Topological Analysis of Wireless Sensor Networks Based on Nodes’ Spatial Distribution

Changle Li, Member, IEEE, Liran Wang, Tingting Sun, Sen Yang, Xiaoying Gan, Feng Yang, and Xinbing Wang, Senior Member, IEEE

Abstract—In this paper, we explore methods to generate optimal network topologies for wireless sensor networks (WSNs) with and without obstacles. Specifically, we investigate a dense network with \( n \) sensor nodes and \( m = n^b \) (\( 0 < b < 1 \)) helping nodes, and assess the impact of topology on its throughput capacity. For networks without obstacles, we find that uniformly distributed sensor nodes and regularly distributed helping nodes have some advantages in improving the throughput capacity. We also explore properties of networks composed of some isomorphic sub-networks. For networks with obstacles, we assume there are \( M = \Theta(n^v) \) (\( 0 < v \leq 1 \)) arbitrarily or randomly distributed obstacles, which block cells they are located in, i.e., sensor nodes cannot be placed in these cells and nodes’ communication cannot cross them directly. We find that the overall throughput capacity is bounded by the transmission burden in areas around these blocked cells and introduce a novel algorithm of complexity \( O(M) \) to generate optimal sensor nodes’ topologies for any given obstacles’ distributions. We further analyze its performance for regularly distributed obstacles, which can be taken to estimate the lower bound of the algorithm’s performance.

Index Terms—Wireless Sensor Network, topology, throughput capacity, nodes’ spatial distribution.

I. INTRODUCTION

Network capacity is a fundamental issue in wireless sensor networks (WSNs). A typical WSN involves little or no infrastructure and sensor nodes may communicate in an ad hoc manner. In Gupta and Kumar’s seminal work, they adopt Protocol and Physical Model to describe a successful transmission and show the per-node throughput capacity scales as \( \Theta(1/\sqrt{n \log n}) \) in random networks, and the per-node transport capacity scales as \( \Theta(1/\sqrt{n}) \) in arbitrary networks, respectively [1]. Extensive research on the broadcast and multicast capacity for ad hoc network has been conducted in [2], [3]. These results provide us not only a theoretical bound but also a foundation in the network optimization and protocol design. Following their work, more researches are conducted to understand the scaling laws in wireless sensor networks.

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Changle Li, Liran Wang and Tingting Sun are with the State Key Laboratory of Integrated Service Networks, Xidian University, Xi’an, Shaanxi, 710071 China.

Sen Yang, Xiaoying Gan, Feng Yang, and Xingbing Wang are with the Department of Electronic Engineering, Shanghai Jiao Tong University, Shanghai, 200240 China.

Xinbing Wang is also with the National Mobile Communications Research Laboratory, Southeast University, Nanjing, 211102 China.

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However, most of works above are for networks with regularly or uniformly distributed sensor nodes. While in practice, sensor nodes may not be placed uniformly, which could have a huge impact on network properties, including the capacity. For example, if lots of nodes are confined in a small region, it would lead to great interference and deteriorate the capacity. Also, if nodes are too sparse in a particular area, communication might get difficult, which also harms network performance. To the best of our knowledge, only a few works have dealt with the capacity of networks with inhomogeneous node density. In [17], [18], [19], [20], [22], capacity of inhomogeneous clustered networks is analyzed. Corresponding scheduling and routing schemes to approach the upper bounds for both Cluster Grid and Cluster Random models are discussed in [21].

On the other hand, almost all the previous works dealt with flat network region. While in practice, sensor networks are often deployed in complex environments, such as battle fields or mountainous areas, and there are often many obstacles distributed in these regions. These obstacles may constrain the distribution of sensor nodes and the transmission of packets. For example, in a building monitoring WSN, electromagnetic wave signal can be attenuated significantly when passing through furniture, walls or floors, which could have a great impact on network performance. Another example is WSNs deployed in a mountainous area, in which both routing strategy design and deployment of sensor nodes should consider the constraint of the complex landscape. Generally, obstacles have a negative impact on the network capacity. However, if we design the network topology appropriately, it could lead to a favorable improvement. For example, in building monitoring WSNs, the capacity can be improved if we place less nodes in areas shadowed by obstacles. Also, in a mountainous region, if we deploy more nodes in open areas, network capacity can be much larger than that we put most of them in valleys or
These motivate us to explore better network topologies for given network regions, especially for networks with obstacles. In this paper, we investigate how nodes’ spatial distributions influence the throughput capacity and explore the optimal nodes distribution on given conditions. We firstly model three typical network spatial topologies, analyze the network capacity respectively and make comparison of the results. Based on intuitive explanation and derivation of the results, we draw some useful conclusions on generating the optimal topology for flat network areas. For networks with obstacles, it’s difficult to derive a general solution for various obstacles distributions. However, a feasible algorithm with linear complexity can be proposed by dividing the whole network region into some small pieces and dealing with them respectively.

Our main contributions are as follows:

- For one overall network consisting of many isomorphic sub-networks, the throughput capacity of the overall network is larger than that of one sub-network that has the same network scale.
- For networks without helping nodes, uniform sensor nodes’ distribution is ordered optimally on maximizing the throughput capacity.
- For networks with non-uniformly distributed user nodes and without helping nodes, if the value range of user nodes distribution’s PDF is limited, the gap in achievable throughput of non-uniform networks and uniform networks is at most a constant time.
- For networks with uniformly distributed sensor nodes, we find that regularly distributed helping nodes are optimal to maximize the network throughput capacity.
- For networks with non-uniformly distributed sensor nodes, though regularly distributed helping nodes are no longer optimal, any improvement on helping nodes’ distribution cannot change the throughput capacity on scale.
- For networks with obstacles, we introduce a novel algorithm of complexity $O(M)$ to generate the optimal sensor nodes’ topology for any given obstacles’ distribution. We further analyze the algorithm’s performance for networks with regularly distributed obstacles.

The rest of the paper is organized as follows. Section II gives the network model. Section III studies the connectivity of networks with different nodes’ distributions. In Section IV, we derive the throughput capacity of networks with different topologies. In Section V and VI, we explore some general properties of network topologies. In Section VII, we investigate the throughput capacity of networks with obstacles and introduce an algorithm to generate the optimal user nodes’ topology for any given obstacles’ distribution. Finally, we conclude the paper in Section VIII.

II. NETWORK MODEL

In this section, we introduce the heterogeneous wireless network model, definition of obstacles, routing strategy and scheduling scheme.

Network Components: A heterogeneous wireless network is a dense network with $n$ user nodes and $m = n^b$ ($0 < b < 1$) helping nodes. Here we assume that the network has asymmetric traffic as that defined in [15], [16], i.e., all the $n$ user nodes are sources while only $n^d (0 < d < 1)$ user nodes are randomly chosen as destinations. User nodes can serve as relays if needed while helping nodes do not have information to transmit or receive and they only help relaying packets from other user nodes. We divide the network traffic into user mode and helping mode, according to whether user nodes’ packets are forwarded by helping nodes. In user mode, packets are forwarded only by user nodes. While in helping mode, packets are firstly sent to the nearest helping nodes, and then forwarded to intended destinations in the helping network. Meanwhile, we assume that all the user nodes have a total bandwidth of 1 and split it into three parts as following

$$W_1 + W_2 + W_3 = 1$$

where $W_1$, $W_2$ and $W_3$ are for ad hoc transmissions in user mode, uplink transmissions in helping mode and downlink transmissions in helping mode, respectively. Besides, we assume that ad hoc transmissions in helping mode have an independent bandwidth of $W_4 = \Omega(1)$.

