Evolution-cast: Temporal Evolution in Wireless Social Networks and Its Impact on Capacity

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***Abstract*—This paper characterizes an evolving network with growing number of nodes, which are associated with each other through social relations but employ transmission via wireless communications. The network exhibits social relations between nodes by attaching a new arriving node to a sample node already existing as well as some nodes the sample directly links to. We study capacity in this evolving network in terms of both geographic distribution of nodes and traffic patterns. Our results show that in heterogeneous geographic distribution where nodes having strong social relations tend to locate more closer, the corresponding capacity is significantly impacted by both social relations and network evolution. In particular, with appropriate control on the initial number of nodes as well as links between them, it is even possible to achieve almost constant per-node capacity except for a poly-logarithmic factor. For homogeneous geographic topology where nodes' positions exhibit uniform distribution, social relations among nodes is useless for improving capacity, which is mainly affected by network evolution and therefore degrades sharply with time. To our best knowledge, this is the first work studying capacity from the perspective of social evolving network.**

keywords: Evolution-cast, Capacity, Social Network

I. Introduction

In the past few years, popularity of social networking has been increasing on a constant rise. As a result, a multitude of new uses for the technology are constantly being observed, including mobile social service [11]- [14], online social network [15]- [17], social business service [18], [19], ad hoc social networks [22]- [23] and pocket switched networks (PSNs) [20], etc. Therefore, social network has emerged as an important direction in research area.

A social network is a structure made up of individuals (or organizations), which are connected by one or more specific types of interdependency, such as friendship, kinship, common interest and financial exchange, etc. Particularly, there have been intense interests for long in modeling real-world social networks. Early observation leading to the attention to social graph comes in the work of Faloutsos *et al.* [1], showing that the degree distribution of Internet graph roughly obeys a "power law". Similar observations are made by Barabasi and Albert [2], who present a model where a network evolves by new nodes attaching themselves to existing nodes with probability proportional to the degrees of those nodes. In another development, Watts *et al.* [3] and Kleinberg *et al.* [4] abstract from experiments graph models dubbed "small-world networks", which captures the phenomenon that if a and b, b and c are friends, then there is a good opportunity that a and c are friends as well. The next significant milestone comes in the work discovering that the real network becomes denser over time with their diameters effectively shrinking over time, presented by a series of mathematical models [5]- [8].

Other than analysis of structural and topological characteristics aforementioned, some initial efforts have been made on studying how social structure can be utilized to impact network performance. The typical work can be found in [9], [10], [24], which aim to design cost-effective routing schemes by taking advantage of social relations to make better forwarding decision. However, these works only reflect partially the impact of social characteristics on network performance while a significant metric, capacity, is not taken into account. Capacity intuitively refers to the average number of packets that can be transmitted between node pairs (The more formal definition is provided in Section III. E). As social networks, especially wireless social networks are rapidly drawing popularity among a wide range of applications, stringent demands on capacity must be imposed to guarantee a better performance of these applications. Also, an important heuristic from these works is that social relations bring new challenges as well as new possibilities to network design. The observations obtained in [9], [10], [24] do reveal that the overhead can be reduced due to intelligent social-based selection on relays. However, this conclusion possibly holds only in pure social networks. Two critical factors, *adjacent interference* and *transmission range* are not considered in these situations while they are non-negligible in wireless networks. A fundamental question therefore arises: Can we see any difference if we study capacity in a network incorporating both the features of social and wireless behaviors? How will capacity be impacted by social network properties, *positively* or *negatively*?

This motivates us to initiate capacity analysis in large scale wireless social networks, where nodes are assigned with social behaviors but obeys the wireless transmission rules. In particular, we focus on an evolving social wireless network. The availability of longitudinal spatio-temporal information about human interactions and social relationships has recently revealed that the networks are not fixed objects at all. The study of dynamical processes taking place on time-evolving graphs has shown that temporal correlations play a fundamental role in diverse settings such as random walks dynamics [26] [27], the spreading of information and diseases [28] [29] [30], etc. Consequently, a network of fixed number of nodes can be only considered as simplified models of real networked systems. For this reason, time-evolving graphs have been lately introduced as a more realistic framework to characterize time-dependent relationships. To capture the features of evolving social networks, we present a novel model which incorporates both nodes' social behaviors and wireless transmission rules. The social behavior in the model is inspired by [25], where Silvio *et al.* first present a simple, realistic and mathematically tractable generative model that intrinsically explains many well-known properties of social networks, as well as *densification* and *shrinking diameter*.

We introduce "*evolution-cast*" to define the traffic dissemination among nodes in an evolving social network and conduct capacity analysis based on both uniform and heterogeneous geographical distribution of nodes. By so doing, we provide a theoretical foundation to the design of routing schemes which corporate both the features of evolving nodes and social relations, analytically showing the impact of such characteristics on capacity.

For uniform geographic distribution, a new node enters the network by locating itself in a uniformly chosen position. We propose a routing scheme where an evolution-cast tree is established to connect a source node with all its destinations. The capacity achieved under the routing algorithm indicate that social relations among nodes does not bring about capacity gain under uniform distribution. Furthermore, the capacity performance degrades sharply with time due to network evolution.

For heterogeneous geographic distribution, new coming node establishes the social relations by attaching to some nodes already existing in the network and locates itself in a place near the node that is geographically closest to the prototype. With our proof of power law property exhibited in geographic distribution of nodes, we propose a routing scheme to send a packet from the source progressively to the destination. The capacity achieved is shown to benefit significantly from the highly correlated distribution due to social relations among nodes. Particularly, with appropriate choice of network parameters, we can achieve an almost constantly bounded per-node capacity, which does not degrade sharply with time.

The capacity results obtained in our model are related to time t, as opposed to those obtained from the network where the number of nodes are assume to be fixed. To our best knowledge, this is the first attempt focusing on time-variant capacity results in an evolving network. The purpose of our work is not to establish optimal information theoretic results, but to show that there are additional aspects to be exploited, i.e., the social behavior of nodes and the network evolution, which have been so far not in extensive study aiming at fundamental capacity scaling laws in wireless ad-hoc networks.

The rest of the paper is organized as follows. A brief review of related work is provided in Section 2. We introduce the network model and list the definitions in Section 3. The routing protocols under both homogeneous and heterogeneous topological distribution and the corresponding capacity analysis are presented in Section 4 and Section 5, respectively. Section 6 provides some discussion on our results and their implications. We conclude the paper in Section 7.

II. Related Work

Over the last decade there have been a flurry of theoretical studies focusing on establishing fundamental scaling laws of large scale wireless ad hoc networks. The seminal work of asymptotic capacity study is initiated by Kumar *et. al.* [31], who show that the maximal per-node unicast throughput achievable in wireless networks is $\Theta(1/\sqrt{n \log n})$ for a uniformly distributed destination.

