Pricing for Uplink Power Control in Cognitive Radio Networks

Hui Yu, Lin Gao, Zheng Li, Xinbing Wang, and Ekram Hossain

***Abstract*—We study the pricing issue in a competitive cognitive radio network in which the secondary users strategically adjust their uplink transmission power levels to maximize their own utilities, and the primary service provider (e.g., base station) charges the secondary users on their transmitted power levels to enhance its own revenue. We model the competitive behavior of the secondary users as a non-cooperative game and address the existence and uniqueness of Nash equilibrium. Based on the unique equilibrium, we formulate the pricing problem for the primary service provider as a non-convex optimization problem. We propose a sub-optimal pricing scheme in terms of revenue maximization of the primary service provider, and we claim that this scheme is not only fair in terms of power allocation among secondary users but also efficient. We perform extensive numerical studies to verify the sub-optimality, fairness, and efficiency of the proposed pricing scheme and investigate the impacts of system parameters on the performance of the proposed pricing scheme.**

Keywords: Cognitive radio, power control, non-cooperative game, profit maximization, fairness, efficiency.

I. INTRODUCTION

Cognitive radio is an emerging technique which has the potential to solve the problem of spectrum under-utilization faced by today's wireless community. The fundamental principle of cognitive radio is to enhance the efficiency and flexibility in spectrum usage by allowing secondary users (unlicensed users) to access the resources owned by primary users (licensed users) in an opportunistic manner. There are many design and implementation issues in the realization of cognitive radio, and one of the most challenging issues is how to achieve "peaceful" coexistence of primary users and secondary users [1].

The "unregulated" and self-organized nature of cognitive radio networks makes resource management problems such as power control quite complicated. Wireless radio devices with "cognitive" ability are intelligent and strategic decision makers who act in accordance with their own self-interest. Secondary users compete with each other to maximize their own utilities[1], while primary users (or primary service providers) charge secondary users on the resource consumption such as transmission power in order to enhance their own revenue. Thus pricing plays an important role in the interaction of primary users and secondary users. In cognitive radio networks, secondary users are price takers who behave strategically considering the price and the competition they face, and primary users are price makers who use the "invisible hand" to allocate resources and to maximize their own revenue.

In this paper, we study the problem of pricing-based power control in cognitive radio networks. Based on the framework of cellular networks, we regard a base station as the primary service provider who prices the uplink power of each secondary user in the same cell, and we model the competition among secondary users using non-cooperative game theory. We formulate the pricing problem for a primary service provider to regulate transmission power by secondary users as a non-convex optimization problem. While it is difficult to solve this problem analytically, we propose a sub-optimal pricing scheme. We show that this scheme is fair as well as efficient in terms of power allocation. We verify its sub-optimality and investigate its properties and the system performance with varying system parameters.

The rest of the paper is organized as follows: we first present the background and related work in Section II. We describe the system model and state the problem formulation in Section III. Then we propose a sub-optimal pricing scheme, followed by relevant derivations and discussions on the properties of the pricing scheme in Section V. We propose an admission control method for the proposed pricing scheme in Section VI. We provide numerical results in Section VII and conclude the paper in Section VIII.

II. BACKGROUND AND RELATED WORK

The power control problem for traditional cellular wireless communications systems was addressed in several classic papers [2], [3] which focused on minimizing total transmitted uplink power subject to maintaining an individual target carrier to interference ratio for each mobile where the assignment of the users to the base stations may not be fixed. The power control problem in wireless networks with strategic nodes was also investigated in the literature. A non-cooperative uplink power control game was formulated in [4] the outcome of which results in a Nash equilibrium that is inefficient. Therefore, a pricing scheme was introduced in order to obtain Pareto improvement. In a similar spirit, [5] also presented a framework of uplink power control based on non-cooperative game, and it analyzed the Nash equilirium as the solution of the game, and proposed two dynamic updating algorithms to achieve the equilibrium. Apart from non-cooperative game, in [6], cooperative game and Nash Bargaining solution were used to model the uplink power control problem in code-division multiple access (CDMA) systems in order to obtain

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[1]We adopt the term "utility" here as in other game-theoretic literature, and it has the same meaning as "net profit".

Pareto-optimal solutions. In [7], the problem of both channel allocation and power control in adaptive wireless networks was modeled by using potential game. Two game-theoretic power allocation schemes were proposed in [8] to achieve efficiency and fairness in power allocation. In these work, price was primarily used as a system control parameter adjusted by the regulator to achieve objectives such as efficiency, whereas revenue-maximization of the price maker was not considered. The problem of power control and rate adaptation for cognitive radios in a CDMA environment was addressed in [9]; however, the pricing issue was not considered.

In [10], the authors addressed the problem of revenue maximization for primary users and presented a Stackelberg game[2] formulation. However, it did not consider the constraints on the resource usage of the primary user and performance guarantees for the secondary users and it ignored the impact of ambient noise. The interaction among spectrum owner, primary users and secondary users was also studied in [12] using Stackelberg game mostly by simulations.

The pricing problem in cognitive radio networks has been addressed in recent literature. In [13], the authors investigated different types of secondary pricing combined with admission control. Spectrum pricing strategies were developed which capture the effects of network-wide interferences. The dynamics of price competition among competitive spectrum providers was analyzed in [14]. The problem of revenue maximization and pricing for the spectrum owners was addressed in [15], [16], however, the resource constraints and performance guarantees were not considered. In [17], a revenue enhancing model for spectrum pricing was proposed where both primary and secondary users share a set of channels using the same protocol. However, the strategic interaction among secondary users was not addressed.