Definition of Obstacles: To describe networks with obstacles, we assume the network area is partitioned into $K = \Theta(n^w)$ ($0 < w \leq 1$) cells. When there is no user node in a cell, we assume at the cell’s center there is a relay working in the same bandwidth as user nodes, which keeps the network’s connectivity. Assume there are $M = \Theta(n^v)$ ($0 < v \leq w$) obstacle nodes in the network area, which can be arbitrarily or randomly distributed. Cells are blocked when there are obstacle nodes in them. Here, “blocked” has two implications: no user nodes can be distributed in blocked cells and communication between user nodes cannot cross blocked cells.

Interference Model: To bound the interference between different nodes, we suppose the system is based on a cellular network model. Assume that the network is a unit square and we divide it into non-overlapping cells. For user mode and helping mode, the length of cells will be different, and we will present a specific pattern study in the following section. Nodes can communicate with each other only when they are in the adjacent cells. Furthermore, we assume that communications between different cells has taken time division multiplexing (TDM) scheme. Therefore, to avoid interference between adjacent cells, we adopt a rotating scheduling scheme as that described in [15], [16]. Thus, at the same time, in all of the adjacent cells there is at most one that can transmit or receive packets and each cell has the same opportunity to be active.

Following the power propagation model introduced in [23], the reception power at node $X_j$ of the signal from node $X_i$ is

$$P_{ij} = C \frac{P_i}{d_{ij}^\gamma}$$  \hspace{1cm} (1)

where $d_{ij}$ is the distance between node $X_i$ and node $X_j$, $\gamma$ is the path loss exponent and $P_i$ is the power emitted by node $X_i$. According to Shannon Theorem, the achievable transmission rate $R_{ij}$ from node $X_i$ to node $X_j$ is:

$$R_{ij} = W \log(1 + SINR_{ij})$$ \hspace{1cm} (2)
where $W$ is the channel bandwidth, and $SINR_{ij} = \frac{C_i P_{ij}}{N + \sum_{x \neq i,j} C_x \frac{P_{ij}}{d_{ij}^2}}$ is the Signal-to-Interference and Noise Ratio of the transmission from node $X_i$ to node $X_j$. In this paper we assume that all the user nodes and all the helping nodes adopt the same transmission power. As derived in [15], [16], we have the following lemma.

**Lemma 1.** Each cell in the network can transmit at a transmission rate $c_1 W_1$, where $c_1$ is a deterministic positive constant.

**Routing Strategy:** As user nodes can only communicate with nodes in neighboring cells, packets from source nodes may need to be forwarded to destination nodes through multi-hop transmissions. For networks with and without obstacles, we adopt the following routing strategies, respectively.

**Routing Strategy I - for networks without obstacles:**
Suppose that a source node is located in cell $S_i$, and that its destination is located in cell $S_j$. The packet sent from the source node firstly is forwarded along the cells in the same horizontal line of cell $S_i$ until it gets to the cell in the same horizontal line of cell $S_j$, then the packet is forwarded along the cells in the same horizontal line of cell $S_j$ until it reaches the destination node.

**Routing Strategy II - for networks with obstacles:**
1) If packet sent from the source node can be relayed to its destination by Routing Strategy I, do it.
2) Otherwise, if there are only convex obstacles polygons, firstly forward the packet along the routing path generated by Routing Strategy I. When it can no longer be forwarded in current direction (vertical or horizontal), change the forwarding direction to another one (horizontal or vertical). Repeat this until it arrives at the destination node.
3) If there exist concave polygons obstacles and neither the source node nor the destination node is in the groove of a concave obstacles polygon, replace the concave obstacles polygons by their convex hulls, respectively, and then following Step 1) and 2) to forward the packet.
4) If source and destination nodes (or either of them) are in the grooves of concave obstacles polygons, we can find cells outside the corresponding convex hulls and nearest to source and destination nodes, respectively. Denote them by $S_A$ and $S_B$, respectively. We firstly transmit the packet from the source node to cell $S_A$, then following Step 1), 2) and 3) to forward this packet from cell $S_A$ to cell $S_B$, and finally we forward the packet from cell $S_B$ to the destination node.

**Network Topology:** We first established coordinate system for the network region. As shown in Fig. 1, the coordinate origin is located at the center of the network. As the edge length of the network region is 1, the maximal scales of x-axis and y-axis are both 1/2.

1) **Uniform Distribution** For networks with uniformly distributed nodes, the probabilities that nodes located at any place of the network is the same with each other, i.e., it has following probability density function:

$$
\begin{align*}
    f(x) &= 1 \quad (-\frac{1}{2} < x < \frac{1}{2}) \\
    f(y) &= 1 \quad (-\frac{1}{2} < y < \frac{1}{2})
\end{align*}
$$

(3)

2) **Centralized Distribution** We firstly consider a simple case of the non-uniform distribution. As shown in Fig. 2, nodes density is large in the center of the network and small at the edge. We call it “centralized distribution”. One of its possible probability density functions can be shown as follows:

$$
\begin{align*}
    f(x) &= (4a - 4) |x| + 2 - a \quad (-\frac{1}{2} < x < \frac{1}{2}) \\
    f(y) &= (4a - 4) |y| + 2 - a \quad (-\frac{1}{2} < y < \frac{1}{2})
\end{align*}
$$

(4)

where $a (0 \leq a \leq 1)$ is centralization coefficient and its value determines the extent that network nodes aggregate to the center. The larger $a$ is, the more uniformly the nodes are distributed, or vice versa. In particular, when $a = 1$ the nodes are uniformly distributed; when $a = 0$, the probability of nodes distributed at the edge of the network would become 0.

3) **Multi-centralized Distribution** In practice, network nodes often aggregate in several locations of the network,
not just the center of the network. We call it “multi-centralized distribution”. As shown in Fig. 3, We can divide the whole network into many small sub-networks according to the aggregation centers and each sub-network has similar network topology. In this paper, we assume that each sub-network is a small centralized distributed network as defined before, i.e., it satisfies the probability density function shown in (4).

III. NETWORK CONNECTIVITY

In the network model, user nodes can communicate with each other only when they are in the adjacent cells. In heterogeneous networks without helping nodes, every user node can only communicate with nodes in neighboring cells. In user mode, the information can be transferred by a relaying node from a cell to another cell to maintain the connectivity of network. So it should be satisfied that there is at least one user node in each cell which can work as a relay node for multiple hop transmission. Therefore the size of a cell should be limited. A blind cell which does not include any nodes should be tried to avoided. In this section, we need to analyze different cells’ length under different probability distributions of user nodes and get the smallest cell size that can maintain the network connectivity.

A. Uniform Network

For uniformly distributed nodes, network connectivity has been adequately researched in various literatures, here we only give a brief result.

Lemma 2. For uniformly distributed nodes, if we divide the network area into cells of length \( l = \sqrt{\frac{c_2 \log n}{n}} \), where \( c_2 > 1 \) is a constant, each cell has at least one node w.h.p. [24].

B. Centralized and Multi-centralized Network

For multi-centralized network, because each of its sub-networks is a small centralized network, we only need to discuss the connectivity of centralized distribution network.

For centralized network, we have the following lemma:

Lemma 3. For centralized, if cell’s length satisfies the following condition, the network is connected w.h.p.

\[
\begin{align*}
 l & \geq \begin{cases} 
 -a + \sqrt{a^2 + 8(1-a)\sqrt{\frac{c_3 \log n}{n}}} & \text{when } 0 \leq a < 1 \\
 \frac{\sqrt{c_3 \log n}}{n} & \text{when } a = 1 
\end{cases} 
\end{align*}
\]

(5)

where \( c_3 \) is a deterministic positive constant.

Proof: Different from uniform network, in non-uniform network, probabilities that nodes distributed in different cells are not the same. For network satisfying probability density function (4), the probability of nodes distributed in the four corners of this network region is the smallest. If there is at least one node in cells located in the four corners, there must be at least one node in every cell for the network w.h.p.