Neely *et al.* [32] later introduce mobility to the nodes and show that there exists a tradeoff between capacity and delay. A series of works [34]- [35] have then followed, focusing on the analysis of throughput-delay tradeoffs under different mobility scenarios, through either the carry-and-forward mode, coding techniques or infrastructure support.

Multicast capacity is also under extensive study in the literature recently. Li *et al.* [33] study the multicast capacity in wireless networks of side length a, with n nodes randomly deployed in it. Through rigorous proof, they claim that, the per-flow multicast capacity (of n multicast flows, each flow with k destinations) is $\Theta(1/\sqrt{nk \log n})$ when $k = O(n/\log n)$ and $\Theta(1/n)$ when $k = \Omega(n/\log n)$. Wang *et al.* [34] further consider multicast capacity in mobile networks, presenting capacity-delay tradeoffs under different routing schemes. Recent application of scaling law analysis also includes energy efficiency [36] and cognitive networks [37].

The numerous papers under Gupta and Kumar's framework are all based on traditional network assumption for investigation of capacity in wireless networks. None of these literatures take into account the impact of social relationship between nodes on network capacity performance.

III. Models and Definitions

In this section, we will present our model with respect to the social-based temporal evolution, network topology, traffic pattern and the interference model in the following subsections. Moreover, we list all the parameters that will be used in later analysis, proofs and discussions in table 1 in the **supplementary file**.

A. Temporal Evolution of Network

A flurry of works have been devoted to modeling the evolution process on both number of nodes and links among them in social networks. Here we adopt the model that captures all the well-known properties of the social networks, as well as the evolution features. The model has been claimed to be realistic and mathematically tractable [25]. Note that the mechanism generates the relationship between nodes in social sense, i.e., two nodes are called *socially related* if there exists a link between them.

Mechanism 1. The evolution process of the network in [25]:

Input: A simple graph $G_0(Q, U)$ at time slot 0 with at least $c_q c_u$ edges, where each node in Q has at least c_q edges and each node in U has at least c_u edges.

Output: A temporal graph $\mathbb{G}(Q_t, U_t)$.

At time slot $t > 0$:

(Evolution of Q)

1: A new node q arrives with probability β and is added to the set Q.

2: A node $q' \in Q$ is chosen as prototype for the new node, with probability proportional to its degree.

3: c_q edges are "copied" from q'. That is, c_q neighbors of q', denoted by $u_1, \ldots, u_{c_q}$, are chosen uniformly at random with out replacement, and the edges $(q, u_1), \ldots, (q, u_{c_q})$ are added to the graph.

4: An edge between q and another node in Q is added for every neighbor they have in common.

(Evolution of U):

5: With probability $1 - \beta$, a new node u is added to U following a symmetrical process in 2, adding c_u edges to u.

6: A new edge is added between two nodes q_1 and q_2 if the new node is a neighbor of both q_1 and q_2.

(Preferentially Chosen Edges):

A set of s nodes $qi_1, \ldots, q_{i_s}$ is chosen, each node independently of the others with replacement, by choosing vertices with probability proportional to their degrees, and the edges $(q, q_{i_1}), \ldots, (q, q_{i_s})$ are added to the graph.

An illustration of the network evolution is provided as Figure 2 in **supplementary file** for better understanding. The intuition behind mechanism 1 is that higher degree nodes are more likely to be chosen as friends of new nodes (step 2), the probability of a node joining a group increases with the number of friends already in the group (step 3), and more social links will be established between nodes in the network due to the continuous arrival of new nodes (step 4 and step 6). This is verified to be rather prevalent in many realistic social network evolution.

B. Geographical Topology

Mechanism 1 merely suffices to reflect the social relations among nodes, whereas the geographical distribution of nodes still needs to be determined. We assume a network as a two dimensional unit square, with time measured at the unit of equal slots[1]. At each time slot, a new node arrives and the network deepens to accommodate it. Later on, we will divide the geographical topology into the cases of *homogeneous* and *heterogeneous* distribution and proceed our respective capacity analysis. Specifically, in homogeneous case the positions of nodes exhibit uniform distribution whereas nodes having strong social relations tend to locate more closer geographically in heterogeneous case. More detailed description is provided in Section V. A as well as Section VI. A.

Remark 1: We assume that arrival of a node is in the same timescale that a packet is transmitted. Hence, it results to a network undergoing rapid evolution. Such a rapid evolution rate can provide us a lower bound of capacity in evolving network. However, we believe that the above issues do not compromise the applicability of our results to a real setting. Notice that assuming one duration of evolution much longer than that of one packet transmission will yield to a network where the number of nodes is almost fixed for a very long period of time. In this situation, we can obtain capacity result by adopting the similar analysis in previous works. We will discuss this case in Section 6.

[1]We assume the length of one slot is sufficient for a packet transmission.

C. Traffic Pattern

We assume the initial number of nodes in the networks as a constant n_0. Consider a specific time slot t. Assume there are $n(t)$ nodes in the network up to time t. When a new node enters the network at time $t+1$, it randomly chooses one or several nodes that already existing prior to $t+1$ as destinations or sources of the new node. As a result, each node at time t may act as a source to multiple nodes while it is also a destination belonging to distinctive sources. Also, $n(t)$ exhibits a function at the same order of t since the number of nodes increases by one per time slot. We define such traffic pattern as *evolution-cast* since the pairs are chosen and determined during evolution process. More specifically, we will further divide the traffic pattern into *evolution unicast* and *evolution multicast* and study capacity performance under these two modes, respectively. At a fixed moment t, the first type represents (private) messaging service between two friends 2 while in the latter pattern information is broadcasted to multiple friends of the source, such as tweets in Twitter and posts in Facebook.

Remark 2: We note that the evolution-cast considered in this paper is not a perfect characterization of general communication manner in all social graphs since it highly depends on the joining pattern of nodes. For example, some of the friendships in LiveJournal appears to be geography independent and may be better explained in other dimensions such as occupations, age, etc. However, a complete reproduction of all features in a realistic evolving social network is too difficult, if at all possible, and we believe it is beneficial to make the proper simplifications towards a tractable model and a meaningful first look into the throughput capacity in evolving social networks.

D. Interference Model

The well-known protocol model is introduced here to roughly represents the behavior of transmission constrained by interference. The model indicates under a fixed total bandwidth W, two node i is allowed to transmit to j at time t if the positions of i and j, denoted by X_i and X_j, satisfy $|X_i - X_j| < r(t)$, where $r(t)$ is a common transmission range employed by all the nodes at time t and for every other node k transmitting, $|X_j - X_k| > (1 + \Delta) r(t)$, being Δ a guard factor.

E. Definitions

In this section, we introduce the definitions of capacity that we will use throughout this paper.

