has a capacity gain of $\Theta(\sqrt{\log n})$ over 1-MPR networks. From these results, we conclude that the main constraints for k-MPR wireless networks to utilize MPR ability are the limited number of transmitters and the limited number of flows served by each node.**

***Index Terms*—Capacity, Scaling law, Multi-packet reception**

I. Introduction

THE seminal work of Gupta and Kumar [1] derived that the wireless network capacity scales as $\Theta(W\sqrt{n})$ bit-meters/sec in arbitrary networks while scales as $\Theta(W\sqrt{\frac{n}{\log n}})$ bits/sec in random networks where n is the number of nodes. For infrastructure wireless mesh networks, P. Zhou et al. [2] derived that the per-client throughput decreases as the number of clients increases. The main reason for these throughput degradations is that all wireless nodes share the same wireless medium and the number of concurrent transmissions is limited.

As the successive interference cancellation (SIC) circuits with simple implementation and low complexity have been introduced, multi-packet reception (MPR) becomes a reality [8], which provides potential to increase the number of concurrent transmissions and improve the network capacity. Past researches on the capacity of MPR-based wireless networks derived their results assuming that all the transmissions within the receiving range of a node can be decoded, however, as the number of transmissions within the receiving range increases, the receiver cannot decode all of them. The reason is as follows. As shown in [8], the main idea of SIC is to cancel each received signal one by one in the decreasing order of the signal strength and the signal cancellation process delay is restricted by the speed of performing Walsh-Hadamard Transform (WHT), so the possible number of cancellations is limited, which leads to the limited number of decoded transmissions.

Using a more practical interference model compared with [5], [6], this paper studies the capacity of 2-D MPR-based wireless ad hoc networks assuming that a wireless interface can decode at most $k, k \geq 1$ transmissions within its receiving range. We call such networks k-MPR wireless networks. For comparison, we call traditional networks 1-MPR wireless networks. The MPR ability, k, depends on the hardware implementation. We study how the capacity of 2-D k-MPR wireless networks scales with k and the number of wireless nodes, n, in both arbitrary and random scenarios.

The remainder of the paper is organized as follows. We summarize the related work in Section II. Section III describes the network model and main results. In Section IV, we prove the results for arbitrary networks. The proofs for the results of random networks are presented in Section V. We discuss the derived results in Section VI. Section VII concludes our work.

II. Related Work

Assuming that a node can concurrently send to and receive from many nodes when using FDMA and CDMA, R. M. D. Moraes et al. [3] studied the upper bound and lower bound of link's Shannon capacity and per source-destination throughput. G. D. Celik et al. [4] presented new backoff mechanisms for MPR-based wireless networks to deal with unfairness and improve network throughput. Assuming that all the transmissions within the receiving range of a receiver can be decoded, J. J. Garcia-Luna-Aceves et al. [5] have shown that 3-D random MPR-based wireless network has a capacity gain of $\Theta(\log n)$. Also assuming that all the transmissions within receiving range can be decoded, J. J. Garcia-Luna-Aceves et al. [6] took SINR and Shannon link capacity into the capacity analysis and gave their results. Using more realistic protocol model compared with [5], [6], X. Wang et al. [7] assumed that at most M simultaneous transmissions within the receiving range of a receiver can be decoded and maximized the aggregate network throughput by formulating an optimization problem.

According to the survey of the related work, we can see that this paper is the first work studying the capacity of 2-D MPR-based wireless networks using a practical model, which assumes that an interface can decode at most k transmissions within its receiving range. Our contributions can be summarized as follows:

- To the best knowledge of us, this paper is the first work studying the capacity of such networks in both arbitrary and random scenarios.
- From the derived results, we get some valuable design implications for k-MPR wirelesss networks.

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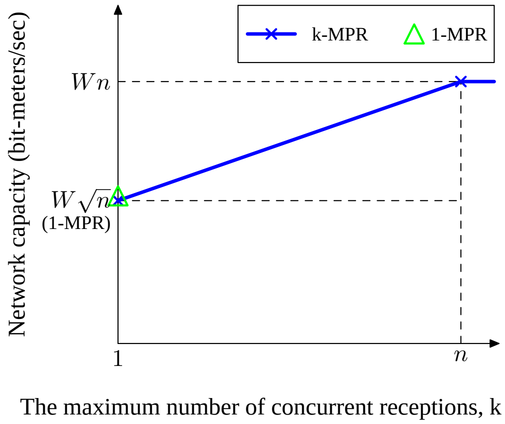

Fig. 1. The capacity of arbitrary k-MPR wireless networks (figure is not to scale). The upper bound and lower bound match exactly.

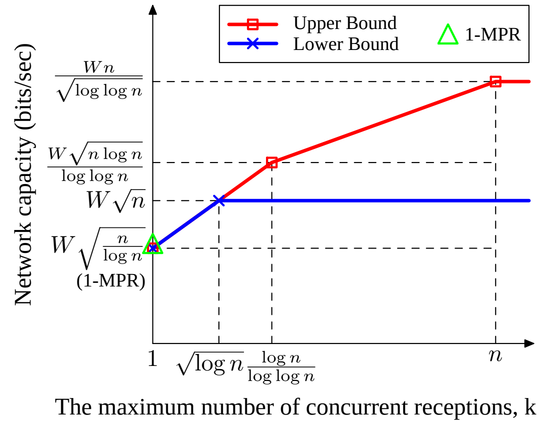

Fig. 2. Upper bound and lower bound for the capacity of random k-MPR wireless networks (figure is not to scale).