For centralized network, the joint probability density function is

\[
f(x, y) = [(4a - 4)|x| + 2 - a] \cdot [(4a - 4)|y| + 2 - a] \quad (6)
\]

where \(-1/2 < x < 1/2 \) and \(-1/2 < y < 1/2 \). Without loss of generality, we consider the cell \( S \) in the the upper right corner, for a user node \( X_i \), the probability that it is in cell \( S \), is

\[
P_i = \int_{\frac{1}{2} - l}^{\frac{1}{2}} \int_{\frac{1}{2} - l}^{\frac{1}{2}} |(4a - 4)x + 2 - a| \cdot |(4a - 4)y + 2 - a| \, dx \, dy
\]

\[
= \int_{\frac{1}{2} - l}^{\frac{1}{2}} ((2a - 2)x^2 + (2 - a)x) \, dx
\]

\[
= \int_{\frac{1}{2} - l}^{\frac{1}{2}} ((2a - 2)y^2 + (2 - a)y) \, dy
\]

\[
= (al - 2(a - 1)l^2)^2
\]

Denote the probability that there is at least one node in cell \( S \) by \( P_S \), when \( n \to \infty \), we have

\[
P_S = 1 - (1 - P_i)^n \to 1 - e^{-nP_i}
\]

(7)

In order to ensure that there is at least one user node in cell, it must be satisfied that \( P_S \to 1 \). Considering the length of cells for uniform network from Lemma 2, we can find a condition of \( P_i \) to guarantee \( P_S \). So we get it.

When \( P_i \) satisfies

\[
P_i \geq \frac{c_3 \log n}{n}
\]

(8)

we can get

\[
P_S \geq 1 - \frac{1}{n^{c_3}}
\]

(9)

i.e., \( P_S \to 1 \) as \( n \to \infty \). Putting (8) into (7), we can get

\[
(al - 2(a - 1)l^2)^2 \geq \frac{c_3 \log n}{n}
\]

(10)

after simplifying the formula, we can get

\[
2(1 - a)^2 + al - \sqrt{\frac{c_3 \log n}{n}} \geq 0
\]
Solving this formula and considering that $0 < l < 1$ in the realistic network which the edge length is 1, we can obtain the result in Lemma 3.

The formulas in Lemma 3 are too complicated; here we give the lower bound of the length for cells. Easy to show that, the probability density functions of networks regions with less user nodes are small, so the scope of cells should be large in order to maintain the network connectivity. We can obtain a high probability to ensure at least one node in the cell. From formula 4, the probability density function of the centralized network, we can get that when $a = 1$, the size for the network cell is the minimum, when $a = 0$, the size of the network cell is the largest. Putting $a = 0$, we can get the following formula

$$l \geq \sqrt{\frac{c_3 \log n}{n}}$$

Conclusions for multi-centralized distribution network is similar to Lemma 3. From Lemma 3, it’s not difficult to get the following corollary.

**Corollary 1.** For centralized network defined by (4), if $l = \sqrt{\frac{c_3 \log n}{n}}$ for any $0 \leq a \leq 1$, there is at least one node in each cell w.h.p.

### IV. The Throughput Capacity of Heterogeneous Wireless Networks without Obstacles

In this section, we explore a lower bound on the throughput capacity of heterogeneous wireless networks without obstacles by deriving the achievable per-node throughput. We further divide the communication process under helping mode into three phases as those in [15], [16]. First, the packet is sent from the source node to the nearest helping node, then forwarded in the helping-network until it reaches the helping node nearest to the destination, and in the final phase the packet is transmitted from that nearest helping node to the destination node. We analyze the throughput capacity in user mode and three phases of helping mode. Denote achievable per-node throughput in user mode and helping mode by $T_u$ and $T_h$, respectively. Thus, the achievable per-node throughput of the heterogeneous wireless networks, denoted by $T$, can be calculated as follows:

$$T = \max\{T_u, T_h\}$$

where

$$T_h = \min\{T_{h1}, T_{h2}, T_{h3}\}$$

Here, $T_{h1}$, $T_{h2}$, $T_{h3}$ are achievable per-node throughputs in three phases of helping mode. We assume that all the helping nodes are placed regularly and only investigate the impact of user nodes’ topology. The impact of helping nodes’ topology will be studied in the following sections.

#### A. Uniform Network

As the uniform network we proposed above is a special case in [15], [16], which assumed the network placed in a rectangular area, here we only give the derivative results briefly.

#### Theorem 1.** An achievable throughput in uniform networks, denoted by $T_{\text{uniform}}$, is

$$T_{\text{uniform}} = \Omega \left( \min \left\{ \frac{1}{\sqrt{n \log n}}, \frac{1}{n^{d-1}} \right\} \right)$$

**B. Centralized Network**

To facilitate the calculation, here we will only consider the case that centralization coefficient is 0, i.e., the density of nodes in the center goes to the maximum. Moreover, results of such extreme case are also easier for us to compare with that of the uniform network. When $a = 0$, probability density function given in (4) becomes

$$\begin{cases}
  f(x) = -4|x| + 2 & (-\frac{1}{2} < x < \frac{1}{2}) \\
  f(y) = -4|y| + 2 & (-\frac{1}{2} < y < \frac{1}{2})
\end{cases}$$

Additionally, due to the non-uniformity, properties of the network are related to cells’ locations. As shown in Fig. 4, we number the rows and columns by integers and let $S(i,j)$ denote the cell located in the $i$th column and the $j$th row, $k$ is the square root of the number of cells in the network. If the number of rows or columns is odd, the numbering of cells should start from 0, i.e., $i, j = 0, \pm 1, \pm 2, \ldots, \pm k$. From the following derivation, we can see that the starting value of the numbering is irrelevant to the result.

1) **Achievable Throughput in User Mode:** Here we use the result for cells of user nodes in Corollary 1. Let $N_s^i$ and $N_d^j$ denote the number of source nodes located in the same column of $S(i,j)$ and the number of destination nodes located in the same row of $S(i,j)$, respectively. Thus, we have

$$E[N_s^i] = n \cdot \int_{(i-1)t}^{it} f(x)dx$$

$$= n \cdot \left( -2n^2 + 2x \right) \bigg|_{(i-1)t}^{it}$$

$$= n \cdot (4n^2 + 2l^2 + 2l)$$

$$= n \cdot \left( 4 \cdot \left[ \frac{1}{\sqrt{n \log n}} \right] \right)$$

**Fig. 4. A numbering of cells**
Similarly,
\[
E[N^i_j] = n^d \cdot \int_{j-1}^{j} f(y) dy
\]
\[
= n^d \cdot (-4j^2 + 2l^2 + 2l)
\]  \hspace{1cm} (16)

According to Corollary 1, we have
\[
E[N^i_j] = 2n(-2i + 1) \left( \frac{c_3 \log n}{n} \right)^\frac{1}{2} + 2n \left( \frac{c_3 \log n}{n} \right)^\frac{1}{4}
\]
\[
E[N^i_j] = 2n^d(-2j + 1) \left( \frac{c_3 \log n}{n} \right)^\frac{1}{2} + 2n^d \left( \frac{c_3 \log n}{n} \right)^\frac{1}{4}
\]

Recall the Chernoff bounds, we can obtain the following lemma.

**Lemma 4.** For each cell, w.h.p.,
1) The number of source nodes which are located in cells with the same x-coordinate is at most \(4nl(-2i + 1) \left( \frac{c_3 \log n}{n} \right)^\frac{1}{2} + 4n \left( \frac{c_3 \log n}{n} \right)^\frac{1}{4}\).
2) The number of destination nodes located in cells with the same y-coordinate is at most \(4n^d(-2j + 1) \left( \frac{c_3 \log n}{n} \right)^\frac{1}{2} + 4n^d \left( \frac{c_3 \log n}{n} \right)^\frac{1}{4}\) when \(1/4 < d < 1\), and at most \(c_4\) when \(0 < d < 1/4\), where \(c_4\) is a constant and \(4\) is a deterministic positive constant.