Definition 1: **Feasible Capacity:** We say that a per-node capacity $\lambda(t)$ at time t is said to be feasible if there exists a spatial and temporal scheduling scheme that yields a per-node capacity of $\lambda(t)$. Consider the case where the network enters stable evolution (the network evolves according to a certain rule over time), for an arbitrary duration$[(i-1)T(t), iT(t)]$, if there are Ψ packets transmitted from source to destination, then, we say the average per-node capacity is

$$\overline{\lambda} = \frac{\Psi}{T(t)}, \tag{1}$$

at time t, after t exceeds a specific value t_0. Here t_0 is the threshold of time after which the network is supposed to enter stable evolution.

Definition 2: **Per-node Capacity:** We say that a per-node capacity at time t in the network is of order $\Theta(f(t))$ if there is a deterministic constant $0 < c_1 < c_2 < +\infty$ such that

$$\liminf_{n \to +\infty} \mathbf{Pr}(\lambda(t) = c_1 f(t) \text{is feasible}) = 1,$$

$$\liminf_{n \to +\infty} \mathbf{Pr}(\lambda(t) = c_2 f(t) \text{is feasible}) \leq 1.$$

And the aggregate capacity at time t is $\Lambda(t) = n(t)\lambda(t)$.
Due to space limitation, we summarize the main results in **supplementary file**.

IV. Evolution-cast in Homogeneous Geographical Topology

In this section, we consider capacity in homogeneous geographical distribution. We will first show how new nodes locate themselves in the network according to a uniform geographical distribution. Then we will propose a routing scheme under our evolving network, which is suitable for both evolution unicast and evolution multicast. Capacity analysis is also provided following each traffic mode.

A. Geographical Distribution of the network

In homogeneous network, assume there are n_0 nodes in the network initially. These n_0 nodes locate uniformly throughout the whole network. At each time slot, when a new node arrives, it also randomly and uniformly chooses a position in the network and then locate itself there. The position remains unchanged once determined. The following lemma indicates that when t is sufficiently long, the geographical locations of nodes follow a uniform distribution.

Lemma 1: Consider the geographical distribution of nodes at time slot t, where there are $n(t)$ nodes in the network. Then, the positions of nodes follow a uniform distribution over the whole network when $t \to \infty$.

Proof: Provided in Section 3.1 in **supplementary file**. ■
The following lemma shows that under homogeneous topology, a pair of nodes that are reachable with a constant number of friends in the sense of social relations cannot guarantee a transmission path that is also bounded by constant number of hops.

Lemma 2: In homogeneous geographical distribution, the probability that a social path (denoted by $S = u_1 \to u_2 \to u_3 \to \ldots \to u_H = D$) composed of a sequence of consecutive links generated in Algorithm 1 are also reachable within constant hop of transmission range goes to zero.

Proof: Notice that here the event that the diameter of two arbitrary nodes are bounded by constant and the event that nodes are reachable within constant hop of the length of transmission range are independent. Denote by $\mathbb{Pr}\{\text{nodes are reachable}\}$ such probability. Hence, we have

$$\begin{aligned}
&\mathbb{Pr}\{\text{two nodes are reachable within constant number of} \\
&\quad \text{transmission hops}\} \\
&\leq \mathbb{Pr}\{\text{All those } H \text{ nodes are bounded within } c \text{ hops} \\
&\quad (c \text{ is a constant})\} \\
&= \prod_{t=t_1}^{t_c} \pi\left(r^2(t)\right) = \prod_{t=t_1}^{t_c} \pi\left(\frac{\log n(t)}{n(t)}\right) \\
&< \pi\left(\frac{\log n(t_1)}{n(t_1)}\right)^c \to 0,
\end{aligned}$$

as $t_1 \to \infty$. This completes our proof. ■
According to Lemma 2, the social relation does not provide help on finding a short routing path. The major factor that will impact capacity may be the evolution of network, which may cause new traffic assignment from bandwidth. We present our routing scheme in next subsection, which performs transmission adjustment according to new arrival at each time slot.

B. Routing Protocol for Our Temporal Evolution Model

A consideration before we design our routing scheme is the transmission range r. Previous investigations have shown that transmission ranges should not be too large to increase power consumption while maintaining network connectivity. A range $r = \Theta\left(\sqrt{\frac{\log n}{n}}\right)$ is commonly adopted to make the whole network fully connected, which has already been verified in [31]. We choose $r = \Theta\left(\sqrt{\frac{\log n(t)}{n(t)}}\right)$ here. Notice that r is time-variable due to the change of $n(t)$ with time.

Note that the network in our work is *quasi-static* because the position of a new arriving node remains unchanged once it is determined. To achieve the asymptotically optimal evolution-cast capacity, we construct an evolution-cast tree (ET) to build routing paths connecting a source with all its destinations. We propose an algorithm similar to that provided in [33], which is a good approximation of minimum Euclidean spanning tree. The difference lies in that the tree in [33] does not change once it is established in advance whereas each ET in our network needs to be updated at each time slot to accommodate the new arriving node as a new leaf node on the tree. The transmission range is also adjusted with time due to the new arrival, resulting in new network partitions at each slot. Hence, the algorithm of ET in our work is to build evolution-cast tree using the algorithm in [33] at each time slot. Obviously, the tree will dynamically expand with time, both in length and number of nodes. An illustration of the routing tree is provided as Figure 3 in our **supplementary file**.

C. Evolution Unicast Traffic Pattern

The evolution unicast traffic represents that at each time slot t, a new arriving node is chosen to be either a source or a destination, with equal probability, of a randomly chosen node that already existing in the network before t. This type of traffic represents messaging sharing between limited number of individuals. The following lemma shows the average number of destinations per source after a sufficiently long time.

Lemma 3: In evolution unicast defined above, the average number of destinations per source is of order $\Theta(\log t)$.

Proof: Provided in Section 3.2 in **supplementary file**. ■

Also, in evolution unicast traffic pattern, the ET has the following property.

Lemma 4: For any $\log t$ nodes placed in a unit square at time t, the length of ET, i.e., the length of the path from the source to the deepest destination node in ET, is bounded by $\Theta(\sqrt{\log t})$.

Proof: It has been proved in [33] that for any k nodes placed in a square region with side-length a, the length of Euclidean Multicast Spanning Tree spanning these k nodes is at least $\tau\sqrt{k}a$ and at most $2\sqrt{2}ka$. Now consider the ET in our network at a specific time slot t. Since we have already proved that the average number of destinations per source at time t is $\Theta(\log t)$, and the square region here is of side-length 1, substituting these terms into $2\sqrt{2}ka$, the result in the lemma then follows. ■

We have mentioned that growing nodes results in decreasing transmission range and new network partition over time. This causes each ET to update its edges and leaf nodes at each time slot. Based on the property of the tree, the following lemma shows that the time to complete one spanning of a total ET is negligible compared to the time t where the source starts to transmit.