Our analysis in this paper is based on the power control framework in [5]. However, we focus on the pricing issue considering not only the system stability provided by the equilibrium but also the profitability of the primary user (or primary service provider). In addition, the fairness and efficiency in power allocation are investigated. Our contribution mainly lies in the modeling of the revenue maximization and pricing problem for a primary service provider as an optimization problem while considering the strategic behavior of the secondary users at the same time. We formulate the pricing problem faced by the primary user as a non-convex optimization problem. The proposed sub-optimal pricing scheme is a generalization of the proportional pricing scheme presented in [10] and provides useful insights into designing cognitive radio networks (e.g., designing admission control method).

III. System Model and Problem Setup

The power control framework is for a CDMA-based cognitive radio cellular network in which primary users (PUs) in a cell are licensed to transmit to the base station over the bandwidth allocated, while the secondary users (SUs) have to pay to the primary service provider (i.e., the base station) for their uplink transmissions. Fig. 1 illustrates the system model where the primary users and the base station constitute the primary network and the secondary users constitute the secondary network which is laid over the primary network in the cell. The secondary users (or cognitive radios) access the channel used by the primary users in a spectrum underlay fashion to transmit to the base station. Each secondary user adjusts its power level in uplink transmission considering the price charged by the primary network and the interference caused by the other secondary users. The primary network aims to maximize its own revenue by charging the secondary users on their resource usage, while taking into account the resource constraints imposed by its own performance requirement. In other words, the primary network must have a threshold on the resource usage of the secondary users so that the performance degradation of primary users is tolerable.

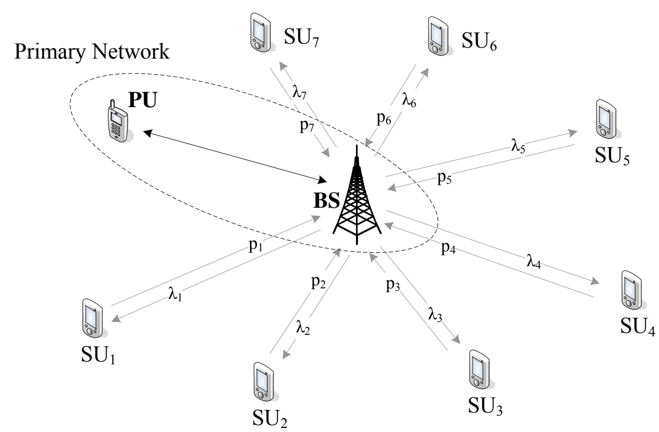

Fig. 1. System model: A single-cell CDMA cognitive radio network.

We consider a network with N cognitive secondary users numbered $1, \cdots, N$. For the ith secondary user, the SNR achieved at the base station can be obtained using the following formula:

$$\gamma_i(\mathbf{p}) = \frac{L p_i h_i}{\sum_{j=1, j\neq i}^{N} p_j h_j + \sigma^2} \tag{1}$$

where $L = W/R$ is the spreading gain of the CDMA system, W is the chip rate and R is the data rate, and $L > 1$; p_i is the transmit power level of the ith secondary user; $\mathbf{p} = \{p_1, p_2, \cdots, p_N\}$ is the transmit power vector for N secondary users; h_i is the uplink channel gain of the ith user, $0 < h_i < 1$; σ^2 is the interference caused by the primary network and the ambient noise in the system.

The base station charges the ith secondary user the amount λ_i per unit of transmit power on its uplink channel. In other words, if the transmit power level of the ith secondary user is p_i, the total amount charged on secondary user i's power usage is $\lambda_i p_i$. For a secondary user, the payment can be seen as its cost of accessing the primary network. For the primary user, the payment can be seen as the compensation of its potential service quality degradation caused by the interfering of secondary user. Thus, the net utility (i.e., the difference between transmission gain and payment) of secondary user i

[2]It is a dynamic model of duopoly proposed by Stackerlberg (1934) in which a dominant (or leader) firm moves first and a subordinate (or follower) moves second [11].

is given by

$$U_i(\mathbf{p}, \lambda_i) = \alpha_i \ln(1 + \gamma_i(\mathbf{p})) - \lambda_i p_i. \quad (2)$$

We note that $\ln(1+\gamma_i(\mathbf{p}))$ is the Shannon capacity [19] for secondary user i and the parameter α_i is the equivalent utility per unit data rate valuation contributing to the overall utility, which is a predefined parameter. In essence, α_i reflects the preference of secondary user i for a given data rate (or channel capacity), and for different secondary users, α_i may be different according to the difference in the type of services. For example, for the service with higher data rate requirement (e.g., multimedia service), the secondary user i may want to set higher α_i, and vice versa. It is easy to see that the secondary users of higher α_i are willing to buy higher amount of transmit power, compared with those of lower α_i. We refer to α_i as the *transmission gain coefficient* of secondary user i.

The total revenue for the base station is given by

$$R(\Lambda, \mathbf{p}) = \sum_{i=1}^{N} \lambda_i p_i \quad (3)$$

where $\Lambda = (\lambda_1, \lambda_2, \cdots, \lambda_N)$ and $\mathbf{p} = (p_1, p_2, \cdots, p_N)$.

We consider the constraints on the resources owned by the primary network and performance guarantees for the secondary users, which characterize the interaction between the primary users and the secondary users. We formulate the revenue maximization problem for a primary service provider as a non-convex optimization problem as follows:

$$\begin{aligned} &\max_{\lambda_i \geq 0} R\left(\Lambda, \mathbf{p}^*(\Lambda)\right) \\ \text{s.t.} \quad & p_i^*(\lambda_i) h_i \leq P_{max}, \quad i = 1, \cdots, N \\ & \sum_{i=1}^{N} p_i^*(\lambda_i) h_i \leq P_{tot\_max} \\ & \gamma_i \geq \Gamma_{min}, \quad i = 1, \cdots, N \end{aligned} \quad (4)$$

where P_{max} is the maximum allowed power received at the base station for a secondary user, P_{tot_max} is the upperbound of the total amount of power received at the base station (i.e., P_{tot_max} is a power threshold below which secondary users' transmission would not cause intolerable interference to the primary users). These constraints reflect the limitation of resources owned by the primary network to accommodate secondary users. Γ_{min} is the minimum SNR required for a secondary user to communicate with the base station. If secondary user i's SNR is below this minimum requirement, it would simply not access the network.