III. Network Model and Main Results

A. Network Model

In this work, we suppose that each node is equipped with one k-MPR wireless interface, which can decode at most k transmissions within its receiving range. We call such ability k-MPR ability. Each node can transmit at W bits/sec over a common wireless channel. We consider a more general scenario where the channel is divided into M subchannels, each of which has a capacity of W_m bits/sec, $1 \le m \le M$ and $\sum_{m=1}^{M} W_m = W$. Transmissions are slotted into synchronized slots of the same length τ. Packets are sent from source to destination in multi-hop.

1. Arbitrary Networks

In an arbitrary k-MPR wireless network, n nodes are arbitrarily located in a disk of unit area in the plane. Each node has an arbitrarily chosen destination to which it could send traffic at an arbitrary rate. Each node can choose an arbitrary power level for each transmission. Under these assumptions, we give the protocol interference model for arbitrary k-MPR wireless networks as follows.

Suppose k nodes, $\{X_{ip}|1 \le p \le k\}$, transmit to node X_j simultaneously. These transmissions are successfully decoded by node X_j if

$$|X_q - X_j| \ge \max_{1\le p\le k} (1+\Delta)\,|X_{ip} - X_j|$$

for any other node X_q simultaneously transmitting over the same subchannel.

Similar as [1], the quantity $\Delta > 0$ models a guard zone that prevents a neighboring node from transmitting on the same subchannel at the same time.

2. Random Networks

In a random k-MPR wireless network, n nodes are randomly located, i.e., independently and uniformly distributed, on the surface of a torus of unit area. Each node has one flow to a randomly chosen destination to which it wishes to send at $\lambda(n)$ bits/sec. Each node employs the same receiving range $r(n)$ for each reception. Under these assumptions, we give the protocol interference model for random k-MPR wireless networks as follows.

Suppose k nodes, $\{X_{ip}|1 \le p \le k\}$, transmit to node X_j simultaneously. These transmissions are successfully decoded by node X_j if

(1) The distance between X_{ip} and X_j is no more than $r(n)$.

$$\max_{1\le p\le k} |X_{ip} - X_j| \le r(n)$$

and

(2) For any other node X_q simultaneously transmitting over the same subchannel, the following inequation holds.

$$|X_q - X_j| \ge (1+\Delta)r(n)$$

B. Main Results

The results of this paper are presented as follows.

1. Arbitrary Networks

In an arbitrary network, the network capacity is measured in terms of “bit-meters/sec” (introduced by [1]). As shown in Fig. 1, the capacity of arbitrary k-MPR wireless networks is as follows:

(1) When $k = O(n)$, the network capacity is $\Theta(W\sqrt{kn})$ bit-meters/sec.
(2) When $k = \Omega(n)$, the network capacity is $\Theta(Wn)$ bit-meters/sec.

Note that Fig. 1 is not to scale. To simplify the illustration, we use piecewise linear curve to represent the capacity scaling law, although the scaling function is not piecewise linear. This figure convention also applys in Fig. 2.

2. Random Networks

In random networks, the network capacity is measured in terms of “bits/sec”. We give a lower bound and an upper bound for the capacity of random k-MPR wireless networks. As shown in Fig. 2, the network capacity exhibits different bounds as the order of k varies.

When $k = O(\sqrt{\log n})$, the lower bound and upper bound match exactly and the network capacity is $\Theta(Wk\sqrt{\frac{n}{\log n}})$ bits/sec.

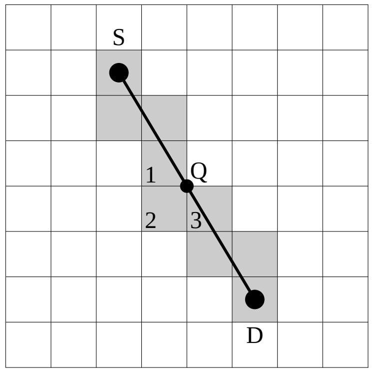

Fig. 3. Routing through cells in a random k-MPR wireless network

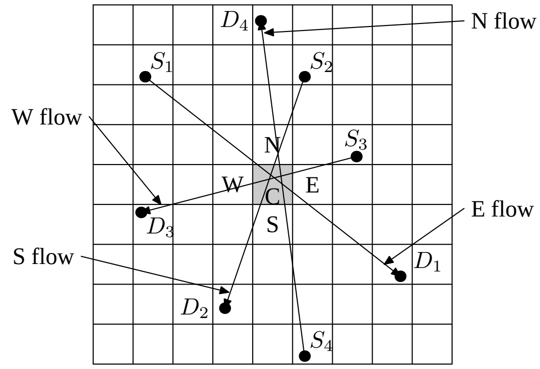

Fig. 4. The flows served by cell C are classified into 4 categories (E, S, W, N flows).

When $k = \Omega(\sqrt{\log n})$, the network capacity is $\Omega(W\sqrt{n})$ bits/sec. The capacity upper bound is presented as follows:

(1) When $k = \Omega(\sqrt{\log n})$ and $k = O(\frac{\log n}{\log \log n})$, network capacity is $O(Wk\sqrt{\frac{n}{\log n}})$ bits/sec.
(2) When $k = \Omega(\frac{\log n}{\log \log n})$ and $k = O(n)$, network capacity is $O(W\frac{\sqrt{kn}}{\sqrt{\log \log n}})$ bits/sec.
(3) When $k = \Omega(n)$, network capacity is $O(\frac{Wn}{\sqrt{\log \log n}})$ bits/sec.

IV. Capacity of Arbitrary Networks

Firstly, we derive the upper bound for the capacity of arbitrary k-MPR wireless networks in Section IV-A. Secondly, in order to illustrate that the upper bound is tight, we give a constructive lower bound, which matches the upper bound exactly, in Section IV-B.

A. Upper Bound

In an arbitrary k-MPR wireless network, we suppose the whole network transports λnT bits over T seconds. The average distance between the source and the destination of a bit is $\overline{L}$ meters. Consequently a capacity of $\lambda n\overline{L}$ bit-meters/sec is achieved. Under the network model we described in Section III-A, we have the following theorem.