Lemma 5: Consider at time t, for an arbitrary source s, suppose Δt as the time it takes for a packet generated from s to span the total ET, then,

$$\lim_{t\to\infty}\frac{\Delta t}{t}=0. \tag{2}$$

Proof: Provided in Section 3.3 in **supplementary file**. ■

Denote $\epsilon(t)$ as the extra time which needs to be spent in spanning a slowly growing tree with slowing shrinking transmission range. Obviously,

$$\begin{aligned}\epsilon(t) &\leq \frac{\sqrt{\log(t+\Delta t)}}{r(t+\Delta t)}-\frac{\sqrt{\log(t)}}{r(t)}\\ &=\sqrt{n(t+\Delta t)}-\sqrt{n(t)}\\ &=\frac{n_0+t+\Delta t-(n_0+t)}{\sqrt{n(t+\Delta t)}+\sqrt{n(t)}}\\ &=\frac{\Delta t}{\sqrt{n(t+\Delta t)}+\sqrt{n(t)}}.\end{aligned} \tag{3}$$

Dividing both sides by Δt, we have

$$\frac{\epsilon(t)}{\Delta t}\leq\frac{1}{\sqrt{n(t+\Delta t)}+\sqrt{n(t)}}. \tag{4}$$

If $\Delta t=\Theta(c)$, where c is a constant, then if we fix t, the right side of Equation (4) can be treated as a constant. Then, we get $\epsilon(t)=\Theta(\Delta t)$. If $\Delta t=\omega(c)$, when $\Delta t\to\infty$, the right side of Equation (4) goes to zero. Hence, $\frac{\epsilon(t)}{\Delta t}\to 0$. Therefore, we can draw the conclusion that no matter $\epsilon(t)=o(\Delta t)$ or $\epsilon(t)=\Theta(\Delta t)$, we can approximate the total time for spanning the whole tree with $\Delta t=\sqrt{\log t}/r(t)=\sqrt{\log t}\cdot\frac{\sqrt{n(t)}}{\sqrt{\log t}}=\sqrt{n(t)}=\sqrt{n_0+t}$. Thus, we deduce that $\Delta t\approx\Theta(\sqrt{t})$.

Based on Lemma 3, 4 and 5, the capacity can be derived by the following theorem.

Theorem 1: With homogeneous geographical distribution of nodes, the per-node capacity for evolution unicast traffic is

$$\lambda_t=\Theta\left(\frac{1}{\sqrt{t}\log t}\right), \tag{5}$$

when t is sufficiently large.

Proof: Adopting our temporal evolving spanning tree routing, we know that if the network stops evolution at t, according to [33], the number of nodes that receive a "copy" of an evolution-cast data during the transmissions of all nodes in the tree up to time t is $\frac{\tau\sqrt{\log t}\cdot r(t)}{c_0}$. Here c_0 is a constant. For our evolution-cast, let $U_i(t)$ be the area of the union region of all transmission disks of all transmitting nodes for task initiated by node v_i, at time t. If the network stops growing at time t, then, the aggregate capacity for this task in the network is at most $\frac{W}{U_i(t)}$, where W is the total bandwidth available. However, according to Lemma 5, the total time Δy for spanning the whole tree is about $\Theta(\sqrt{t})$. Thus the data copies during the duration of $\Theta(\sqrt{t})$ can be approximated by $\frac{\tau\sqrt{\log\sqrt{t}}\cdot r(\sqrt{t})}{c_0}=\Theta(\sqrt{\log\sqrt{t}}\cdot r(\sqrt{t}))$. The number of nodes added into the network during this period is $\Theta(\sqrt{t})$. Denoting $C(t)$ as the number of nodes that receive a "copy" of data during the transmission up to time t, we have

$$\frac{W}{C(t+\sqrt{t})\cdot n(t+\sqrt{t})}<\bar{\lambda}_t<\frac{W}{C(t)\cdot n(t)}. \tag{6}$$

Since $\sqrt{t}=o(t)$, we know

$$\frac{W}{C(t+\sqrt{t})\cdot n(t+\sqrt{t})}\to\frac{W}{C(t)\cdot n(t)}. \tag{7}$$

Hence, both sides of Equation (6) are bounded by $\Theta\left(\frac{1}{\sqrt{t}\log t}\right)$. ■

D. Evolution Multicast Traffic Pattern

The evolution multicast represents a traffic mode where at each time t, a new arrival randomly chooses $k(t)$ out of $n(t)$ nodes that already existing before t, acting as a source or destinations with equal probability of these $k(t)$ nodes. This traffic represents message broadcast between the new node and a large number of old nodes. Similar to the case of evolution unicast, we present the following lemma showing the average number of destinations per source when t is sufficiently long.

Lemma 6: In evolution mutlicast traffic, the average number of destinations per source is of order $\Theta(t^\alpha)$, $0<\alpha\leq 1$.

Proof: Provided in Section 3.4 in **supplementary file**. ■

Lemma 7: Consider at time t, for an arbitrary source s, suppose Δt as the time it takes for a packet generated from the s to span the total ET, then, $\lim_{t\to\infty}\frac{\Delta t}{t}=0$.

Proof: In evolution multicast traffic pattern, using the same technique as that in Lemma 4, the total length of a spanning tree is $\Theta(\sqrt{t^\alpha})$ for a source having t^α destinations.

Similarly,

$$c_2 \cdot \sqrt{(t+\Delta t)^{\alpha}} = \sum_{i=t}^{t+\Delta t} r(i) \geq 2\sqrt{\log t} \cdot \frac{\Delta t}{\sqrt{t+\Delta t}+\sqrt{t}}, \quad (8)$$

where c_2 is a constant. Suppose $\Delta t = \omega(t)$, then as $t \to \infty$, the last line of Equation (8) yields $\sqrt{\Delta t} \cdot \sqrt{\log t}$, and the left side of the first line of Equation (8) yields $\sqrt{(\Delta t)^{\alpha}}$. Obviously, $\sqrt{\Delta t} \cdot \sqrt{\log t} > \sqrt{(\Delta t)^{\alpha}}$. However, this result contradicts with Equation (8). Hence, we deduce that $\Delta t = o(t)$. ■

Denote $\epsilon(t)$ as the extra time which needs to be spent in spanning an slowly growing tree with slowing shrinking transmission range. Obviously,

$$\begin{aligned}
\epsilon(t) &\leq \frac{\sqrt{(t+\Delta t)^{\alpha}}}{r(t+\Delta t)} - \frac{\sqrt{t^{\alpha}}}{r(t)} \\
&= \frac{\sqrt{(t+\Delta t)^{1+\alpha}}}{\sqrt{\log(t+\Delta t)}} - \frac{\sqrt{(t)^{1+\alpha}}}{\sqrt{\log(t)}} \\
&= \frac{\frac{(t+\Delta t)^{1+\alpha}}{\log(t+\Delta t)} - \frac{(t)^{1+\alpha}}{\log(t)}}{\frac{\sqrt{(t+\Delta t)^{1+\alpha}}}{\sqrt{\log(t+\Delta t)}} + \frac{\sqrt{(t)^{1+\alpha}}}{\sqrt{\log(t)}}} \\
&< \frac{1}{\log t} \cdot \frac{(t+\Delta t)^{1+\alpha} - t^{1+\alpha}}{\frac{\sqrt{(t+\Delta t)^{1+\alpha}}}{\sqrt{\log(t+\Delta t)}} + \frac{\sqrt{(t)^{1+\alpha}}}{\sqrt{\log(t)}}}.
\end{aligned} \quad (9)$$