IV. Analytical Model for Power Control and Revenue Maximization

The strategic interaction among secondary users can be characterized as a non-cooperative game, in which each secondary user is a decision maker who behaves selfishly to maximize its own utility. Specifically, for a given unit price λ_i and the transmit power vector of all secondary users except the ith user denoted by $\mathbf{p}_{-i}$, secondary user i's objective function can be represented by[3]

$$\max_{p_i \geq 0} \; U_i(p_i, \mathbf{p}_{-i}, \lambda_i). \quad (5)$$

We regard the *Nash equilibrium (NE)* as a desirable outcome of this game, and we formally state the following definition:

Definition 1. *A power vector $\mathbf{p}^* = (p_1^*, \cdots, p_N^*)$ is a Nash equilibrium of the uplink power control game if for every user $i \in \{1, \cdots, N\}$,*

$$U_i(p_i^*, \mathbf{p}_{-i}^*, \lambda_i) \geq U_i(p_i, \mathbf{p}_{-i}^*, \lambda_i), \quad \forall p_i \neq p_i^*.$$

That is, at the NE, given the power levels of the other secondary users, no secondary user can benefit by changing its transmission power unilaterally.

We derive the best response for each secondary user as a function of the price set by the primary network and the power levels used by the other secondary users in the network. For notational simplicity, we denote the ith secondary user's power level received at the base station as $y_i := p_i h_i$ and introduce the quantity $y_{-i} := \sum_{j=1, j\neq i}^{N} y_j$. To facilitate the presentation and analysis, we further introduce a user specific notation θ_i for each secondary user i, as shown in the following equation:

$$\theta_i = \frac{\alpha_i h_i}{\lambda_i} - \frac{\sigma^2}{L}. \quad (6)$$

We can easily find that the objective function in (5), i.e., the net utility $U_i(p_i, \mathbf{p}_{-i}, \lambda_i)$, is concave in p_i. Thus we can obtain the best response of secondary user i when the first derivative of $U_i(p_i, \mathbf{p}_{-i}, \lambda_i)$ with respect to p_i equals to 0, i.e.,

$$\begin{aligned} \frac{\partial U_i}{\partial p_i} &= \frac{\alpha_i}{1+\gamma_i(\mathbf{p})} \cdot \frac{\gamma_i(\mathbf{p})}{p_i} - \lambda_i \\ &= \frac{\alpha_i}{\frac{1}{\gamma_i(\mathbf{p})} + 1} \cdot \frac{1}{p_i} - \lambda_i \\ &= \frac{\alpha_i}{\frac{y_{-i}+\sigma^2}{L h_i} + p_i} - \lambda_i = 0. \end{aligned}$$

The last line follows because $\gamma_i(\mathbf{p}) = \frac{L p_i h_i}{\sum_{j=1, j\neq i}^{N} p_j h_j + \sigma^2} = \frac{L p_i h_i}{y_{-i}+\sigma^2}$. Thus we can write the best response of secondary user i as follows:

$$p_i^* = \Phi_i(\lambda_i, \mathbf{p}_{-i}) \triangleq \begin{cases} \frac{1}{h_i}\left[\theta_i - \frac{y_{-i}}{L}\right], & \text{if } y_{-i} \leq L\theta_i \\ 0, & \text{else.} \end{cases} \quad (7)$$

We note that the second choice is due to the non-negativity constraint on the transmit power in (5). Then we adopt the result of [5] and give the following theorem:

Theorem 1. *In the power game with N secondary users, we index the secondary users such that $\theta_i < \theta_j \Rightarrow i > j$, with the ordering picked arbitrarily if $\theta_i = \theta_j$. Let M^* be the largest integer $M \leq N$ for which the following condition is satisfied:*

$$\theta_M > \frac{1}{L+M-1} \sum_{i=1}^{M} \theta_i. \quad (8)$$

[3]Note that $U_i(p_i, \mathbf{p}_{-i}, \lambda_i)$ is the net utility of the ith secondary user which is defined in (2), i.e., $U_i(p_i, \mathbf{p}_{-i}, \lambda_i) = U_i(\mathbf{p}, \lambda_i)$, where $\mathbf{p} = (p_i, \mathbf{p}_{-i})$. Here, we rewrite the ith secondary user's net utility as $U_i(p_i, \mathbf{p}_{-i}, \lambda_i)$ to better bring forth the relation between her net utility and strategy (i.e., p_i).

Then, the power game admits a unique NE, which has the property that users $M^*+1,\cdots,N$ have zero power levels, $p_j^*=0, j\geq M^*+1$. The equilibrium power levels of the first M^* users are obtained uniquely by

$$p_i^* = \frac{1}{h_i}\left\{\frac{L}{L-1}\left[\theta_i - \frac{1}{L+M^*-1}\sum_{j=1}^{M^*}\theta_j\right]\right\} \quad (9)$$

for each $i=1,2,\cdots,M^*$.

If there is no M for which (8) is satisfied, then the NE solution is again unique, but assigns zero power level to all N users.

Proof: The proof can be found in [5]. ■

We present *system efficiency* as another metric of system performance, and we give the following definition:

Definition 2. *A power vector $\mathbf{p}^\dagger=(p_1^\dagger,\cdots,p_N^\dagger)$ is system efficient (or simply efficient) if it maximizes the system capacity, i.e., the sum of the channel capacity of all the secondary users. That is,*

$$\sum_{i=1}^{N}\alpha_i\ln(1+\gamma_i(\mathbf{p}^\dagger)) \geq \sum_{i=1}^{N}\alpha_i\ln(1+\gamma_i(\mathbf{p})), \quad \forall \mathbf{p}\neq\mathbf{p}^\dagger.$$

In many cases, the Nash equilibrium is not efficient and it is difficult to achieve an efficient power allocation. The strategic interaction among secondary users naturally leads to a Nash equilibrium which is stable, whereas system efficiency is not guaranteed. We base our pricing scheme on the establishment of a Nash equilibrium, and we integrate the consideration of system efficiency into our design as well.