Theorem 1: Under the protocol model, the upper bound for the capacity $\lambda n\overline{L}$ of arbitrary k-MPR wireless networks is presented as follows:

1. When $k = O(n)$, $\lambda n\overline{L} = O(W\sqrt{kn})$ bit-meters/sec.
2. When $k = \Omega(n)$, $\lambda n\overline{L} = O(Wn)$ bit-meters/sec.

Proof: When k is $O(n)$, we use similar techniques used in [1] to prove the result. There are two main differences in our proof. Firstly, different from [1], the n nodes should be grouped to utilize the k-MPR ability. Secondly, in [1], the disks centered at receivers should be disjoint and at one time slot there is at most one transmission in each disk. In our proof, these disks are also disjoint but there are at most k transmissions in each disk. When $k = \Omega(n)$, since there are not enough transmitters to further improve the throughput, the network capacity is at most the capacity when $k = \Theta(n)$, which has been given. Due to space constraint, we omit the full proof. □

B. A Constructive Lower Bound

In this section, we will show that the upper bound in the previous section is tight by achieving it. For deriving the achieved lower bound, we present the following lemma.

Lemma 1: Suppose $k_1 = O(k_2)$, C_1 is the capacity of k_1-MPR wireless networks, C_2 is the capacity of k_2-MPR wireless networks, then $C_1 = O(C_2)$.

Proof: The k_2-MPR wireless networks can imitate k_1-MPR wireless networks by restricting k_2-MPR ability to k_1-MPR ability. Hence the capacity of k_2-MPR wireless networks is at least the capacity of k_1-MPR wireless networks. Hence we get the lemma. □

Note that Lemma 1 applys in both arbitrary networks and random networks.

Theorem 2: The lower bound for the capacity of arbitrary k-MPR wireless networks is presented as follows:

1. When $k = O(n)$, $\lambda n\overline{L} = \Omega(W\sqrt{kn})$ bit-meters/sec.
2. When $k = \Omega(n)$, $\lambda n\overline{L} = \Omega(Wn)$ bit-meters/sec.

Proof: When $k = o(n)$, we use the similar node deployment used in [1] to prove the result. The only difference is that at each transmitting location we place k nodes rather than only deploy one node. A receiver and its k nearest transmitters form a transmitting group. There are k transmissions for each group. When $k = \Theta(n)$, for construction we suppose that $n = k + 1$ and there is only one transmitting group. The result when $k = \Omega(n)$ can be deduced from the result when $k = \Theta(n)$ by Lemma 1. Due to space constraint, we omit the full proof. □

V. Capacity of Random Networks

In a random k-MPR wireless network, we suppose that each node sends at $\lambda(n)$ bits/sec to its destination. The highest value of $\lambda(n)$ that can be supported by every source-destination pair with high probability is defined as the per-node throughput of the network. The traffic between a source-destination pair is referred to as "flow". Since there are n flows in total, the network capacity is defined to be $n\lambda(n)$. In Section V-A, we give a constructive lower bound for the capacity of random k-MPR wireless networks. In Section V-B, we give a loose upper bound.

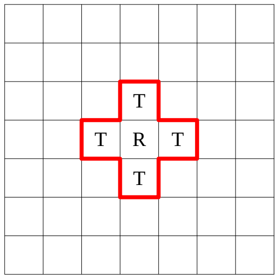

Fig. 5. Structure and its R-Cell and T-Cells in scheduling scheme

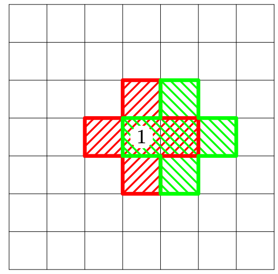

Fig. 6. 1-Conflict Structures

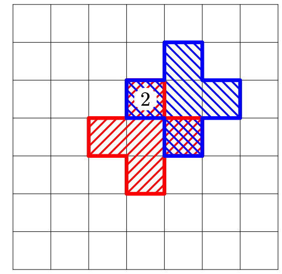

Fig. 7. 2-Conflict Structures

A. A Constructive Lower Bound

In order to establish a lower bound, we construct a routing scheme and a transmission scheduling scheme for any random k-MPR wireless network.

1. Cell Construction

Utilizing the approach used in [9], we divide the surface of the unit torus into square cells, each of which has an area of $a(n)$. We choose the receiving range $r(n)$ of each node to be $\sqrt{8a(n)}$. With this range, a node in one cell can communicate with any node in its eight neighboring cells. The area of each cell, $a(n)$, should be carefully chosen to satisfy multiple constraints, which will be described later. We set $a(n) = min(max(\frac{100 \log n}{n}, \frac{k}{n}), (\frac{\log \log n}{\log n})^2)$. The first constraint is connectivity constraint. The cell size $a(n)$ should be large enough, say, $a(n) \geq \frac{100 \log n}{n}$, to gurantee network connectivity. The second constraint is used to gurantee that each node has at least k neighbors to utilize the k-MPR ability. The cell size should be large enough, say, $a(n) \geq \frac{k}{n}$, to ensure that each node has at least k neighbors. The third constraint is to ensure that the network can at least accommodate the terminating flows of each node. The cell size should be chosen small enough, say, $a(n) \leq (\frac{\log \log n}{\log n})^2$ to satisfy this constraint. We will discuss the third constraint in detail in the discussion on the routing scheme. For construction, we use the following lemma to bound the number of nodes in each cell. This lemma has been proved in [11].

Lemma 2: If $a(n) > \frac{50 \log n}{n}$, then each cell has $\Theta(na(n))$ nodes, with high probability.