Techniques used to prove Lemma 4 are similar to that of Lemma 3 in [15], [16] and we would not discuss them in detail here. Interested reader can refer to [15], [16].

On the other hand, according to [15], [16], for the network with \(n\) source nodes and \(n^d\) destination nodes, we also have

**Lemma 5.** For each destination node, w.h.p., there are at most \(2n^3/d\) source nodes destined to it [15], [16].

Let \(F^{ij}_k\) denote the number of data flows crossing cell \((i,j)\). For each cell, we have
\[
F^{ij}_k \leq N^i_j + 2n^1/d N^j_y
\]
\[
\leq 4nl(-2i + 1) \left( \frac{c_3 \log n}{n} \right)^\frac{1}{2} + 4n \left( \frac{c_3 \log n}{n} \right)^\frac{1}{4}
+ 2n^1/d \left( \frac{c_3 \log n}{n} \right)^\frac{1}{2} - \left( \frac{c_3 \log n}{n} \right)^\frac{1}{4}
+ 4n^d(-2j + 1) \left( \frac{c_3 \log n}{n} \right)^\frac{1}{2} + 4n^d \left( \frac{c_3 \log n}{n} \right)^\frac{1}{4}
+ 2c_4n^1/d \left( \frac{c_3 \log n}{n} \right)^\frac{1}{2} - \left( \frac{c_3 \log n}{n} \right)^\frac{1}{4}
\]  \hspace{1cm} (17)

Notice that the right part of (17) are monotonically decreasing functions, we have
\[
F^{ij}_{k,max} = F^{ij}_{k,1} = 12n \left[ \left( \frac{c_3 \log n}{n} \right)^\frac{1}{2} - \left( \frac{c_3 \log n}{n} \right)^\frac{1}{4} \right]
\]
\[
= \begin{cases} 
12n \left[ \left( \frac{c_3 \log n}{n} \right)^\frac{1}{2} - \left( \frac{c_3 \log n}{n} \right)^\frac{1}{4} \right] & \text{when } 1/4 < d < 1 \\
4n \left[ \left( \frac{c_3 \log n}{n} \right)^\frac{1}{2} - \left( \frac{c_3 \log n}{n} \right)^\frac{1}{4} \right] + 2c_4n^1/d & \text{when } 0 < d < 1/4
\end{cases}
\]

i.e.,
\[
F^{ij}_{k,\max} = O \left( \max \left\{ n \cdot \left( \frac{\log n}{n} \right)^{\frac{1}{2}}, n^{1-d} \right\} \right)
\]  \hspace{1cm} (18)

From Lemma 1, each cell can achieve a constant transmission rate. Denote the achievable throughput in user mode by \(T^{central}_u\), from (18), we can obtain that
\[
T^{central}_u = \Omega \left( \min \left\{ n^{-1}, \left( \frac{n}{\log n} \right)^{\frac{1}{4}}, n^{d-1} \right\} \right)
\]  \hspace{1cm} (19)

2) **Achievable Throughput in Helping Mode:** Recall that we have divided communication in helping mode into three phases, in this section we derive the throughput capacity in these three phases, respectively, and synthesize them to get the achievable throughput in helping mode.

*Phase I: transmitting from sources to the helping nodes*

In helping mode, we can divide the network into many cells according to helping nodes’ distribution. We assume source nodes have low transmission power and they need to transmit packets to the helping node in the same cell via multiple hops. So there must be at least one helping node in each cell. Since helping nodes are regularly placed, we can re-divide the network into \(m\) cells of length \(l = \sqrt{1/m} = n^{-\frac{1}{2}}\). There is exactly one helping node in each cell. Then we can obtain the following lemma.

**Lemma 6.** There are at most \(8n \left(-n^{-b} + n^{-\frac{1}{2}}\right)^2\) user nodes in each cell.

*Proof:* Denote the number of user nodes located in cell \((i,j)\) by random variable \(N_{ij}\). Thus, the expectation of \(N_{ij}\) is
\[
E[N_{ij}] = n \int \int f(x) f(y) dx dy
\]
\[
= n \left( (-4i + 2)n^{-b} + 2n^{\frac{b}{2}} \right) \left( (-4j + 2)n^{-b} + 2n^{\frac{b}{2}} \right)
\]  \hspace{1cm} (20)

Notice that the right part of (20) is a monotonically decreasing function, thus we have
\[
E[N_{ij}] = E[N_{i1}] = 4n \left(-n^{-b} + n^{-\frac{1}{2}}\right)^2
\]  \hspace{1cm} (21)

Similar to the proof of Lemma 4, we can easily show that \(P(N_{ij} < 2E[N_{ij}]_{\max} \forall S_{ij}) \to 1\) as \(n \to \infty\).

Let \(T^{central}_{h1}\) denote the achievable throughput in phase I. We can obtain that
\[
T^{central}_{h1} = \Omega \left( \frac{W_2}{8n \left(-n^{-b} + n^{-\frac{1}{2}}\right)^2} \right) = \Omega(n^{b-1})
\]  \hspace{1cm} (22)

*Phase II: forwarding in helping network*

Notice that packets forwarding in helping network is also ad hoc transmission. Similarly, we can obtain the following lemmas.

**Lemma 7.** Each cell in the network can transmit at a transmission rate \(c_5 W_4\), where \(c_5\) is a deterministic positive constant.
Lemma 8. For every cell, w.h.p.,

1) There are at most $4n \left(\frac{-2i+1}{b}n^{-b} + n^{-\frac{b}{2}}\right)$ source nodes located in the same column.
2) The number of destination nodes located in the same row is at most $4n^{d_i} \left(\frac{-2j+1}{b}n^{-b} + n^{-\frac{b}{2}}\right)$ when $d_i > \frac{b}{2}$, and at most $c_6$ when $d < \frac{b}{2}$, where $c_6$ is a constant.

Thus, similar to the derivation of achievable throughput capacity in user mode, we can obtain the throughput capacity in phase II, denoted by $T_{h1}^{central}$, as follows:

$$T_{h1}^{central} = \Omega \left(\min \left\{n^{\frac{b}{2}-1}, n^{d-1}\right\}\right)$$ (23)

Phase III: transmitting from the helping nodes to destinations

For the re-divided network in helping mode, we have the following lemma.

Lemma 9. For centralized distribution network, in each cell, w.h.p., there are at most $8n^{d_i} \left[-n^{-b} + n^{-\frac{b}{2}}\right]$ destination nodes when $0 < b < d < 1$, and at most $c_7$ when $0 < d < b < 1$, where $c_7 > b$.

Proof: Denote the number of destination nodes located in cell $S(i, j)$ by random variable $D_{ij}$. Thus, the expectation of $D_{ij}$ is

$$E[D_{ij}] = n^{d_i} \int_{(i-1)b}^{ib} \int_{(j-1)b}^{jb} f(x)f(y)dydx$$

$$= n^{d_i} \left(-4i+2\right)n^{-b} + 2n^{-\frac{b}{2}}$$

$$= \left(-4j+2\right)n^{-b} + 2n^{-\frac{b}{2}}$$

$$\leq 4n^{d_i} \left(-n^{-b} + n^{-\frac{b}{2}}\right)^2$$ (24)

Similar to the proof of Lemma 4, we can easily show that

$$P(D_{ij} \leq 8n^{d_i} \left[-n^{-b} + n^{-\frac{b}{2}}\right]) \forall S(i,j) \to 1 as n \to \infty.$$

Similar to Lemma 1, we can obtain that there can be a constant transmission rate $c_7 W_3$ in phase III, where $0 < c_7 < \infty$ is a deterministic constant. Furthermore, we find that w.h.p. the number of data flows from each helping node to its destination nodes is at most $8n^{d} \left[-n^{-b} + n^{-\frac{b}{2}}\right] \times 2n^{-d} = \Omega \left(n^{1-b}\right)$ when $0 < b < d < 1$, and at most $c_8 \times 2n^{-d} = 2c_8 n^{1-d}$ when $0 < d < b < 1$. Then, we can obtain the throughput capacity in phase III, denoted by $T_{h3}^{central}$, as follows:

$$T_{h3}^{central} = \left\{ \begin{array}{ll}
\Omega \left(n^{b-1}\right) & \text{when } 0 < b < d < 1 \\
\Omega \left(n^{d-1}\right) & \text{when } 0 < d < b < 1
\end{array} \right.$$ (25)

Combining (22), (23) and (25), we can get

$$T_h^{central} = \Omega \left(\min \left\{n^{\frac{b}{2}-1}, n^{d-1}\right\}\right)$$ (26)

Substituting (19) and (26) into (11), we can get the following theorem.