Provided that we obtain the Nash equilibrium as a stable outcome of the competition among secondary users, we now focus on the revenue maximization problem faced by the primary user. By substituting (6) and (9) into the total revenue given by (3), we obtain

$$R(\Lambda,\mathbf{p}^*(\Lambda)) = \frac{L}{L-1}\sum_{i=1}^{M^*}\left(\alpha_i - \frac{\sigma^2\lambda_i}{Lh_i} - \frac{1}{L+M^*-1}\frac{\lambda_i}{h_i}\sum_{j=1}^{M^*}\left(\frac{\alpha_j h_j}{\lambda_j}-\frac{\sigma^2}{L}\right)\right). \quad (10)$$

We observe that the above optimization problem is non-convex, and we must use the Karush-Kuhn-Tucker (KKT) condition [20] in a general case to solve the optimal point, which makes the problem very complex. Therefore, we propose a pricing scheme which can approach the optimal solution. We verify its sub-optimality by numerical studies. The proposed pricing scheme also has nice properties such as fairness and efficiency. We present this pricing scheme and investigate its properties in the following section.

V. PRICING SCHEME

The pricing scheme is inspired by [10], in which the authors addressed the first and second order optimality conditions for the revenue maximization problem while the effect of noise σ^2 was ignored. Also, in [10], the constraints specified in (4) above were not considered. We claim that the pricing scheme presented below is sub-optimal for the non-convex optimization problem mentioned in the previous section and we will verify its sub-optimality later by numerical studies. We denote by $\lambda^*=(\lambda_1^*,\lambda_2^*,\cdots,\lambda_N^*)$ the solution price vector for the proposed pricing scheme.

Pricing Scheme. *The primary service provider first sets the prices proportionally, which satisfy the following conditions:*

$$\frac{\lambda_i^*}{h_i\sqrt{\alpha_i}} = \frac{\lambda_j^*}{h_j\sqrt{\alpha_j}} = K, \quad \forall i,j=1,\cdots,N. \quad (11)$$

Then the primary service provider adjusts the parameter K to maximize its total revenue under the constraints in (4).

For the pricing scheme, the optimization problem can be reduced to the problem of finding the optimal value of K under the constraints in (4). Considering the three constraints in (4), we give the following three Lemmas:

Lemma 1. *Let $i^+=\arg\max_{i\in\{1,\cdots,M^*\}}\alpha_i$. Then the constraints $p_i^*(\lambda_i)h_i\leq P_{max}, i=1,\cdots,M^*$ suggest that*

$$K \geq \widetilde{K}_1 = \frac{L}{L-1}\frac{\sqrt{\alpha_{i^+}} - \frac{1}{L+M^*-1}\sum_{j=1}^{M^*}\sqrt{\alpha_j}}{P_{max}+\frac{\sigma^2}{L+M^*-1}}. \quad (12)$$

*Proof: See the **Appendix**.* ■

Lemma 2. *The constraint $\sum_{i=1}^{M^*}p_i^*(\lambda_i)h_i\leq P_{tot\_max}$ suggests that*

$$K \geq \widetilde{K}_2 = \frac{\frac{L}{L+M^*-1}\sum_{j=1}^{M^*}\sqrt{\alpha_j}}{P_{tot\_max}+\frac{M^*\sigma^2}{L+M^*-1}}. \quad (13)$$

*Proof: See the **Appendix**.* ■

Lemma 3. *The constraints $\gamma_i\geq\Gamma_{min}, i=1,\cdots,M^*$ will be satisfied only when the following conditions are satisfied:*

$$\sqrt{\alpha_i} \geq \frac{L\Gamma_{min}+1}{(\Gamma_{min}+1)(L+M^*-1)}\sum_{j=1}^{M^*}\sqrt{\alpha_j}, \quad \forall i=1,\cdots,M^*. \quad (14)$$

Let $i^-=\arg\min_{i\in\{1,\cdots,M^*\}}\alpha_i$. Then the constraints $\gamma_i\geq\Gamma_{min}, i=1,\cdots,M^*$ suggest that

$$K \leq \widehat{K} = \frac{L}{L-1}\frac{\frac{(\Gamma_{min}+1)(L+M^*-1)}{L\Gamma_{min}+1}\sqrt{\alpha_{i^-}} - \sum_{j=1}^{M^*}\sqrt{\alpha_j}}{\sigma^2}. \quad (15)$$

*Proof: See the **Appendix**.* ■

We assume that the primary network is able to obtain all the information required to calculate the upperbound and lowerbounds of K according to the above three Lemmas. Thus, the optimal K lies in the interval as follows:

$$\max\{\widetilde{K}_1,\widetilde{K}_2\} \leq K \leq \widehat{K}. \quad (16)$$

We now seek the optimal value of K in the interval specified above. Before we proceed, we state the following Lemma:

Lemma 4. *The total revenue $R(\Lambda,\mathbf{p}^*(\Lambda))$ is monotonically decreasing with respect to K in the interval specified in* (16).

*Proof: See the **Appendix**.* ■

With the proposed pricing scheme, the optimal value of K for the optimization problem in (4) can be defined in the following theorem:

Theorem 2. *The optimal value of K for the pricing scheme stated above is*

$$K^* = \max\{\widetilde{K}_1, \widetilde{K}_2\} \tag{17}$$

where $\widetilde{K}_1$ and $\widetilde{K}_2$ are given in Lemma 1 and Lemma 2, respectively.