By construction, we gurantee that $a(n) \geq \frac{100 \log n}{n}$ for large n by setting $a(n) = min(max(\frac{100 \log n}{n}, \frac{k}{n}), (\frac{\log \log n}{\log n})^2)$. Consequently, Lemma 2 holds and each cell has $\Theta(na(n))$ nodes.

2. Routing Scheme

The routing scheme consists of two steps, cell assignment, and then node assignment.

Cell Assignment – Cells are assigned to serve each flow of the network. As shown in Fig. 3, packets of a flow are routed through the cells that lie along the straight line joining the source and the destination node. For each intersected cell of a flow (source-destination line), we choose a node to relay the traffic of this flow (we will describe the node assignment scheme later). We should consider a special case wherein the line passes a grid point exactly, say, Q, in Fig. 3. In this case, we require the cell on the right side of Q to serve this flow. Hence in Fig. 3 the packets of flow S-D should be firstly relayed from cell 1 to cell 2 and then be relayed from cell 2 to cell 3.

Node Assignment – For each flow served by a cell, we select a node from the cell to serve this flow. According to the cell assignment scheme, the next hop for any flow must be within one of the four neighboring cells: the northern, southern, eastern and western neighboring cell. Each flow served by a cell can be classified by direction into one of four cardinal categories. A served flow whose next hop is within the northern neighboring cell is called a N flow served by this cell. Similarly, we can define S, E, W flows served by a cell. The examples of N, S, E and W flows served by cell C can be found in Fig.4.

The node assignment scheme has two steps. In step one, the source node and destination node of a flow are assigned to serve the flow. In step two, we assign nodes to serve those flows that pass through a cell. For each passing flow of a cell, the node, which has been assigned to the least number of flows of the same category so far, is assigned to serve the flow. This step evenly distributes the flows of each category among the nodes of a cell. Hence for each flow category, all nodes serve nearly the same number of flows. The node assigned to a flow will receive packets from the assigned node in the previous cell and send the received packets to the assigned node in the next cell.

We present the following lemma to bound the number of source-destination lines that pass through any cell. This lemma is proved in [9].

Lemma 3: The number of source-destination lines passing through any cell (including lines originating and terminating in the cell) is $O(n\sqrt{a(n)})$ with high probability.

From Lemma 2 and Lemma 3, we conclude that each node serves $O(\frac{1}{\sqrt{a(n)}})$ flows.

After introducing the routing scheme, we discuss the third constraint for cell size. Because each node picks a destination node randomly, a node may be the destination of multiple flows. Let $D(n)$ be the maximum number of flows that have the same destination node. We use the following lemma to bound $D(n)$. This lemma is proved in [11].

Lemma 4: The maximum number of flows for which a node in the network is a destination, $D(n)$, is $\Theta(\frac{\log n}{\log \log n})$, with high probability.

Because each node should at least accommodate the flows that have the node as destination, we should ensure $\frac{1}{\sqrt{a(n)}} = \Omega(D(n))$. Hence the cell size $a(n)$ should not exceed $\Theta((\frac{\log\log n}{\log n})^2)$.

3. Scheduling Transmissions

The scheduling scheme aims to schedule each flow with equal opportunity (i.e., all flows in a local region are served with same time slots) while satisfying the following constraints.

(1) A node cannot transmit and receive simultaneously, since each node has only one k-MPR wireless interface.
(2) A node cannot transmit to more than one receivers simultaneously. The reason is the same as that of constraint (1).
(3) Any two simultaneous transmissions should not interfere with each other under the protocol model.
(4) In order to utilize k-MPR ability, the transmissions for the same receiver should be concurrent.

For scheduling transmissions, we define a new scheduling unit "Structure". In Fig. 5, the five cells surrounded by the red line compose a structure. We call the centering cell receiving cell (R-Cell). Each of the four neighboring cells of a R-Cell is called transmitting cell (T-Cell). For calculating the achieved network capacity, we consider the transmission scheduling in one second. We divide one second into several big slots, "Structure Slots". Then we further divide each structure slot into smaller slots, "Flow Slots".

We build a schedule using a two-layer process. The first layer is "Structure Layer" and the second layer is "Flow Layer". On the structure layer, we schedule the structures with structure slots to avoid conflicts and interferences between any two structures. On the flow layer, we schedule the individual flows with flow slots. After illustrating the two layer scheduling scheme, we will prove that the scheduling scheme schedules each flow with equal opportunity while satisfying the four constraints. Hence this scheme is feasible.

Structure Layer – On the structure layer, we schedule the structures satisfying the following requirements

(1) In any structure slot, a cell cannot act as both a T-Cell and a R-Cell.
(2) In any structure slot, there is only one R-Cell for each T-Cell.
(3) In any structure slot, the transmissions of one structure should not interfere with the transmissions of any other structure under the protocol model.

Based on the three requirements, we introduce the definition of *i-conflict structure*, $1 \leq i \leq 3$. If structure A and B cannot be scheduled in same structure slot due to requirement (i), $1 \leq i \leq 3$, we say that structure A is a *i-conflict structure* of structure B, and vice versa. Structure A and B are i-conflict structures with each other.

Theorem 3: Under the protocol model, there is a schedule such that in every c structure slots, each structure in the tessellation gets one structure slot such that the three requirements are satisfied where c depends only on Δ.

Proof: To prove the theorem, we should satisfy each of the three requirements.

Firstly, to satisfy the requirement (1), we should ensure that the 1-conflict structures are scheduled in different structure slots. As shown in Fig. 6, cell 1 acts as a R-Cell in the red structure (the left structure) while it acts as a T-Cell in the green structure (the right structure). Hence red and green structures are 1-conflict structures with each other. In our grid tessellation, the number of 1-conflict structures for a structure is a constant, say, c'_1 (in fact, it is four).