Dividing both sides by Δt and suppose $\Delta(t) = \Theta(c)$ where c is a constant, following the similar proof of $\epsilon(t)$ in last subsection, we get $\epsilon(t) = \Theta(\Delta t)$. And if we suppose $\Delta(t) = \omega(c)$, we get

$$\frac{\epsilon(t)}{\Delta t} < \frac{1}{\sqrt{(\Delta t)^{1-\alpha}}} \to 0$$

as $\Delta t \to \infty$. Hence, we conclude that $\epsilon(t) = O(\Delta t)$. And we can approximate Δt with

$$\Delta t = \frac{\sqrt{t^{\alpha}}}{r(t)} = \Theta\left(\sqrt{\frac{t^{1+\alpha}}{\log t}}\right).$$

Lemma 8: When $0 < \alpha < 1$, when t is sufficiently large, $t^{\alpha} < \frac{t}{\log t}$.

Proof: When $0 < \alpha < 1$, if $t^{\alpha} < \frac{t}{\log t}$, then it means $\log t < t^{1-\alpha}$. That is to say, $(1-\alpha)\log t > \log\log t$. Namely, $\alpha < 1 - \frac{\log\log t}{\log t}$. When t is sufficiently large, $1 - \frac{\log\log t}{\log t} \to 1$. This holds as long as t is sufficiently large. Then we can get the conclusion $t^{\alpha} < \frac{t}{\log t}$ when $0 < \alpha < 1$. ■

Combining Lemma 6, 7 and 8, the capacity results can be derived, shown in the following theorem.

Theorem 2: The per-node capacity of evolution multicast traffic under uniform geographical distribution is

$$\lambda_t = \Theta\left(\max\left\{\frac{1}{\sqrt{t^{\alpha+1}\log t}}, \frac{1}{t}\right\}\right), \quad (10)$$

when t is sufficiently large.

Proof: When $\alpha = 1$, the average number of destinations per source is $\Theta(t) = \Theta(n(t))$. This is similar to the broadcast case in fixed number wireless networks. And the per-node capacity is thus $\lambda_t = W/\frac{n(t)}{n(t+\Delta t)} \approx W/n(t) = \Theta(1/t)$.

When $0 < \alpha < 1$, according to Lemma 8, $t^{\alpha} < \frac{t}{\log t}$. Using the same techniques in proof of Theorem 1, we have Using the same techniques in proof of Theorem 1, we have

$$\frac{W}{C(t+\sqrt{t^{\alpha}})} < \bar{\lambda}_t < \frac{W}{C(t)}. \quad (11)$$

We have proved in Lemma 5 that $\Delta t = o(t)$. Then, both sides of Equation (11) are bounded by $\Theta\left(\frac{1}{\sqrt{t}\log t}\right)$. ■

V. Evolution-cast in Heterogeneous Topology

In this section, we will proceed to analyze capacity in heterogeneous topology. We will first show the way in which new nodes choose positions in the network according to the social relations between other nodes and themselves. Then we study evolution unicast capacity under our proposed routing scheme which takes advantage of geographic distribution.

A. Generation of Heterogeneous Topology

The central thesis in developing heterogeneous geographic distribution is that geographical community is usually formed by people sharing similar interests and hobbies. For example, pop music fans usually meet at music bars while golf lovers will go to golf club for more communication. Mapping it into an abstract graph, we propose the following mechanism to generate a heterogeneous topological distribution based on the social relation evolution described in mechanism 1.

Mechanism 2. The generation process of network topology:

Input: An initial graph $G_0(Q,U)$ with at least $c_q c_u$ edges, where each node in Q has at least c_q edges and each node in U has at least c_u edges. And for an arbitrary node i in $Q(U)$, the minimum distance between i and one of the $c_q(c_u)$ nodes is of $\Theta(cr(t))$.

Output: A temporal graph $\mathbb{G}(Q_t, U_t)$ having $n(t)$ nodes at time t, which follows a topological distribution $\mathfrak{T}(n(t))$.

At time slot $t > 0$:

Set the common transmission range of all nodes in the network as $r(t)$. The position of a new arriving node can be determined *either through*

1. the nodes in Q with probability β: Randomly choose a position that is away from at least those c_q nodes q is linked to in Algorithm 1 of distance $\Theta(cr(t))$, then locate q here;

or through

2. the nodes in U with probability $1-\beta$**:** Follow the symmetric process of Q;

or: Locate the new node in a position with distance $\Theta(cr(t))$ from one of the randomly chosen q_{i_s} nodes in Algorithm 1.

We still employ a transmission range $r(t) = \Theta\left(\sqrt{\frac{\log n(t)}{n(t)}}\right)$ at each time slot t. The following two lemmas show that the geographic distance between nodes generated from Mechanism 2 follows a power law distribution in some condition constraints.

Lemma 9: If the topological generation of the network evolves according to Mechanism 2, then, when t is sufficiently large, the distribution of geographic distance between nodes will yield as follows:

The spatial stationary distribution of a node is assumed to be rotationally invariant with respect to another node called *support*, which can be described by a function $\phi(l)$ decaying as a power law of exponent σ, i.e., $\phi(l) \sim l^{-\sigma}$, $\sigma = 2 + \frac{c_u(1-\beta)}{c_q\beta}$. And here l ranges from $\Theta(1/\sqrt{t})$ to $\Theta(1)$, representing the distance between the node and the support.

Proof: Provided in Section 3.5 in **supplementary file**. ■

The geographic distribution also exhibits the following feature, which will be used in later proof.

Lemma 10: If there is a node u in $\mathbb{G}(Q, U)$ of distance $g(t)$ from another node v at time t, then, v had, with high probability, another node w of distance $\Theta(g(t))$ also at time ϵt from v, for any constant $\epsilon > 0$.

B. Routing Protocol for Temporal Evolution Model

1) Temporal Dynamic Routing Path: We propose a routing scheme, aiming to make messages advance along a chain of relay nodes whose home points become progressively closer to that of the destination. The routing proceeds until the *support* of the target node falls within distance $d_0(t)$ from the support of the last relay node at t. The detailed routing scheme is presented as follows:

The temporal evolution routing scheme:

node a handles a packet designated to destination d.

At each time slot $t > 0$:

STEP 1: Node a first computes the distance between the support nodes of a and d: $\|S_a - S_d\|$.

STEP 2: If $\|S_a - S_d\| \leq d_0$, then a directly forwards the packet to d.