Theorem 2. An achievable throughput in centralized distribution networks, denoted by $T_{h}^{central}$, is

$$T_{h}^{central} = \Omega \left(\max \left\{\min \left\{n^{-1}, \left(\frac{n}{\log n}\right)^{\frac{b}{2}}, n^{d-1}\right\}, \min \left\{n^{\frac{b}{2}-1}, n^{d-1}\right\}\right\}\right)$$ (27)

C. Multi-centralized Network

Following similar trace of derivation in centralized distribution networks, we can obtain the following theorem.

Theorem 3. An achievable throughput in multi-centralized distribution networks, denoted by $T_{u}^{multi}$, is

$$T_{u}^{multi} = \Omega \left(\min \left\{\frac{k}{n}, \left(\frac{n}{\log n}\right)^{\frac{b}{2}}, n^{d-1}\right\}, \min \left\{kn^{\frac{b}{2}-1}, n^{d-1}\right\}\right)$$ (28)

Proof: We need to find an achievable throughput in user mode and helping mode, respectively. An achievable throughput in multi-centralized distribution networks can be obtained by choosing the maximum.

1) user mode

For multi-centralized distribution network, in every sub-network, there are $n/k^2$ nodes. Thus, in user mode, cells’ length is $\frac{1}{k} \cdot \sqrt{\frac{c_3 W_3}{\log n}}$. Similar to that in Section IV-B (1), we can obtain that

$$T_u^{multi} = \min \left\{\frac{k}{n}, \left(\frac{n}{\log n}\right)^{\frac{b}{2}}, n^{d-1}\right\}$$

2) helping mode

The cell length in helping mode is still $\sqrt{\frac{c_3 W_3}{\log n}}$ however, the area of each centralized sub-network is only $\frac{1}{k}$. Thus, the relative coverage of each cell is increased. Similar to that in Section IV-B (2), we can obtain that

$$T_h^{multi} = \Omega \left(\min \left\{\frac{c_7 W_3}{\log n}, n^{1-b}\right\}, \min \left\{kn^{\frac{b}{2}-1}, n^{d-1}\right\}\right)$$

Thus, an achievable throughput in multi-centralized distribution networks is

$$T_{multi} = \Omega \left(\max \left\{\min \left\{\frac{k}{n}, \left(\frac{n}{\log n}\right)^{\frac{b}{2}}, n^{d-1}\right\}, \min \left\{kn^{\frac{b}{2}-1}, n^{d-1}\right\}\right\}\right)$$
V. General Properties of “Combined Networks”

From the results in Section IV, we can see that compared to sub-networks of the same network scales, the overall networks have a larger achievable throughput. Here, “scale” means the size of network area, number of nodes and size of cells.

To explain this phenomenon, we can divide the impacts of combination into two categories:

1) The interference of different sub-networks
2) Flows passing across different sub-networks

For impact 1), from the proof of Lemma 1 in [15], [16], we can see that since there is only one node that can transmit packet in one time slot of each cell, the interference between cells is only relative to the size of cells and irrelevant to the number of nodes in it. Thus, the interference in combined network is the same as that in sub-networks of the same scales.

Impact 2) is not so easy to explore, however, from the derivation in Section IV, we can see that the achievable throughput of a network without obstacles is determined by the cells and rows of the largest nodes density, i.e., by variables \( N_{x,\text{max}} N_{y,\text{max}} \).

For the combined network consisting of \( k \times k \) sub-networks, denote the corresponding variables by \( \hat{\cdot} \). We have \( \hat{C}_{\text{max}} = C_{\text{max}}, \hat{D}_{\text{max}} = D_{\text{max}}, \hat{N}_{x,\text{max}} = kN_{x,\text{max}}, \hat{N}_{y,\text{max}} = kN_{y,\text{max}} \).

Furthermore, we have

\[
\hat{F}_{ij,\text{max}} = \tilde{N}_{x,\text{max}} + 2n^{1-d}N_{y,\text{max}} = kN_{x,\text{max}} + 2n^{1-d}kN_{y,\text{max}} = kF_{ij,\text{max}}
\]

Substituting \( \hat{C}_{\text{max}}, \hat{D}_{\text{max}}, \hat{F}_{ij,\text{max}} \) into (31), (32), (34) and (35), we can obtain that

\[
\tilde{T}_u = \frac{c_1 W_1}{F_{ij,\text{max}}} = \frac{\hat{c}_1 W_1}{kF_{ij,\text{max}}}
\]

\[
\tilde{T}_{h1} = \frac{\hat{c}_1 W_2}{\hat{C}_{\text{max}}} = \frac{\hat{c}_1 W_2}{k \cdot \frac{1}{2} \cdot F_{ij,\text{max}}}
\]

\[
\tilde{T}_{h2} = \frac{W_4}{F_{ij,\text{max}}} = \frac{W_4}{kF_{ij,\text{max}}}
\]

\[
\tilde{T}_{h3} = \frac{\hat{c}_1 W_3}{D_{\text{max}}} = \frac{\hat{c}_1 W_3}{kF_{ij,\text{max}}}
\]
Thus, we can obtain that
\[ \hat{T} = \max\{\tilde{T}_u, \tilde{T}_h\} \]
\[ = \max\{\tilde{T}_u, \min(\tilde{T}_{h1}, \tilde{T}_{h2}, \tilde{T}_{h3})\} \]
\[ \geq \max\{\tilde{T}_u, \min(\tilde{T}_{1}, \tilde{T}_{2}, \tilde{T}_{3})\} \]
\[ = \tilde{T} \]
(38)

Then, we can obtain that the overall networks have a larger achievable throughput.

VI. IMPACT OF NETWORK TOPOLOGY ON THROUGHPUT CAPACITY

A. Impact of User Nodes’ Topology

Comparing the results in Section IV with each other, we can find that achievable throughput in networks with the three different topologies have similar scales (take k as a constant). In general, we have the following theorem.

Theorem 5. For the topology of user nodes, if the value range of nodes distribution’s probability density function (PDF) is limited, the gap in achievable throughput of non-uniform networks and uniformly networks is at most a constant time.