Proof: *See the* ***Appendix***. ■

Since it is difficult to quantify the sub-optimality of the proposed pricing solution to the non-convex optimization problem in (4), we will present numerical results in the next section to verify that the prices obtained for the proposed pricing scheme approach very well to the optimal prices obtained by brute force searching.

In addition to its sub-optimality, the proposed pricing scheme is fair in terms of allocation of power. The primary network charges higher prices those secondary users who have higher uplink channel gain (i.e., h_i) and/or have higher transmission gain coefficient (i.e., α_i). Moreover, the pricing scheme is system efficient in the framework of proportional price setting, and we formally state it in the following theorem:

Theorem 3. *Given the proportional price setting, the optimal K obtained from Theorem 2 is system efficient, i.e., the resulting power allocation maximizes the sum of the channel capacity of all the secondary users. That is,*

$$\sum_{i=1}^{M^*} \alpha_i \ln(1+\gamma_i(K^*)) \geq \sum_{i=1}^{M^*} \alpha_i \ln(1+\gamma_i(K)) \quad for \quad \max\{\widetilde{K}_1, \widetilde{K}_2\} \leq K \leq \widehat{K} \tag{18}$$

Proof: *See the* ***Appendix***. ■

VI. Admission Control Method for the Proposed Pricing Scheme

From Lemma 3, we notice that in order to meet the performance requirement, we need to guarantee not only that the condition in (14) is satisfied, but also to make sure that $\max\{\widetilde{K}_1, \widetilde{K}_2\} \leq \widehat{K}$ so that a feasible value of K exists. This involves fairly complicated admission control to regulate the number of secondary users (N) allowed to share the radio spectrum with primary users.

For illustration purpose, we assume that $M^* = N$ and $\alpha_i = \alpha_j = \alpha, \forall i, j = 1, \cdots, N$, which is a symmetric case. We further assume that there exists feasible K and we find the optimal K^*. Then, every secondary user has the same SNR γ and it can be expressed as a function of N as follows:

$$\gamma(N) = \frac{L\sqrt{\alpha} - \sigma^2 K^*(N)}{L(N-1)\sqrt{\alpha} + L\sigma^2 K^*(N)} \tag{19}$$

where $K^*(N) = \max\{\widetilde{K}_1(N), \widetilde{K}_2(N)\}$.

From (17), we obtain $\widetilde{K}_1(N)$ and $\widetilde{K}_2(N)$ as follows:

$$\widetilde{K}_1(N) = \frac{L\sqrt{\alpha}}{(L+N-1)P_{max} + \sigma^2}$$

$$\widetilde{K}_2(N) = \frac{LN\sqrt{\alpha}}{(L+N-1)P_{tot\_max} + N\sigma^2}. \tag{20}$$

Based on the above equations, we can derive the maximum admissible number of secondary users based on the performance requirement $\gamma \geq \Gamma_{min}$. These manipulations are omitted due to the brevity of the paper.

We notice that the total revenue gained by the base station is also a function of N given that we can always obtain an optimal K^*. The formula for revenue is as follows:

$$R(N, K^*(N)) = \frac{-K^*(N)\sigma^2 N\sqrt{\alpha}}{L+N-1} + \frac{LN\alpha}{L-1}\left(1 - \frac{N\alpha}{L+N-1}\right) \tag{21}$$

where $K^*(N) = \max\{\widetilde{K}_1(N), \widetilde{K}_2(N)\}$ as specified in (20).

Note that the relationship between the revenue R and N is quite complicated, and the maximum allowed N might not be optimal in terms of revenue maximization.

VII. Numerical Study

In this section, we first verify the sub-optimality of the proposed pricing scheme in terms of revenue maximization by comparing it to brute-force searching, and we discuss fairness among secondary users as well. Then we investigate the impact of several system parameters on the performance of the proposed pricing scheme. Further, we present simulation results and discussions on system efficiency. Last, we verify the relationship between the total revenue and K, and that between the system capacity and K.

A. Revenue Maximization

Considering that the complexity of brute-force searching grows exponentially with the number of users, we begin with a simple scenario with $N = 2$ secondary users in a cognitive radio network. We set the system parameters as follows: $L = 8$, $\sigma^2/L = 0.1$, $P_{max} = 5$, $P_{tot\_max} = 8$, $\Gamma_{min} = 0.01$. We present Figs. 2-3 showing the revenue contour of the two prices both ranging from 0 to 0.8 and the contours are obtained by brute-force searching. These two figures differ in user parameters (i.e., $(\alpha_1, \alpha_2), (h_1, h_2)$). In Fig. 2, two secondary users have the same transmission gain coefficient and different channel gain, while in Fig. 3, they have the same channel gain and different transmission gain coefficients. In both cases, the dotted lines representing the proportional price setting according to the proposed pricing scheme are at the top of the contours. And the optimal prices on the dotted lines corresponding to the optimal K in the two figures are circle-marked, which lie very near to the top of the contours. Thus, we conclude that for the assumed system settings the optimal prices obtained by the proposed pricing scheme approach very well to those obtained by brute-force searching. Later we will vary system parameters (i.e., L, σ^2) to see how this sub-optimality will be affected.

In addition, Figs. 2-3 reveal that the pricing scheme is fair. In Fig. 2 the primary network charges more a secondary user with a higher channel gain than her opponent, while in Fig. 3 the primary network charges more the secondary user with a higher transmission gain coefficient than her counterpart.