Secondly, as shown in Fig. 7, cell 2 acts as a T-Cell in the red structure (the lower left structure) while it acts as a T-Cell in the blue structure (the upper right structure). Hence the red and blue structures are 2-conflict structures with each other. In our grid tessellation, the number of 2-conflict structures for a structure is constant, say, c'_2 (in fact, it is eight).

Thirdly, we will satisfy the requirement (3). Let N_1 denote the number of interfering cells of each cell in a random 1-MPR network (the cell tessellation is the same with that we described above). Let N_2 denote the number of interfering cells for each cell in a random k-MPR network. Let N_3 denote the number of interfering structures of each structure in a random k-MPR network. Since only the R-Cell of a structure can be interfered by other cells, we have $N_3 \leq N_2$. Since the only difference between the protocol models of 1-MPR and k-MPR networks is k concurrent reception, we have $N_2 \leq N_1$. Hence we have $N_3 \leq N_1$. [11] has shown that N_1 is upper bounded by a number, say, c'_3, which depends only on Δ. Hence the number of 3-conflict structures for a structure is less than c'_3.

Letting each structure denote a node, we build an interference graph. There is an edge between two nodes, if the corresponding two structures of the two nodes are i-conflict structures, $1 \leq i \leq 3$, with each other. From the analysis above, we conclude that the maximum vertex degree of this interference graph is at most $c'_1 + c'_2 + c'_3$. Let $c' = c'_1 + c'_2 + c'_3$. According to [12], the interference graph can be vertex-colored with at most $(c' + 1)$ colors. Letting $c = c' + 1$, we get the theorem. □

Flow Layer – On the flow layer, we build a two-phase scheme to schedule each flow served by R-Cell using the k-MPR ability when a structure is scheduled for a structure slot. To achieve the scheduling objective, flow layer scheduling should ensure that each flow is served with equal number of flow slots in a structure slot. To achieve this goal and to be feasible, flow layer scheduling should satisfy the following requirements.

(1) In both phases, the number of concurrent receptions cannot exceed the number of transmitters.
(2) In the second phase, the number of concurrent receptions cannot exceed the number of flows each node serves.

We will show that these requirements are necessary conditions for feasible flow layer scheduling later.

Because of these requirements, we cannot always fully utilize the k-MPR ability and the feasible number of concurrent receptions cannot always be k. Hence for flow layer scheduling, we define "g" as the feasible number of concurrent receptions where $g \leq k$.

First Phase – In this phase, each flow gets a flow slot to be transmitted from the T-Cell to the R-Cell. For each node in the T-Cells, each of the served flows whose next hops are in the R-Cell is transmitted for one flow slot. Since we use multi-packet reception, the number of receivers is less than the number of transmitters (ideally, the ratio between transmitters and receivers is k, when k-MPR ability is fully utilized). The other nodes in R-Cell do nothing in first phase. By construction, we arbitrarily select a portion (this portion, which is dependent on g, will be discussed later) of nodes from each cell and use them as receivers when the cell acts as a R-Cell in the first phase. We call these receivers of a R-Cell in first phase "Busy Nodes" and call all of the other nodes "Free Nodes". In the first phase, busy nodes receive transmissions on behalf of a R-Cell. Additionally, the flows served by free nodes but received by busy nodes in first phase should be distributed uniformly among the busy nodes for utilizing multi-packet reception in second phase.

Second Phase – In this phase, each flow served by free nodes is assigned a flow slot to be transmitted from busy nodes to free nodes using the k-MPR ability. The transmitters in this phase are busy nodes.

Theorem 4: Using the two-phase flow layer scheduling, for a scheduled structure, each flow served by the R-Cell can be assigned one effective flow slot to be transmitted from the serving node in the T-Cell to the serving node in the R-Cell.

Proof: The flows served by the R-Cell can be classified into two categories. The first category includes all the flows served by the busy nodes of the R-Cell. The second category includes all the flows served by free nodes of R-Cell. For the first category, each of the flows is assigned only one flow slot in the first phase. Hence each flow is transmitted from the serving node in the T-Cell to the serving node in the R-Cell for one effective flow slot. For the second category, each of the flows is assigned two flow slots (one in the first phase and one in the second phase). Since in the first assigned flow slot, the flow is transmitted from T-Cell to a temporary receiver, the number of effective assigned flow slots is only one (the one assigned in second phase). Hence in flow layer scheduling, each of the flows served by the R-Cell gets one effective flow slot to be transmitted from the serving node in the T-Cell to the serving node in the R-Cell. □

After introducing the two-phase process, we present the following theorem to show that the two requirements are necessary for feasible flow layer scheduling.

Theorem 5: If flow layer scheduling with g concurrent receptions is feasible, then requirement (1) and (2) are satisfied.

Proof: Firstly, we prove that requirement (1) is a necessary condition for feasible flow layer scheduling with g concurrent receptions. If requirement (1) is not satisfied, g is larger than the number of transmitters in the first or second phase. Hence we have not enough transmitters to fully use g concurrent receptions and flow layer scheduling with g concurrent receptions is infeasible, which contradicts the assumption. Secondly, we prove that requirement (2) is a necessary condition for feasible flow layer scheduling with g concurrent receptions. If requirement (2) is not satisfied, g is larger than the number of flows each node serves in second phase. As a result, to make g concurrent receptions feasible, some or all of the flows served by free nodes are assigned more than one flow slot because there are not enough flows for assigning each flow one flow slot to fully use g concurrent receptions. However, each of these flows is assigned only one flow slot in first phase and in each flow slot, a node sends at a data rate which is at most W bits/sec, so the maximum number of bits of each flow to be transmitted in second phase is at most $W \cdot$ (*length of flow slot*). Hence in order to keep flow conservation, in second phase, in all the assigned flow slots for each flow (including the added flow slots), the total number of transmitted bits of the flow cannot exceed $W \cdot$(*length of flow slot*). Consequently the added flow slots due to g concurrent receptions contribute nothing to the capacity. Hence g concurrent receptions is infeasible, which contradicts the assumption. □

We give the following theorem to justify the proposed two-layer scheduling scheme.