**Proof:** Firstly, from the analysis in Section V, we can see that if size of cells stay unchanged, the interference between cells can not be changed by network topologies. Secondly, in the proof of Theorem 4, we have concluded that for networks without obstacles the achievable throughput is determined by cells and rows of the largest nodes density, i.e., by variables \( N_{x,\text{max}} N_{y,\text{max}}, C_{\text{max}}, D_{\text{max}}, N'_{x,\text{max}} \) and \( N'_{y,\text{max}} \). Let \( \bar{N}_{x,\text{max}} \) and \( \bar{N}^{\prime}_{x,\text{max}} \) denote the the maximal number of source nodes located in cells with the same x-coordinate in uniform and non-uniform, respectively. Since the value range of the nodes distribution’s PDF is limited, i.e., \( \exists M \in R^+, \text{for} \forall x, y \text{we have} |f(x)| \leq M, |f(y)| \leq M \). Thus, we have
\[ E[\bar{N}_x] = n \cdot \frac{l}{4} \]
(39)
\[ E[\bar{N}^\prime_x] = n \cdot \int_{l}^{d} f(y)dy \]
\[ \leq n \cdot \int_{l}^{d} Mdy \]
\[ = Mnl \]
\[ = ME[\bar{N}_x] \]

Using Chernoff Bounds, we can prove that w.h.p. \( \bar{N}_x \leq 2E[\bar{N}_x] \) and \( \bar{N}^\prime_x \leq 2E[\bar{N}^\prime_x] \). Thus, we have
\[ \bar{N}_{x,\text{max}} \leq M\bar{N}^{\prime}_{x,\text{max}} \]
(40)

Similar results can be proved for \( N_{y,\text{max}}, C_{\text{max}}, D_{\text{max}}, N'_{x,\text{max}} \) and \( N'_{y,\text{max}} \). Denote the achievable throughput in uniform and non-uniform network by \( \tilde{T} \) and \( \hat{T} \), respectively. Following the trace of derivation in Theorem 4’ proof, we can conclude that
\[ T' \geq C(M)\hat{T} \]
(41)

B. Impact of Helping Nodes’ Topology

We only considered regularly distributed helping nodes above. We will explore the impact of different helping nodes’ distributions in this subsection. Then we have the following theorem.

Theorem 6. For networks with uniformly distributed user nodes, regularly distributed helping nodes are optimal to maximize the network throughput capacity.

**Proof:** Firstly, we analyze the impact of helping nodes’ distribution on interference. For the non-uniform distribution of helping nodes, we can adopt domain decomposition to divide the network domain into a set of square regions and the boundaries of which are the perpendicular bisectors of the lines joining the points, similar to the Voronoi tessellation. We can use a square surrounding one helping node to split the network, and finally need to adjust the length and number of squares to ensure that each cell contains at least one helping node. If there are more than one helping nodes in one cell, we randomly choose one helping node as the relay node of the cell. So the squares with different lengths will not overlap with each other and the length of each cell is not the same. In network with non-uniformly distributed helping nodes, cells’ sizes in helping mode are also different. Let \( l' \) denote length of cells in network with regularly distributed helping nodes and \( l'' \) denote the maximal length of cells of network with non-uniformly distributed helping nodes, respectively. We can easily obtain that \( l'' \geq l' \). Denote the achievable rate of cells in networks with regularly and non-uniformly distributed network by \( W' \) and \( W'' \), respectively. From the derivation of Lemma 1, we can obtain that \( W' \geq W'' \).

Secondly, according to the proof of Theorem 4, achievable throughput in helping mode is determined by variables \( C_{\text{max}}, D_{\text{max}}, N_{x,\text{max}} \) and \( N_{y,\text{max}} \). Let \( C'_{\text{max}}, D'_{\text{max}}, N'_{x,\text{max}} \) and \( N'_{y,\text{max}} \) denote the corresponding variables in network with regularly distributed helping nodes, and \( C''_{\text{max}}, D''_{\text{max}}, N''_{x,\text{max}} \) and \( N''_{y,\text{max}} \) denote corresponding variables in network with regularly distributed helping nodes, respectively. In networks with non-uniformly distributed helping nodes, there must be some cells of length larger than the average value. Thus, we can obtain that \( C''_{\text{max}} > C'_{\text{max}}, D''_{\text{max}} > D'_{\text{max}}, N''_{x,\text{max}} > N'_{x,\text{max}}, \) and \( N''_{y,\text{max}} > N'_{y,\text{max}} \).

Denote the achievable throughput capacity of network with regularly and non-uniformly distributed network by \( T' \) and \( T'' \), respectively. Since that throughput capacity in helping mode is inversely proportional to these variables, from above we can obtain the following conclusion
\[ T' \geq T'' \]
(41)

Theorem 7. For networks with non-uniformly distributed user nodes, though regularly distributed helping nodes topology is no longer optimal, any improvement on the helping nodes’ topology cannot change the network throughput capacity on scale.

**Proof:** Firstly, considering interference, according to the proof of Theorem 6, non-uniformly distributed helping nodes
can only increase the interference between cells and decrease the achievable rate of cells. Thus, it has a negative impact on the network throughput capacity.

Secondly, if we do not consider change of interference, network throughput capacity in helping mode is determined by variables $C_{max}, D_{max}, N_{x, max}$ and $N_{y, max}$. Thus, if we change the helping nodes’ topology, network throughput capacity will achieve the maximal value when each cell has the same number of nodes. However, similar to that in the proof of Theorem 5, we can easily show that this improvement is not larger than a constant time.

Combining conclusions above together, we can conclude that improvement on the helping node topology cannot change the network throughput capacity on scale.

VII. OPTIMAL TOPOLOGY FOR NETWORKS WITH OBSTACLES

In this section, we introduce a novel algorithm to generate the optimal network topology for any given obstacles distributions and analyze its performance.

A. Algorithm to Obtain the Optimal Network Topology

To design the optimization algorithm, we first consider a simple scenario. As shown in Fig. 5, assume that there is a “wall” with a “gate” in the network, which divides the network area into two parts. In this case, the area around the gate is the “hot spot” and the bottleneck of the network achievable throughput since any data flow passing from one side of the wall to another side must pass through the gate. To maximize the network throughput capacity, we can minimize the transmission burden of this area by the following algorithm.

Algorithm - “Wall with Gate”:

1) Assume that there are $\hat{n}, n_1$ and $n_2$ number of nodes in the gate area, the left and the right part of the network, respectively, where $\hat{n} + n_1 + n_2 = n$. The expected number of data flows passing through the gate is $u = f(\hat{n}, n_1, n_2)$, where function $f(\cdot)$ can be decided using the methods given in Section IV. Thus, the transmission burden of the gate area is $B_0 = u/k_0$, where $k_0$ is the number of cells in the gate area (the nodes’ distribution in the gate area is assumed to be uniform since this area is relatively small).

2) Ignore the wall and the right part of the network. Put $\varphi_1 = g_s(\hat{n}, n_1, n_2)$ number of virtual source nodes and $\varphi_1 = g_d(\hat{n}, n_1, n_2)$ number of virtual destination nodes uniformly in front of the gate (i.e., the area illustrated in Fig. 5) to replace the ignored user nodes. Virtual source and destination nodes work as sources and destinations, respectively, generating virtual date flows. Functions $g_s(\cdot)$ and $g_d(\cdot)$ are determined by the routing strategy so that this number of virtual nodes have the same influence on the left part of the network as the ignored parts. Then we obtain a degraded sub-network without any obstacles.

3) For the degraded sub-network, use methods and conclusions given in Sections IV - VI to generate an optimal topology $T_1 = T_1(\hat{n}, n_1, n_2)$ and calculate the corresponding transmission burden $B_1 = h_1(\hat{n}, n_1, n_2)$.

4) Repeat Step 2 and 3 to the right part of the network, respectively. Generate the optimal topology $T_2 = T_2(\hat{n}, n_1, n_2)$ and calculate the transmission burden $B_2 = h_2(\hat{n}, n_1, n_2)$.

5) Use appropriate optimization methods to minimize the cost function $B = \max(B_0, B_1, B_2)$. Calculate corresponding $\hat{n}, n_1$ and $n_2$. Since the topologies obtained in Step 3 and 4 are functions of $\hat{n}, n_1$ and $n_2$, the optimal topology for the whole network can thus be determined by combining $T_1$ and $T_2$.

This “Wall with Gate” algorithm can be generalized to obtain optimal topology for any given networks with obstacles. Firstly, divide the network area into pieces by the following method.

Divide the network by walls - Method I: As shown in Fig. 6, take blocked cells in a row (vertical or horizontal) as a wall and cells without obstacles in this row as gates. Then the network is divided into some sub-networks by these walls.