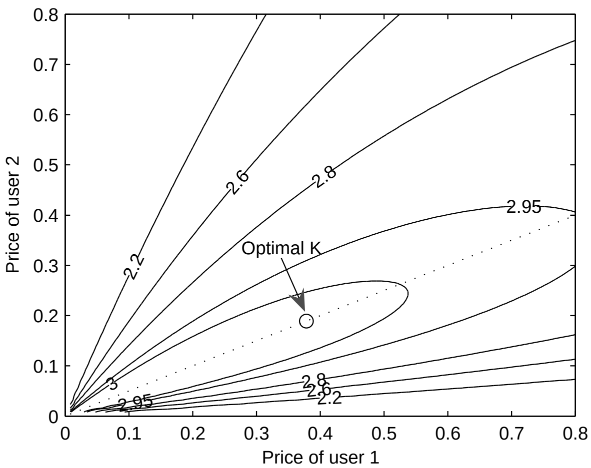

Fig. 2. The revenue contour of the prices of secondary user 1 and 2 when $(\alpha_1, \alpha_2) = (2, 2)$, $(h_1, h_2) = (1, 0.5)$.

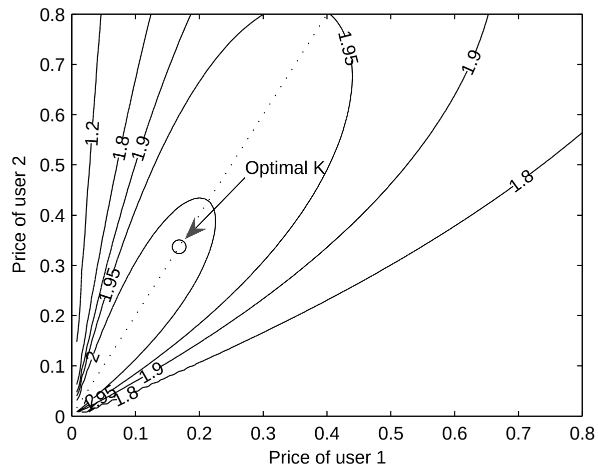

Fig. 3. The revenue contour of the prices of secondary user 1 and 2 when $(\alpha_1, \alpha_2) = (0.5, 2)$, $(h_1, h_2) = (1, 1)$.

B. Impact of Parameters on System Performance

We now investigate how the sub-optimality of the proposed pricing scheme will be affected by the system parameter σ^2/L, which characterizes the impact of ambient noise on the CDMA system. From Fig. 4, we can see that the gap between the total revenue obtained by the proposed pricing scheme and that obtained by brute-force searching increases when σ^2/L becomes higher. Fig. 4 shows that the revenue gap between the proposed pricing scheme and the optimal approach decreases from 100% to 90% as the noise σ^2/L varies from 0.1 to 3. In other words, the sub-optimality of the proposed pricing scheme is guaranteed when σ^2/L is small. This is actually a reasonable assumption. For example, with IS-95 standard [19], $L = 128$ and $\sigma^2 = 1 \sim 10$.

We then observe the impact of other system parameters (i.e., P_{max}, P_{tot_max}, Γ_{min}) on the performance of the proposed pricing scheme. We now consider a network consisting of $N = 10$ secondary users, and the system setup is as follows: $L = 16$, $\sigma^2/L = 0.1$, α_is are uniformly distributed in $[0.5, 1]$, and h_is are uniformly distributed in $[0, 1]$.

We fix $\Gamma_{min} = 0.01$ in Fig. 5, and we can see that the optimal total revenue of primary network obtained by our pricing scheme increases as P_{max} or P_{tot_max} increases. In other words, the primary network can gain a higher revenue by relaxing the constraints on its resources. Moreover, P_{max} and P_{tot_max} are correlated with each other. Fig. 5 shows that when P_{max} is small, the revenue increases dramatically as P_{max} increases. When P_{max} is large, P_{tot_max} becomes the bottleneck to the increase in revenue. At this point, increasing P_{tot_max} will effectively increase the revenue.

In Fig. 6, we fix $P_{max} = 5$, $P_{tot\_max} = 50$, and we observe that the optimal total revenue of the primary network is not affected by Γ_{min} when the number of secondary users in the network remains the same. This is because, from (17) we know that Γ_{min} has no effect on the optimal K and on the optimal total revenue. However, Γ_{min} impacts the constraint on K in (16). If the value of Γ_{min} is raised to an extent where the primary network cannot accommodate N secondary users any more, it must exercise admission control to satisfy the minimum SNR requirements for the admitted secondary users. We assume that the base station always excludes the secondary user with the minimum α_i in the network when admission control is exercised. As we can see in Fig. 6, due to the admission control exercised by the primary network, the number of secondary users admitted into the network decreases as Γ_{min} increases.

C. System Efficiency

We compare the system capacity, namely, the sum of channel capacity of all the secondary users obtained by the proposed sub-optimal pricing scheme to the optimal system capacity obtained by brute-force searching. For illustration purpose, we also make the comparison in a simple scenario with $N = 2$ secondary users, and the system setup is as follows: $P_{max} = 5$, $P_{tot\_max} = 7$, $\Gamma_{min} = 0.2$, $\alpha = (1, 1)$, and $\mathbf{h} = (1, 0.5)$. We present Figs. 7-9 showing the capacity contour of the power of two users when $\sigma^2/L = 0.5, 0.1, 0.05$, and these contours are obtained by brute-force searching. The shadow lines in these figures represent the three constraints in (4), namely, the constraints on P_{max}, P_{tot_max}, and Γ_{min}. The shaded regions bounded by the shadow lines in these figures are feasible regions. The circle-marked points are the optimal power resulting from the proposed pricing scheme. In Fig. 7, the sub-optimal point obtained by the proposed pricing scheme is located in the contour curve with the highest capacity, while in Figs. 8 and 9, the sub-optimal points obtained by the proposed pricing scheme are located in the second and third highest contour curves, respectively.

As we can see in Fig. 7, the sub-optimal point obtained by the proposed pricing scheme approximates to the optimal point in the feasible region, that is the tangent point between the shadow line and the curve. In Fig. 8, some curves in the upper and right part become oppositely oriented and the sub-optimal point is smaller than the optimal. The sub-optimal pricing performs worse than the optimal scheme in terms of system efficiency (Fig. 9).