STEP 3: Otherwise, a compares $\|S_a - S_d\|$ with the set of thresholds $\{d_i = 2^i d_0, i \in \mathbb{N}\}$, finding $i_* \geq 1$ such that $2^{i_*-1}d_0 < \|S_a - S_d\| < 2^{i_*}d_0$.

STEP 4: a forwards the packet to any node whose support satisfies condition in STEP 3. Otherwise, a stays idle.

Figure 4 is a simple illustration showing how to send a packet progressively to destination. Notice that the steps described in the scheme are counted backward, starting from the destination. And the packet p has to go through i_* relay nodes before finally being delivered to the destination. Due to the shrinking diameter phenomenon, when t is sufficiently long, for an arbitrary pair in all but $o(t)$ nodes, they have the location information about each other. And we adopt a scheduling scheme similar to that in [35], in charge of selecting at each time slot t the set of non-interfering transmitter-receiver pairs to be enabled in the network, as well as the message to be transmitted over each enabled pair. Here we choose $d_0 = \Omega\left(\sqrt{\frac{\log t}{t}}\right)$. Thus, each node has $\Theta(d_0^2) = \Theta(\log t)$ destinations, which is consistent with evolution unicast traffic pattern.

2) Capacity Analysis under the Proposed Schemes: Based on the routing scheme we proposed in Section V.B, we study the network capacity for evolution-cast here. Before that, we first show some following lemmas.

Note that in step 3 of our routing scheme, once i has been selected, the total network is divided into squarelets of area denoted by A_i, forming a regular square tessellation. According to the protocol model, at most one transmission can be enabled in each squarelet. The choice of A_i is critical since too large A_i will result in large waiting delay while too small A_i will lead to inefficient capacity. The following lemma gives useful directions of dimensioning A_i.

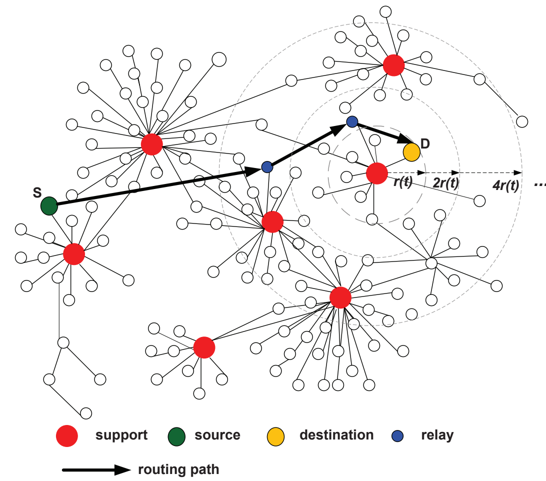

Fig. 1: The illustration of how to route a packet from the source to destination utilizing powerlaw location distribution and transmission range adjustment.

Lemma 11: At routing step i, the minimum value of A_i which guarantees that $\liminf_{t\to\infty} \mathbb{P}\{\text{populated squarelet}|i\}$ is given by:

$$A_i = \Theta\left(\min\left\{\frac{\log t}{t}, \max\left(d_i^{\sigma-2}, \sqrt{\frac{\log t}{t}}\right)\right\}\right). \tag{12}$$

Proof: The sketch of the proof is presented as follows. First, using the similar techniques in the proof of Lemma 2 in [35] and change the network size from $\Theta(\sqrt{n})$ to $\Theta(1)$, we can get that in a network of n nodes, $A_i = \log n/n$ when $\delta = 2$ and $A_i = \Theta\left(\max\left(d_i^{\sigma-2}, \sqrt{\frac{\log n}{n}}\right)\right)$ when $\sigma > 2$. Then we turn to our evolving network where the number of nodes increases with time as $n(t) = \Theta(t)$. According to Lemma 10, during time interval $[t, \epsilon t]$ for any $\epsilon > 0$, the topology distribution of nodes do not change in order sense. Hence, we can simply treat the number of nodes as $\Theta(t)$ during this period. Substituting it into the results obtained in n, we then get the results presented in the lemma. ■

Another significant consideration is how to set d_0, the maximum required distance within which a node carrying packet can directly transmit to the destination. The lemma below shows the appropriate choice of d_0.

Lemma 12: The parameter d_0 in our temporal routing scheme must satisfy $d_0 = \Omega\left(\sqrt{\frac{\log t}{t}}\right)$.

Proof: Consider the network at time t. We observe that $A_i(t)$ depends both on $d_i = 2^i d_0(t)$ of the associated routing step and the power law exponent σ characterizing nodes' location distribution. Since the spatial distribution of node follows power law with exponent $\sigma \geq 2$, the first step is the one that requires the largest value of $A_i(t)$. Thus, the constraint is $d_0(t) = \Omega\left(\sqrt{\frac{\log t}{t}}\right)$. ■

The following lemma shows the average number of hops a packet has to go through before it eventually arrives at destination.

Lemma 13: Under the proposed routing scheme, for a destination arriving at the network at time t, the average hop counts $H(t)$ of sending a packet from source to the destination is

$$H(t) = \Theta\left(\max\left\{\log^4 t, t^{\frac{\sigma}{2}-1}\log t\right\}\right). \tag{13}$$

Proof: We also present a sketch of proof here. Adopting the similar technique in Section VI.C in [35] and scale down the network size by a factor of $\Theta(\sqrt{n})$, we can get $H(t) = \Theta\left(\log^4 t\right)$ when $\sigma = 2$ and $\Theta\left(t^{\sigma/2-1}\log t\right)$ when $\sigma > 2$, for a network composed by n nodes. Now we turn to our evolution network. Using the similar techniques in Lemma 11, the result then follows. ■

After we obtain the conclusions from the above lemmas, we now have a basic way to compute capacity. That is to compute the average number of cells that are transmitting simultaneously, denoted by $N_s(t)$. Also, in our scheme, for a specific time slot, we need to compute the probability that it is assigned to send the node to the next node at a distance between $[2^{i_*-1}d_0, 2^{i_*}d_0]$, denoted it by $p_i(t)$.