After the network is divided into several sub-networks and gates, the optimal topology for this network area can be obtained by applying the idea of “Wall with Gate” algorithm for each sub-network and gate area. It is important to note that in this situation the gate areas may be relatively large, we can no longer assume that nodes’ distribution for gate areas is uniform and also need to perform the algorithm in these gate areas.

The optimal topology for the whole network can be no longer obtained by simply integrating the optimal topology for sub-networks and gate areas using the “Wall with Gate” algorithm. For this network, a sub-network may be adjacent to different gates with other sub-networks. If per pair of adjacent sub-networks is applied by the “Wall with Gate” algorithm to obtain the optimal topology for the pair sub-networks and corresponding gate. It’s very likely that there are different optimal topologies for one sub-network with different adjacent sub-network and corresponding gate. The optimal topology of the whole network can’t be obtained in this case.

The optimal topology of the whole network can be gotten by applying the idea of “Wall with Gate” algorithm, and
we should apply the algorithm to the whole network. After
the network is divided into several sub-networks and gate
areas by walls, we can assume that there are \( S \) sub-networks
and \( R \) gates. Step 2 is applied to all sub-networks and gate
areas, then we can obtain a degraded network without any
obstacles, only including several degraded sub-networks and
gates. The optimal topology and the transmission burden for the
optimal topology and transmission burden for the
sub-network is divided into several sub-networks and gate
areas can be obtained by using methods and conclusions given in Sections IV - VI.

We use \( \mathcal{T}_i,k, k = 1, \ldots, S \) and \( B_i,k, k = 1, \ldots, S \) as the
optimal topology and transmission burden for the \( k \)th sub-

Finally, we can select appropriate optimization
methods to minimize the cost function \( B = \min(B_0,B_1,\ldots,B_S,B_1,\hat{B}_1,\ldots,\hat{B}_R) \), and calculate the corresponding parameters of \( n_1,n_2,\ldots,n_S, \hat{n}_1,\hat{n}_2,\ldots,\hat{n}_R. \)

area into two parts. For each part, repeat this step iteratively
until all the blocked cells are crossed by at least one wall.

**Lemma 10.** The algorithm complexity is \( O(M^2) \) when using
network dividing method I and is \( O(M) \) when using method II.

**Proof:** Here we consider the worst case that all cells with
obstacles are not collinear. There are \( 2M \) walls with method I
to divide the network. Denote the number of sub-networks and
number of gates by \( S_I \) and \( R_I \), respectively. A new blocked
cell can add 2 walls to the network considering the worst
case. Towards a new blocked cell with two new walls, there are
\( 2M + 1 \) new sub-networks than the previous network with
\( M + 1 \) blocked cells. So in the worst case, these \( 2M \) walls
divide the network into \( (M + 1)^2 \), so \( S_I = (M + 1)^2 \). Because
there is a gate between any neighboring sub-network areas,
there are at least \( (M + 1) \) \( 2M \) gates, i.e., the number of

denote the number of sub-networks and number of gates by \( S_{II} \) and \( R_{II} \),
respectively. These walls divide the network into \( M + 1 \) areas,
thus, \( S_{II} = M + 1 \). Furthermore, as shown in Fig. 8, each
cell with obstacles can generate at most four additional gates
to the network, i.e., \( R_{II} \leq 4M \). So the algorithm complexity is

\[
\eta_{II} = O(S_{II} + R_{II}) = O(M) \quad (43)
\]

**C. Performance of the Algorithm for Network with Regularly
Distributed Obstacles**

Generally, the more uniform the obstacles’ distribution is,
the larger the achievable throughput is, and thus the less
room for improvement there is. Therefore we take regular
obstacles distribution, one of the most uniform distributions,
as an example to estimate the lower bound of the algorithm
performance.
1) Achievable throughput of network with uniformly distributed user nodes: Since obstacle nodes are regularly distributed, cells blocked by obstacles are also regularly distributed. And there are no user nodes to be distributed in these blocked cells. Denote the number of blocked cells by $\Omega$. To maintain the network connectivity, we have $M \leq \frac{K}{2}$ and $\Omega = M$. As shown in Fig. 9, the marked cells are the busiest ones in the network since they have to relay extra data flows from or to the nodes located in the adjacent columns or rows, respectively. Denote the number of data flows that cross the $i$th cell by $F_i$. Following similar trace of derivation in Section IV, we can obtain that for all $i$

$$F_i \leq \begin{cases} 6 \left(1 + C \sqrt{\frac{K - M}{K}}\right) \cdot \frac{n}{K-M} \cdot \sqrt{K} & \text{when } \frac{w}{T} < d < 1 \\ \left(1 + C \sqrt{\frac{K - M}{K}}\right) \left(\frac{2n}{K-M} \cdot \sqrt{K} + 4c_0n^{1-d}\right) & \text{when } 0 < d < \frac{w}{T} \end{cases}$$

(44)

where $c_0$ is a deterministic constant, $C = 1/2$ when $M < K/4$ and $C = 1$ when $M = K/4$. The achievable throughput is $T_r = c_1W_1/F_i$. For comparison, in network without obstacles, the number of data flows that cross the $i$th cell, denoted by $\hat{F}_i$, is

$$\hat{F}_i \leq \begin{cases} 6 \frac{n}{\sqrt{K}} & \text{when } \frac{w}{T} < d < 1 \\ 2 \frac{n}{\sqrt{K}} + 4c_0n^{1-d} & \text{when } 0 < d < \frac{w}{T} \end{cases}$$

(45)

The corresponding achievable throughput is $\hat{T} = c_1W_1/\hat{F}_i$. Considering that $\frac{n}{K-M} \cdot \sqrt{K} = o(n^{1-d})$ and $\frac{n}{\sqrt{K}} = o(n^{1-d})$ when $0 < d < w/2$, as $n$ goes to infinity, the gap in achievable throughput between these two networks is

$$\frac{\hat{T}}{T_r} = \begin{cases} \frac{K}{K-M} \cdot \left(1 + C \sqrt{\frac{K - M}{K}}\right) & \text{when } \frac{w}{T} < d < 1 \\ \left(1 + C \sqrt{\frac{K - M}{K}}\right) & \text{when } 0 < d < \frac{w}{T} \end{cases}$$

(46)

From equation (46), we can obtain the following theorem.

Theorem 8. Uniform user nodes’ distribution is order optimal for throughput capacity of networks with regularly distributed obstacles.

Proof: Since $\hat{T}$ is the upper bound of achievable throughput for networks with obstacles, no matter how user nodes and obstacles are distributed, we only need to prove that $T_r = O(\hat{T})$. According to equation (46), when $M = o(K)$, we have $\hat{T}/T_r = 1 + C$. When $M = A \cdot K$ (0 < $A < 1/4$), we have

$$\frac{\hat{T}}{T_r} = \begin{cases} 1 + C - C\sqrt{A} & \text{when } \frac{w}{T} < d < 1 \\ 1 - A & \text{when } 0 < d < \frac{w}{T} \end{cases}$$

(47)

Since $1/2 \leq C \leq 1$ and $0 < A < 1/4$, for any $0 < M \leq K/4$, we can obtain that $1 \leq \frac{\hat{T}}{T_r} \leq 2$, i.e., $T_r = \Theta(\hat{T})$. ■

2) Performance of the topology optimization algorithm: A better network topology is shown in Fig. 10, whose achievable throughput can be taken as the lower bound of the optimal network topology. User nodes are distributed uniformly in the marked areas. Denote the number of data flows crossing the $i$th cell by $F_i'$, we have that for all $i$

$$F_i' \leq \begin{cases} 6 \frac{n}{\sqrt{K}} & \text{when } \frac{w}{T} < d < 1 \\ 2 \frac{n}{\sqrt{K}} + 4c_0n^{1-d} & \text{when } 0 < d < \frac{w}{T} \end{cases}$$