Theorem 6: The proposed two-layer scheduling scheme is feasible and schedules each flow with equal opportunity.

Proof: As each cell can act either as a R-Cell or a T-Cell and according to the routing scheme all the flows served by a cell are from the four T-Cells, each flow in the network can be served. From Theorem 4, we conclude that each flow is served with equal opportunity. Next, we prove that the scheme satisfies the four constraints to conclude that the scheme is feasible. Firstly, on the structure layer, a cell cannot act both as a R-Cell and act as a T-Cell in any structure slot and on the flow layer, a node cannot both transmit and receive in any flow slot, so constraint (1) is satisfied. Secondly, on the structure layer, there is only one R-Cell for each T-Cell in any structure slot and on the flow layer, a transmitter has only one receiver in any flow slot, so constraint (2) is satisfied. Thirdly, on the structure layer, the transmissions of a structure do not interfere with transmissions of any other structure under the protocol model in any structure slot and on the flow layer, at most k flows are scheduled in any flow slot, so there is no interference between any two transmissions under the protocol model and constraint (3) is satisfied. Fourthly, on the flow layer, g receptions of a receiver is scheduled in same flow slot, so the constraint (4) is satisfied. Consequently we get the theorem. □

4. The achieved capacity lower bound

After introducing the whole scheduling scheme, we discuss the capacity lower bound for random k-MPR wireless networks. Recalling that each node serves $O(\frac{1}{\sqrt{a(n)}})$ flows, for requirement (2) of flow layer scheduling, we should ensure that the number of concurrent receptions is $O(\frac{1}{\sqrt{a(n)}})$ in the second phase.

a. When $k = O(\sqrt{\log n})$, we have the following lemmas for the capacity lower bound.

Lemma 5: When $k = O(\sqrt{\log n})$, we can fully utilize k-

MPR ability and the number of concurrent receptions is k.

Proof: To prove the lemma, we need to show that with k concurrent receptions, the requirement (1) and (2) are satisfied. Note that when $k = O(\sqrt{\log n})$, the cell size $a(n)$ is $\frac{100\log n}{n}$. In first phase, it can be concluded from the node assignment scheme that the number of transmitters is $\Theta(na(n)) = \Theta(\log n) = \Omega(k)$. In second phase, the number of transmitters (busy nodes), $\Theta(\frac{na(n)}{k})$, is $\Omega(k)$. Hence requirement (1) is satisfied. Since $k = O(\sqrt{\log n})$, k is $O(\frac{1}{\sqrt{a(n)}})$. Hence requirement (2) is satisfied. □

Based on Lemma 5, we present the following lemma for the capacity lower bound when $k = O(\sqrt{\log n})$.

Lemma 6: When $k = O(\sqrt{\log n})$, $n\lambda(n) = \Omega(kW\sqrt{\frac{n}{\log n}})$ bits/sec.

Proof: In the first phase, since the number of flows is $O(n\sqrt{a(n)})$ and the number of concurrent receptions is k, we need $O(\frac{n\sqrt{a(n)}}{k})$ flow slots to assign each flow one flow slot. In the second phase, the number of flows to be redistributed from busy nodes to free nodes is $O((\frac{k-1}{k})n\sqrt{a(n)}) = O(n\sqrt{a(n)})$, so we also need $O(\frac{n\sqrt{a(n)}}{k})$ flow slots to assign each flow one flow slot. Consequently, for flow layer scheduling, we need $O(\frac{n\sqrt{a(n)}}{k})$ flow slots to schedule the flows served by R-Cell such that each flow is assigned one effective flow slot to be transmitted from the serving node in T-Cell to the serving node in R-Cell.

We divide one second into c structure slots and divide each structure slot into $O(\frac{n\sqrt{a(n)}}{k})$ flow slots, each of which has a length of $\Omega(\frac{k}{cn\sqrt{a(n)}})$ seconds. Since each node can transmit at the rate of W bits/sec, in each flow slot, $\lambda(n) = \Omega(\frac{kW}{cn\sqrt{a(n)}})$ bits can be transported. Hence $\Omega(\frac{kW}{cn\sqrt{a(n)}})$ bits of each flow can be transmitted in one second. According to Theorem 3, c is a number dependent only on Δ. Since $a(n) = \frac{100\log n}{n}$, we get the lemma. □

b. When $k = \Omega(\sqrt{\log n})$ and $k = O(\sqrt{n})$, we have the following lemmas for the capacity lower bound.

Lemma 7: When $k = \Omega(\sqrt{\log n})$ and $k = O(\sqrt{n})$, we can partially use the k-MPR ability and the number of concurrent receptions is $\Theta(\sqrt{na(n)})$.