At a specific time t, the maximum number of steps equals to $i_{\max}(t) = \lfloor\frac{1}{2}\log_2 n(t) - \log_2 d_0\rfloor$. Given $i_{\max}$(t), we have

$$p_i(t) = \frac{1/N_s(t)}{\Sigma_{i=0}^{i_{\max}} 1/N_s(t)} = \frac{A_i(t)}{\Sigma_{i=0}^{i_{\max}} 1/A_i(t)}.$$

Now we consider a particular step i, the average number of packets stored at i_{th} relay nodes to be delivered to the next relay at one time slot is $p_i(t)N_s(t)$. Note that the capacity is determined by the step having the minimum $p_i(t)N_{is}(t)$. To avoid any step i to become the bottleneck of the total capacity, $p_i(t)$ and $N_{is}(t)$ should be distributed equally at each step. Under equal distribution, the aggregate capacity can be bounded by

$$\frac{1}{\Sigma_{i(t)=0}^{i_{\max}(t)} A_i(t)} < \Lambda(t) = p_e(t)N_e(t) < \frac{1}{\Sigma_{i(t)=0}^{i_{\max}(t+H(t))} A_i(t)}. \tag{14}$$

According to Lemma 13, when $\sigma = 2$, the number of new nodes added into network during the period $[t, t+H(t)]$ is $o(t)$. For $\sigma > 2$, that number is also $O(t)$ if $2 < \sigma \leq 4$. This guarantees that the capacity loss caused by new bandwidth consumption of these extra nodes is negligible. When $\sigma > 4$, the hop counts is $H(t) = \omega(t)$, during which the number of new arriving nodes increases as $\omega(t)$. In this case, the extra capacity loss can no longer be negligible. Now we can present the results on capacity in heterogeneous topology as follows:

Theorem 3: For heterogeneous topology distribution, under our proposed routing scheme, the achievable per-node capacity of evolution-cast, under uniform traffic pattern, is

$$\lambda_t = \Theta\left(\min\left\{\frac{1}{\log^2 t}, t^{-\frac{1}{2}\frac{c_u(1-\beta)}{c_q\beta}}\right\}\right). \tag{15}$$

Proof: When $\sigma = 2$, i.e., $\frac{c_u(1-\beta)}{c_q\beta} = 0$, $A_i = \log t/t$. Thus the per-node capacity is

$$\frac{1/n(t+H(t))}{\Sigma_{i(t)=0}^{i_{\max}(t+H(t))} A_i(t)} < \lambda_t < \frac{1/n(t)}{\Sigma_{i(t)=0}^{i_{\max}(t)} A_i(t)}. \tag{16}$$

When $\sigma > 2$, first, if $\sigma \leq 4$, then, $\sigma/2 - 1 < 1$. Hence, $H(t) = O(t)$. In this case, it is equivalent to $2 > \frac{c_u(1-\beta)}{c_q\beta} > 0$. Since $A_i = \Theta\left(\max\left(d_i^{\sigma-2}, \sqrt{\frac{\log t}{t}}\right)\right)$, we have

$$\frac{1/n(t)}{\Sigma_{i(t)=0}^{i_{\max}(t)}(2^i d_0)^{\sigma-2}} < \lambda_t < \frac{1/n(t+H(t))}{\Sigma_{i(t)=0}^{i_{\max}(t+H(t))}(2^i d_0)^{\sigma-2}}. \tag{17}$$

The left side of Equation (17) can be approximated with $\Theta\left(t^{1-\sigma/2}\right)$ while the right side can be approximated with $\Theta\left((t+\Theta(H(t)))^{1-\sigma/2}\right)$. If $\sigma > 4$, i.e., $\frac{c_u(1-\beta)}{c_q\beta} > 4$, then, according to Equation (13), $H(t) = \omega(t)$. Thus, the term $\frac{n(t)}{n(t+H(t))}$ cannot be approximated as 1 anymore. Rewriting the term, we have $\frac{n(t)}{n(t+H(t))} = \frac{n(t)}{n(t)+\Delta n(H(t))} = \frac{\Theta(t)}{\Theta(t)+\omega(t)} = \frac{1}{1+\omega(t)/t} = o(1)$. Using the similar technique, we get $\lambda_t = o\left(t^{1-\sigma/2}\right)$. Rewriting σ into $2 + \frac{c_u(1-\beta)}{c_q\beta}$, the result in the lemma then follows. ■

VI. Discussion of Results and Implications

A. Impact of Evolution-cast on Capacity

1) The Impact in Homogeneous Topology: From our results, it can been seen that the social relations alone cannot provide much help for capacity improvement without topological cooperation. The results obtained under uniform topology imply that the transmission between a socially tightly related source-destination pair must be executed through opportunistic routing, if they are geographically far apart. This is because in wireless environments, transmission can only occur between two nodes that are within a certain transmission range. In uniform topology distribution, the average distance between an arbitrary source-destination pair is $\Theta(1)$. Hence, the source has to send a packet through opportunistic selection of relay nodes along the routing path. Moreover, due to the new arriving node at each time slot, new traffic assignments should be determined for it. The total bandwidth distribution also needs to be updated according the current number of nodes located in the network.

2) The Impact in Heterogeneous Topology: Since we only study evolution unicast in heterogeneous topology, we will make a comparison of evolution unicast capacity between homogeneous and heterogeneous topology.

The per-node capacity of evolution unicast is $\frac{1}{\sqrt{t}\log t}$ with uniform topology whereas it is $\Theta\left(t^{-\frac{1}{2}\frac{c_u(1-\beta)}{c_q\beta}}\right)$ with heterogeneous topology. If $\frac{c_u(1-\beta)}{c_q\beta} < 1$, the per-node capacity will be larger than that in uniform topology. Moreover, if setting $-\frac{1}{2}\frac{c_u(1-\beta)}{c_q\beta} = 0$, we can achieve a constant capacity (expect for a polylogarithmic factor) in heterogeneous geographic distribution. With further observation, we can find setting $-\frac{1}{2}\frac{c_u(1-\beta)}{c_q\beta} = 0$ means $\beta = 1$ (It is impossible to set c_u as zero since this also causes c_q to be zero, which leads to an invalid fraction, i.e., $\frac{0}{0}$.). Looking at the bipartite evolution graph $B(Q, U)$ when $\beta = 1$, a new node at each time slot only enters the set Q, and establishes some links with the nodes in U. As time goes by, the number of nodes in Q increases while that in U remains the same but with increasing number of links for each of them. This causes the nodes in Q to be highly-centralized. From social perspective, these nodes keep collecting the information of all the other nodes during evolution. In geographical perspective, these nodes act as infrastructure support, helping location information sharing among nodes. Note that the location information will be helpful in routing paths establishment, since good relay candidates can be picked out based on their locations such that the hop counts between source-destination pair will be largely reduced.

B. Relationship with Capacity in Fixed Random Network

In this subsection, we make a comparison of capacity scaling between our evolving network and the wireless random network where the number of nodes is fixed. We will associate our results with those obtained in fixed networks by letting our network stop evolution at a sufficiently large t_s. We will discuss in the following two cases.

1) Fixing t in Uniform Topology: In uniform topological distribution, if setting $t_s = n$ (This means that at time t_s the evolving network contains $\Theta(n)$ nodes already.). Now we consider evolution unicast traffic pattern first. Substituting $t_s = n$ into the capacity obtained in Theorem 1, we get a new capacity result for a network with fixed number $\Theta(n)$ of nodes, $\lambda = \Theta\left(\frac{1}{\sqrt{n}\log n}\right)$. There is a capacity loss of $\sqrt{\log n}$ compared to the results $\lambda = \Theta\left(\frac{1}{\sqrt{n\log n}}\right)$ obtained under unicast, done by Kumar *et. al.* [31]. This is easy to understand since there is only one destination per source in [31] while that number is $\Theta(\log n)$ in our traffic pattern. As a result, extra redundancies is introduced to disseminate the information to those $\log n$ destinations. However, despite of the small gap, our evolution-cast result presents a good approximation for unicast traffic in fixed networks.