(45)

The corresponding achievable throughput is $T_r' = c_1W'_1/F_i'$. When $n$ goes to infinity, the improvement in achievable throughput is

$$\frac{T_r'}{T_r} = \begin{cases} \frac{(1+C)\sqrt{K-C-M}}{\sqrt{K+M}} & \text{when } \frac{w}{T} < d < 1 \\ \frac{(1+C)\sqrt{K-C-M}}{\sqrt{K}} & \text{when } 0 < d < \frac{w}{T} \end{cases}$$

(46)

When $M = o(K)$, we have

$$\frac{T_r'}{T_r} = \frac{\hat{T}}{T_r} = 1 + C$$

(48)
When $M = A \cdot \mathcal{K}$ ($0 < A < 1/4$), we have

$$
T'_r = \begin{cases} 
\frac{1 + C - C\sqrt{A}}{1 + \sqrt{A}} & \text{when } \frac{w}{r} < d < 1 \\
1 + C - C\sqrt{A} & \text{when } 0 < d < \frac{w}{r} \\
\frac{\hat{T}}{T_r} & \text{when } \frac{w}{r} < d < 1 \quad (49) \\
\frac{\hat{T}}{T_r} & \text{when } 0 < d < \frac{w}{r} 
\end{cases}
$$

From equations (48) and (49) we can find that when the number of obstacles is small (i.e., $M = o(\mathcal{K})$), or when the number of destinations is small (i.e., $0 < d < w/2$), the asymptotic throughput capacity of networks with regularly distributed obstacles and optimal network topology can reach the that of networks without obstacles. When the number of obstacles and number of destinations are both large (i.e., $M = \Theta(\mathcal{K})$ and $w/2 < d < 1$), the asymptotic throughput capacity of networks with regularly distributed obstacles and optimal network topology may smaller than that of networks without obstacles by at most a constant time. The worst case happens when $M = \mathcal{K}/4$, in which $T'_r = T/2$.

**VIII. CONCLUSION AND FUTURE WORKS**

In this paper, we investigate the throughput capacity of heterogeneous wireless networks with different network topologies and analyze the impact of topologies on the network properties. We find that compared to the sub-networks with the same network scales, overall networks have a larger network achievable throughput. We analyze the impact of user nodes’ topology and helping nodes’ topology on the network capacity. We find that for non-uniformly distributed user nodes, if the value range of nodes distribution’s PDF is limited, the gap in achievable throughput of non-uniform networks and uniform networks is at most a constant time. Compared to regularly distributed helping nodes, any change of helping nodes’ topology cannot improve the network achievable throughput on scale. We further investigate the impact of obstacles. An algorithm is introduced to divide the network with obstacles into several sub-networks and gate areas. Then we study the performance of the algorithm for the regularly distributed obstacles. Moreover the in-depth analysis for network areas with different distributions is needed. Centralized network with node density which has a power law distribution, normal or poisson distribution may be more in line with reality and should be analyzed in the future. Then the optimal topology for these network models or the characteristic conditions for ensuring the optimal solution can be obtained according to these analysis. The optimal nodes’ spatial distribution should be studied for specific obstacles’ distributions on the basis of practice with the algorithm and these optimal solutions for different nodes’ distributions.

**REFERENCES**


wireless body area network (WBAN).


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Changle Li (M'09) received the B.Eng. degree in Microwave Telecommunication Engineering, M.Eng. and Ph.D. degrees in Communication and Information System, all from Xidian University, China, in 1998, 2001 and 2005, respectively. From 2006 to 2007, he was with Computer Science Department at University of Moncton, Canada as a postdoctoral researcher. From 2007 to 2009, he was an expert researcher at National Institute of Information and Communications Technology (NICT), Japan. He is currently a professor in State Key Laboratory of Integrated Service Networks at Xidian University, China. His research interests include cross-layer design and communication protocols for cellular network, mobile ad hoc network, wireless sensor network, vehicular network and wireless body area network (WBAN).

Liran Wang received the B.Eng. degree in Electronic and Information Engineering from Xidian University, Xi’an, Shaanxi, China, in 2012, and is currently pursuing the M.Eng. degree in Communication Engineering, at Xidian University. Her current research interests is in the areas of capacity analysis in wireless ad hoc networks and the communication protocols for vehicular ad hoc networks.

Sen Yang received the B.S. degree in Electronic Engineering from Shanghai Jiao Tong University, China in 2010. He received the M.S. degree in Electronic and Communication Engineering of Shanghai Jiao Tong University and another M.S. degree in Electrical and Computer Engineering of Georgia Institute of Technology both in 2013. He is currently pursuing his Ph.D. degree in Electrical and Computer Engineering of Georgia Institute of Technology, Atlanta, GA, USA. His research interests focus on the data streaming algorithms in computer networking and online social networks.

Feng Yang received his Ph.D. degree in Information and Communication from Shanghai Jiao Tong University, China in 2006. She is currently with Institute of Wireless Communication Technology, at Department of Electronic Engineering, Shanghai Jiao Tong University (SJTU), where she is an Associate Professor. From 2009 to 2010, she worked as a visiting researcher at California Institute for Telecommunications and Information Technology (Calit2), University of California San Diego, CA, U.S. Her current research interests include Heterogenous Cellular Network, Software Defined Wireless Networks, Intelligent Wireless Networks, Cognitive Network, and Dynamic Radio Resource Management.

Xiaoying Gan received her Ph. D degrees in Electronic Engineering from Shanghai Jiao Tong University, Shanghai, China in 2006. She is currently with Institute of Wireless Communication Technology, at Department of Electronic Engineering, Shanghai Jiao Tong University (SJTU), where she is an Associate Professor. From 2009 to 2010, she worked as a visiting researcher at California Institute for Telecommunications and Information Technology (Calit2), University of California San Diego, CA, U.S. Her current research interests include Heterogenous Cellular Network, Software Defined Wireless Networks, Cognitive Network, and Dynamic Radio Resource Management.

Tingting Sun received the B. Eng. degree in Communications Engineering from Hefei University of Technology, Hefei, Anhui, China in 2011. She is currently pursuing for the M. Eng. degree in Communication and Information System from Xidian University, Xian, Shaanxi, China. Her current research interests include wireless network capacity and medium access control for wireless personal area networks.

Changle Li (M'09) received the B.Eng. degree in Microwave Telecommunication Engineering, M.Eng. and Ph.D. degrees in Communication and Information System, all from Xidian University, China, in 1998, 2001 and 2005, respectively. From 2006 to 2007, he was with Computer Science Department at University of Moncton, Canada as a postdoctoral researcher. From 2007 to 2009, he was an expert researcher at National Institute of Information and Communications Technology (NICT), Japan. He is currently a professor in State Key Laboratory of Integrated Service Networks at Xidian University, China. His research interests include cross-layer design and communication protocols for cellular network, mobile ad hoc network, wireless sensor network, vehicular network and wireless body area network (WBAN).
Xinbing Wang (M’06-SM’12) received the B.S. degree (with hon.) from the Department of Automation, Shanghai Jiaotong University, Shanghai, China, in 1998, and the M.S. degree from the Department of Computer Science and Technology, Tsinghua University, Beijing, China, in 2001. He received the Ph.D. degree, major in the Department of electrical and Computer Engineering, minor in the Department of Mathematics, North Carolina State University, Raleigh, in 2006. Currently, he is a professor in the Department of Electronic Engineering, Shanghai Jiaotong University, Shanghai, China. Dr. Wang has been an associate editor for IEEE/ACM Transactions on Networking and IEEE Transactions on Mobile Computing, and the member of the Technical Program Committees of several conferences including ACM MobiCom 2012, ACM MobiHoc 2012, 2013, IEEE INFOCOM 2009-2014.