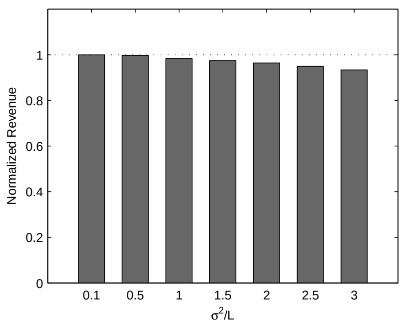

Fig. 4. The gap between the total revenue obtained by the proposed pricing scheme and the optimal revenue obtained by brute force searching when σ^2/L varies.

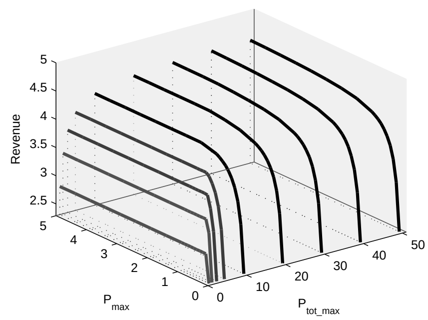

Fig. 5. The optimal total revenue obtained by the proposed pricing scheme with varying P_{max} and P_{tot_max}.

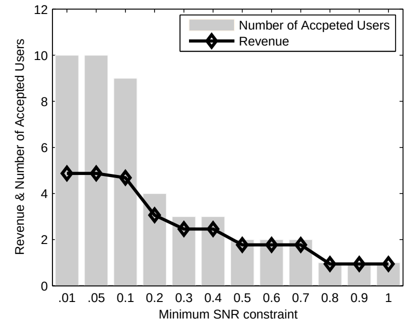

Fig. 6. The optimal total revenue obtained by the proposed pricing scheme and the number of accepted secondary users with varying Γ_{min}.

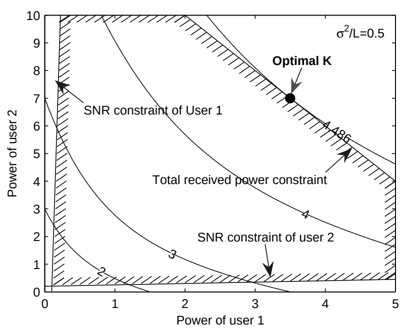

Fig. 7. The capacity contour of the power of secondary user 1 and 2 when $\sigma^2/L = 0.5$.

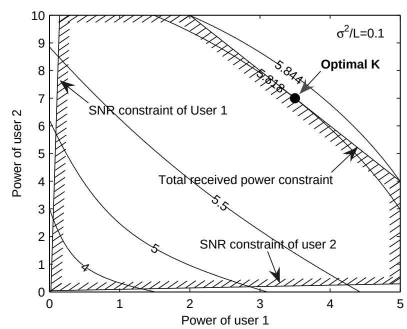

Fig. 8. The capacity contour of the power of secondary user 1 and 2 when $\sigma^2/L = 0.1$.

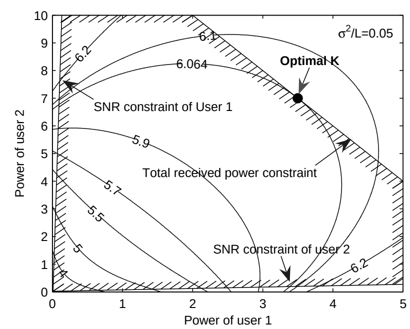

Fig. 9. The capacity contour of the power of secondary user 1 and 2 when $\sigma^2/L = 0.05$.

Fig. 10 shows how ambient noise (σ^2/L) affects the gap between the system capacity obtained by the sub-optimal pricing scheme and the optimal scheme. Interestingly, the gap becomes smaller when the noise grows. The gap is primarily due to the fact that the proportional price setting in the proposed pricing scheme might not be optimal in terms of system efficiency. When the noise is large enough, from (6) and (9) we notice that the pricing would not have much effect on the power allocation and the power allocation can approach the optimal one in this case. Fig. 10 shows that the capacity gap between the proposed pricing scheme and the optimal approach increases from 80% to 100% as the noise σ^2/L varying from 0.01 to 0.5.

D. Revenue and Capacity vs. K

In Theorem 2 and 3 and relevant proofs in the Appendix, we explore the relationship between the revenue and K, and that between the system capacity and K. Here we verify these relationships through Figs. 11-12, where the circles denote the optimal point of K obtained by Theorem 2 and 3. We see that both the total revenue and system capacity are decreasing functions of K in various system settings and these results also justify our choice of the optimal K as we mentioned earlier.

VIII. Conclusion

We have formulated a non-convex optimization problem for the pricing issue in a competitive cognitive radio network. In this case the primary service provider charges on the uplink power level of multiple strategic secondary users to maximize its own revenue, considering constraints on its resource and performance guarantees for the secondary users (in terms of SINR). We have adopted the unique Nash equilibrium of the game among the secondary users as desirable outcome. Based on the equilibrium we have proposed a sub-optimal, fair, and efficient pricing scheme to address the price optimization problem for the primary service provider. The proposed pricing scheme is sub-optimal in terms of the revenue maximization of the primary user and fair and efficient in terms of the power allocation among the secondary users. Numerical results have verified the sub-optimality of our pricing scheme in general system settings and have provided insights into how system parameters would affect the performance of the pricing scheme.

The model can be extended to a multiple-primary-user scenario where the primary service providers compete with each other by tuning their prices to attract the secondary users. On the other hand, the secondary users strategize on both cell selection and power level adjustment. In this paper we have assumed that the private information of secondary user i such as α_i is known to the primary network. However, a secondary user can possibly report its private information untruthfully to the primary user for its own self-interest. This “misbehavior” problem should be also addressed.