Next we consider evolution multicast traffic pattern. Also setting $t_s = n$, we obtain another new capacity result for a fixed network with $\Theta(n)$ nodes. That is $\lambda = \Theta\left(\frac{1}{\sqrt{n^{\alpha+1}\log n}}\right)$ for $0 < \alpha < 1$ and $\lambda = \Theta\left(\frac{1}{n}\right)$ for $\alpha = 1$. Note that α represents an exponent where $n^{\alpha}(t)$ nodes are picked out at each time t, assigned to be either sources or destinations with equal probability. Replacing n^{α} with k, the results yields

$$\lambda_t = \Theta\left(\max\left\{\frac{1}{\sqrt{nk}\log n}, \frac{1}{n}\right\}\right). \tag{18}$$

With comparison of the result obtained by Li. *et. al.* [33], we find there is a capacity gain of $\sqrt{\frac{n}{k\log n}}$ in the case where $n^{\alpha} < k < \frac{n}{\log n}$. The gain becomes larger as t_s increases. The improvement is due to the social evolution process during the t_s period.

2) Fixing t in Non-uniform Topology: In non-uniform topology, we only study evolution unicast traffic. Also setting $t_s = n$, we can get the capacity shown as follows:

$$\lambda_t = \Theta\left(\min\left\{\frac{1}{\log^2 n}, n^{-\frac{1}{2}\frac{c_u(1-\beta)}{c_q\beta}}\right\}\right). \tag{19}$$

It can be seen that the per-node capacity $\Theta\left(\frac{1}{\log^2 n}\right)$ is achievable when $\frac{c_u(1-\beta)}{c_q\beta} = 0$. This is close to $\Theta(1)$ per-node capacity achieved in [38] except for a poly-logarithmic factor. However, note that we do not seek for the sophisticated hierarchical cooperation with MIMO communications proposed in [38] to improve capacity. The capacity improvement in our results comes from the impact of social evolution on network topology. Moreover, when $\frac{c_u(1-\beta)}{c_q\beta} = 0$, the shrinking diameter in social evolution makes the corresponding geographically mapped nodes which know the locations of each other separated by distance consistent with transmission range adopted in Algorithm 3. Hence, the routing path between an arbitrary source-destination pair can be chosen in advance. Unlike opportunistic routing, the social-aid path helps to save the extra transmission resource consumed on looking for the candidate next relay. And it can also be seen that when $\frac{c_u(1-\beta)}{c_q\beta} > 4$, the capacity is severely degraded. This is caused by the sparseness between different groups of nodes in the network. In this scenario, the cross-group transmission becomes the bottleneck of capacity.

3) Flexibility in Adjustment between the Rate of Evolution and Packet Transmission: Now we try to refine our model a bit more realistic by varying the rate between network evolution and packet transmission. Let the network evolve at a lower rate than that of packet transmission, i.e., the time interval of node arrival (denoted by t_e) is no shorter than that of packet transmission (denoted by t_p). We denote $f(t_e, t_p)$ as a function characterizing the number of nodes resulted from the difference between t_e and t_p. This will subtly change all the capacity results derived in previous sections, shown as follows:

Given t_e and t_p, the per-node capacity of evolution-cast is

$$\lambda = \Theta\left(\frac{1}{\sqrt{f(t_e, t_p)}\log f(t_e, t_p)}\right) \tag{20}$$

for evolution unicast with homogeneous geographical distribution and

$$\lambda = \begin{cases} \Theta\left(\frac{1}{\sqrt{f^{\alpha+1}(t_e,t_p)\log f(t_e,t_p)}}\right) & k = f(t_e, t_p)^{\alpha} \\ \Theta\left(\frac{1}{f(t_e,t_p)}\right)) & k = f(t_e, t_p) \end{cases} \tag{21}$$

for evolution multicast with heterogeneous geographic distribution.

Also with t_e and t_p, the per-node capacity of evolution-cast gives rise to

$$\lambda = \begin{cases} \Theta\left(\frac{1}{\log^2 f(t_e,t_p)}\right) & \frac{c_u(1-\beta)}{c_q\beta} = 0 \\ \Theta\left(f(t_e, t_p)^{1-\frac{1}{2}\frac{c_u(1-\beta)}{c_q\beta}}\right) & \frac{c_u(1-\beta)}{c_q\beta} < 4 \\ o\left(f(t_e, t_p)^{1-\frac{1}{2}\frac{c_u(1-\beta)}{c_q\beta}}\right) & \frac{c_u(1-\beta)}{c_q\beta} \geq 4 \end{cases} \tag{22}$$

for evolution unicast with heterogeneous geographic distribution.

If $t_e \gg t_p$, $f(t_e, t_p)$ is simplified as a value only dependent of t_e. As long as t is sufficiently long, $f(t_e, t_p)$ will also be sufficiently large and we can simply treat it as a parameter $n(t_e)$, which is scale-free from t_p; The corresponding capacity in Equation (20)-(22) can be further derived by substituting $f(t_e, t_p)$ with $n(t_e)$ into all the three equations. If t_e scales at the same order as t_p, $f(t_e, t_p)$ increases sharply over time, at a constant increase rate. The capacity scalings are in order sense equal to those presented in previous sections. Finally, if t_e and t_p obey a certain relationship such that the number of nodes slowly increase with time, where time slots are measured at the unit of t_p, then the corresponding capacity may yield a value between the results obtained in the two cases above.

C. Applying to the Birth & Death Model

In reality, both arrival of new nodes and death of old nodes occur in the network. A typical example is the Logistic population increase model, which can be expressed as follows:

$$n(t) = \frac{N*}{1 + \left(\frac{N*}{N_0} - 1\right) \cdot e^{-r* \cdot (t - t_0)}}.$$

Here N_0 represents the initial number of nodes in the network and $N*$ is the maximum limitation the the number of nodes can reach on the earth. $r*$ is the exponent of exponential part, indicating the increase ratio. Obviously, when t is sufficiently long, $n(t)$ goes to $N*$, regardless of t.

VII. Conclusion

We present a mathematically tractable model where nodes are associated with each other through social relations but employ transmission through wireless communications. The model captures all the features of social network as well as desification and shrinking diameter. Assuming that at each time slot, a new node enters the network with edge copying from an old node already existed in the network, we discuss capacity with regard to both topological distribution and traffic patterns. We discuss in terms of homogeneous and heterogeneous topological distribution and uniform and non-uniform traffic patterns. This is, to our best knowledge, the first work studying capacity in an evolving and socially-behaved network.

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