Proof: Note that when $k = \Omega(\sqrt{\log n})$ and $k = O(\sqrt{n})$, the cell size $a(n)$ is $\frac{k}{n}$. Firstly, we prove that when $k = \Omega(\sqrt{\log n})$ and $k = O(\sqrt{n})$, we cannot fully use the k-MPR ability and the number of concurrent receptions, g, cannot be k. To see this, if the number of concurrent receptions is k, in second phase, the number of transmitters (busy nodes) is $\Theta(\frac{na(n)}{k}) = \Theta(1) = O(k)$. Hence the number of concurrent receptions exceeds the number of transmitters, which contradicts requirement (1). Hence we cannot fully use the k-MPR ability. Secondly, we prove that we can partially use the k-MPR ability and the number of concurrent receptions is $\Theta(\sqrt{na(n)})$. To see this, if the number of concurrent receptions, g, is $\Theta(\sqrt{na(n)})$, in the first phase, it's easy to see that the number of concurrent transmissions does not exceed the number of transmitters. In second phase, the number of transmitters is $\Theta(\frac{na(n)}{g}) = \Theta(\sqrt{na(n)})$, which has the same order compared with the number of concurrent receptions, g. Hence requirement (1) is satisfied. Since $k = O(\sqrt{n})$, we have $g = \Theta(\sqrt{na(n)}) = O(\frac{1}{\sqrt{a(n)}})$ in the second phase. Hence requirement (2) is satisfied. □

Based on Lemma 7, we present the following lemma for the capacity lower bound when $k = \Omega(\sqrt{\log n})$ and $k = O(\sqrt{n})$.

Lemma 8: When $k = \Omega(\sqrt{\log n})$ and $k = O(\sqrt{n})$, $n\lambda(n) = \Omega(W\sqrt{n})$ bits/sec.

Proof: When $k = \Omega(\sqrt{\log n})$ and $k = O(\sqrt{n})$, the number of concurrent receptions, g, is $\Theta(\sqrt{na(n)})$. Substituting k with $\Theta(\sqrt{na(n)})$, using the similar techniques used in proof for Lemma 6, we get the lemma. □

c. When $k = \Omega(\sqrt{n})$, we have the following lemma for the capacity lower bound.

Lemma 9: When $k = \Omega(\sqrt{n})$, $n\lambda(n) = \Omega(W\sqrt{n})$ bits/sec.

Proof: According to Lemma 1, when $k = \Omega(\sqrt{n})$, the achieved capacity is at least the achieved capacity when $k = \Omega(\sqrt{\log n})$ and $k = O(\sqrt{n})$. Since $n\lambda(n) = \Omega(W\sqrt{n})$ bits/sec when $k = \Omega(\sqrt{\log n})$ and $k = O(\sqrt{n})$, when $k = \Omega(\sqrt{n})$, $n\lambda(n) = \Omega(W\sqrt{n})$ bits/sec. □

Based on Lemmas 6, 8 and 9, the capacity lower bound can be presented as follows.

Theorem 7: The lower bound for the capacity of random k-MPR wireless networks is as follows:

1. When $k = O(\sqrt{\log n})$, $n\lambda(n) = \Omega(kW\sqrt{\frac{n}{\log n}})$ bits/sec.
2. When $k = \Omega(\sqrt{\log n})$, $n\lambda(n) = \Omega(W\sqrt{n})$ bits/sec.

B. Upper Bound

In this section, we give the upper bound for the capacity of random k-MPR wireless networks. The capacity of random k-MPR wireless networks is constrained by three constraints, destination bottleneck constraint, general constraint and interference constraint respectively. For each of these constraints, there is an upper bound for the network capacity. These upper bounds together define the upper bound for the capacity of random k-MPR wireless networks.

Destination Bottleneck Constraint – The capacity of random k-MPR wireless networks is constrained by the data that can be received by a destination node. Using similar techniques used in [11], we can get following upper bound for this constraint.

1. When $k = O(n)$, $n\lambda(n) = O(\frac{Wnk\log\log n}{\log n})$ bits/sec.
2. When $k = \Omega(n)$, $n\lambda(n) = O(\frac{Wn^2\log\log n}{\log n})$ bits/sec.

General Constraint – Since a random k-MPR wireless network is a special kind of arbitrary k-MPR wireless network, the capacity upper bound for arbitrary networks is also applicable in random k-MPR wireless networks. Because the distance between the source and destination of a flow is $\Theta(1)$, we have the following capacity upper bound.

1. When $k = O(n)$, $n\lambda(n) = O(W\sqrt{kn})$ bits/sec.
2. When $k = \Omega(n)$, $n\lambda(n) = O(Wn)$ bits/sec.

Interference Constraint – Since we need to avoid the interference, we present the upper bound for this constraint below.

Theorem 8: To satisfy the interference constraint, the capacity of random k-MPR wireless networks is bounded as follows

1. When $k = O(\frac{\log n}{\log\log n})$, $n\lambda(n) = O(Wk\sqrt{\frac{n}{\log n}})$ bits/sec.
2. When $k = \Omega(\frac{\log n}{\log\log n})$ and $k = O(n)$, $n\lambda(n) = O(W\sqrt{\frac{kn}{\log\log n}})$ bits/sec.
3. When $k = \Omega(n)$, $n\lambda(n) = O(\frac{Wn}{\sqrt{\log\log n}})$ bits/sec.

Proof: To prove the theorem, we use similar techniques used in [1]. Firstly, based on the area consuming observation, we can bound the total data rate of the whole network, D_1. Secondly, by lower bounding the number of hops for a flow, we can lower bound the data rate served by the whole network, D_2, which is dependent on the receiving range, $r(n)$. Obviously, we have $D_2 \leq D_1$. To make the upper bound tight, we should ensure that the network is connected and each node has at least k neighbors to utilize the k-MPR ability. According to the results of [10], we can take the receiving range, $r(n) \geq \sqrt{\frac{\log n + 2k\log\log n}{\pi n}}$ to satisfy these two requirements. Discussing the order of k, we can get the theorem based on the inequation $D_2 \leq D_1$. Due to the space constraint, we omit the full proof. □

Because the first two upper bounds are loose compared with the third one, the third upper bound is the upper bound for the capacity of random k-MPR wirelesss networks.