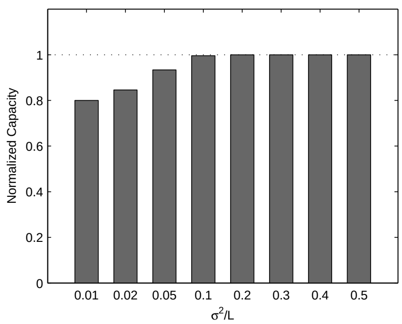

Fig. 10. The gap between system capacity obtained by the proposed scheme and the optimal capacity obtained by brute force searching when σ^2/L varies.

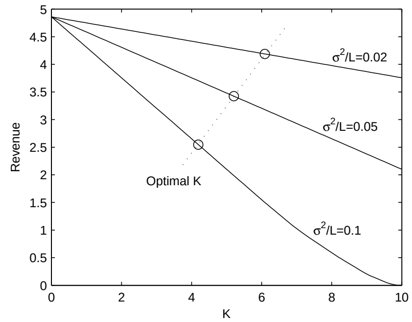

Fig. 11. The relationship between total revenue and K where $\Gamma_{min} = 0.01, P_{max} = 0.1, P_{tot\_max} = 1$.

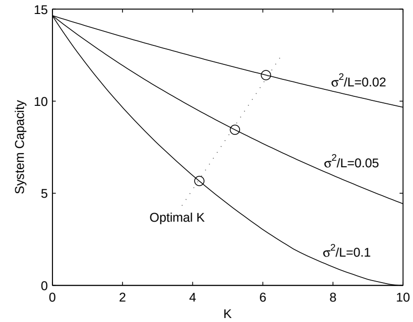

Fig. 12. The relationship between system capacity and K where $\Gamma_{min} = 0.01, P_{max} = 0.1, P_{tot\_max} = 1$.

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APPENDIX

Proof of Lemma 1:

Proof: Let $i^+ = \arg\max_{i\in\{1,\cdots,M^*\}} \alpha_i$. Then $p^*_{i^+}h_{i^+} = \max\{p^*_i h_i, i = 1, \cdots, M^*\}$, where

$$p^*_i h_i = \frac{L}{L-1}\frac{1}{K}\left(\sqrt{\alpha_i} - \frac{1}{L+M^*-1}\sum_{j=1}^{M^*}\sqrt{\alpha_j}\right) - \frac{\sigma^2}{L+M^*-1}.$$

Then we derive the first lowerbound of K (i.e., $\widetilde{K}_1$) from the inequality below:

$$p^*_{i^+}h_{i^+} \le P_{max}. \tag{22}$$

■

Proof of Lemma 2:

Proof: We derive the second lowerbound of K (i.e., $\widetilde{K}_2$) from the relationship below:

$$\sum_{i=1}^{M^*} p^*_i h_i = \frac{L\beta}{K} - \frac{M^*\sigma^2}{L+M^*-1} \le P_{tot\_max} \tag{23}$$

where $\beta = \frac{1}{L+M^*-1}\sum_{j=1}^{M^*}\sqrt{\alpha_j}$.

■

Proof of Lemma 3:

Proof: Let $i^- = \arg\min_{i\in\{1,\cdots,M^*\}} \alpha_i$. Then $\gamma_{i^-} = \min\{\gamma_i, \text{ for } i = 1, \cdots, M^*\}$, where

$$\gamma_i = \frac{\frac{L}{L-1}\left(\sqrt{\alpha_i} - \beta\right) - \frac{K\sigma^2}{L+M^*-1}}{L\beta - \frac{L}{L-1}\left(\sqrt{\alpha_i} - \beta\right) + \frac{LK\sigma^2}{L+M^*-1}} \tag{24}$$

in which $\beta = \frac{1}{L+M^*-1}\sum_{j=1}^{M^*}\sqrt{\alpha_j}$.

Then we derive the upperbound of K (i.e., $\widehat{K}$) from the the inequality below:

$$\gamma_{i^-} \ge \Gamma_{min}. \tag{25}$$

The condition in (14) is due to the nonnegativity of $\widehat{K}$ represented in (15).

■

Proof of Theorem 2:

Proof: By substituting (11), the proportional price setting in the proposed pricing scheme into (10), the total revenue of the primary user, we obtain the total revenue as a function of K as follows:

$$R(K) = -K\sigma^2\beta + \frac{L}{L-1}\left(\sum_{i=1}^{M^*}\alpha_i\right)\left(1 - \frac{\sum_{i=1}^{M^*}\alpha_i}{L+M^*-1}\right) \tag{26}$$

where $\beta = \frac{1}{L+M^*-1}\sum_{j=1}^{M^*}\sqrt{\alpha_j}$. Thus $R(K)$ is a decreasing linear function of K and by combining with the constraint on K imposed by (16) we obtain the optimal K as in (17).

■

Proof of Theorem 3:

Proof: By substituting (24) into the Shannon capacity formula, we obtain the sum of the channel capacity of all the secondary users as follows:

$$\begin{aligned}&\sum_{i=1}^{M^*}\alpha_i\ln(1+\gamma_i(K)) = \\ &\sum_{i=1}^{M^*}\alpha_i\ln(1 + \frac{\frac{L}{L-1}\left(\sqrt{\alpha_i} - \beta\right) - \frac{K\sigma^2}{L+M^*-1}}{L\beta - \frac{L}{L-1}\left(\sqrt{\alpha_i} - \beta\right) + \frac{LK\sigma^2}{L+M^*-1}})\end{aligned} \tag{27}$$

where $\beta = \frac{1}{L+M^*-1}\sum_{j=1}^{M^*}\sqrt{\alpha_j}$.

From the above equation, we can see that the sum of the channel capacity of all the secondary users is a monotonically increasing function of $\gamma_i, \forall i = 1, \cdots, M^*$, and therefore, it is a decreasing function of K. Thus, the conclusion naturally follows. ■

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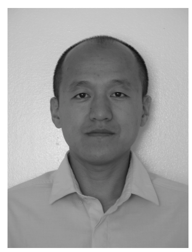

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