VI. Discussions

We analyze the derived results in this section. As shown in Section IV, the capacity of arbitrary k-MPR wireless networks becomes $\Theta(Wn)$ bit-meters/sec when $k = \Omega(n)$, which means the network is truly scalable. The main reasons for this scalable network are the arbitrary deployment of nodes and arbitrary traffic pattern, which allow us to deploy nodes and plan the traffic pattern properly in order to fully utilize the k-MPR ability.

On the other hand, in random k-MPR wireless networks, both the deployment of nodes and the traffic pattern are random, so we cannot intentionally deploy the nodes and plan the traffic pattern to fully utilize the k-MPR ability. Receivers should redistribute the received flows to more nodes in order to allow more transmitters to transmit the flows, which utilizes the k-MPR ability better. Recalling our constructive procedure for the lower bound, when k is small enough, say, $k = O(\sqrt{\log n})$, the random network has enough transmitters and flows to fully utilize the k-MPR ability so the capacity gain over 1-MPR networks is exactly $\Theta(k)$. When $k = \Omega(\sqrt{\log n})$, since k is too large, there are not enough transmitters or enough flows to fully utilize the k-MPR ability. Consequently we can only employ part of the k-MPR ability and the capacity gain of the lower bound over 1-MPR networks is $\Theta(\sqrt{\log n})$.

Satisfying the destination bottleneck constraint, the general constraint and the interference constraint, we give a loose upper bound. We see that this upper bound is loose by noting that in the deriving process of this upper bound, the receivers do not redistribute flows to more nodes for future transmissions using the k-MPR ability. Consequently, for later transmissions, there are not enough transmitters to utilize the k-MPR ability. Hence this upper bound cannot be achieved by any random k-MPR wireless network in fact.

In summary, the main constraints for random k-MPR wireless networks to utilize the k-MPR ability are the limited number of transmitters and the limited number of flows each node serves. To address the first constraint, the receivers need to distribute the received flows to more nodes for future transmissions. To address the second constraint, we should assign more flows to each node (i.e., assign the same flow to more nodes to balance the traffic load and improve the robustness of the network).

VII. Conclusions

In this work, we study the capacities of both arbitrary k-MPR wireless networks and random networks. We equip each node in the network with one k-MPR wireless interface. Each interface is able to decode at most k concurrent transmissions within its receiving range. Under these assumptions, we have shown that for arbitrary networks, when $k = O(n)$, there is a $\Theta(\sqrt{k})$ capacity gain over 1-MPR wireless networks. When $k = \Omega(n)$, the capacity of arbitrary k-MPR wireless networks is $\Theta(Wn)$ bit-meters/sec and the network is scalable. For random networks, we give a constructive lower bound and an upper bound for the network capacity. We have shown that even the lower bound has a capacity gain of $\Theta(\sqrt{\log n})$ over 1-MPR random wireless networks when k is large enough. From these results, we conclude that the main constraints for the capacity of random k-MPR wireless networks are the limited number of transmitters and the limited number of flows served by each node.

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References

[1] P. Gupta and P. R. Kumar, "The Capacity of Wireless Networks", *IEEE Transaction on Information Theory*, vol. 46, no. 2, pp. 388–404, March 2000.

[2] P. Zhou, X. Wang and R. R. Rao, "Asymptotic Capacity of Infrastructure Wireless Mesh Networks," *IEEE Transaction on Mobile Computing*, vol. 7, no. 8, pp. 1011–1024, Aug. 2008.

[3] R. M. D. Moraes, H. R. Sadjadpour, and J. J. Garcia-Luna-Aceves, “Many-to-Many Communication: A New Approach for Collaboration in MANETs”, *IEEE INFOCOM*, 2007.

[4] G. D. Celik, G. Zussman, W. F. Khan and E. Modiano, “MAC for Networks with Multipacket Reception Capability and Spatially Distributed Nodes”, *IEEE INFOCOM*, 2008.

[5] J. J. Garcia-Luna-Aceves, H. R. Sadjadpour, and Z. Wang, “Challenges: Towards Truly Scalable Ad Hoc Networks”, *ACM MobiCom*, 2007.

[6] J. J. Garcia-Luna-Aceves, H. Sadjadpour and Z. Wang, “Extending the Capacity of Ad Hoc Networks Beyond Network Coding”, *ACM IWCMC*, 2007.

[7] X. Wang and J. J. Garcia-Luna-Aceves, “Embracing Interference in Ad Hoc Networks Using Joint Routing and Scheduling with Multiple Packet Reception”, *IEEE INFOCOM*, 2008.

[8] P. Patel and J. Holtzman, “Analysis of a Simple Successive Interference Cancellation Scheme in a DS/CDMA System”, *IEEE Journal on Selected Areas in Communications*, vol. 12, no. 5, pp. 796–807, June 1994.

[9] A. E. Gamal, J. Mamen, B. Prabhakar, and D. Shah, “Throughput-Delay Trade-Off in Wireless Networks”, *IEEE INFOCOM*, 2004.

[10] P. J. Wan, C. W. Yi, “Asymptotic Critical Transmission Radius and Critical Neighbor Number for k-Connectivity in Wireless Ad Hoc Networks”, *ACM MobiHoc*, 2006.

[11] P. Kyasanur and N. H. Vaidya, “Capacity of Multi-Channel Wireless Networks: Impact of Number of Channels and Interfaces”, *ACM MobiCom*, 2005.

[12] D. B. West, “Introduction to Graph Theory”, *Prentice Hall*, 2nd edition, 2001.

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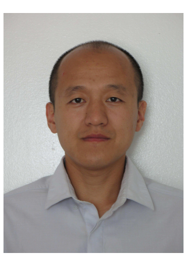

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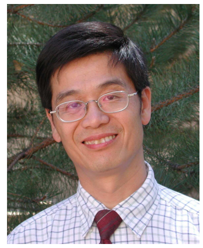

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