Asymptotic Analysis on Throughput and Delay in Cognitive Social Networks

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***Abstract*—In this paper, we study the throughput and delay in wireless cognitive social networks. Specifically, we consider a common scenario for cognitive radio networks (CRN) where the primary and secondary networks operate at the same time, space and share the spectrum. On this basis, we integrate social relationship into CRN where each source node selects its destination upon an rank based model which captures the social characteristic well. By applying a cellular time-division multiple access (TDMA) scheduling scheme, we first characterize the distinct traffic pattern caused by the social relationships between nodes. Then we derive the achievable throughput and delay for both primary and secondary networks under the new network setting. In addition, we also study the cognitive social networks with infrastructure where $l = o^1(n)$ base stations are regularly deployed within the primary network. Given a probabilistic routing strategy, throughput of the proposed network is recalculated. Particularly, due to the social relationships between nodes, we reveal that a larger l is required if we expect a significant capacity gain within the primary network compared with previous works.**

***Index Terms*—Throughput, delay, cognitive radio networks (CRN), social characteristics, hybrid networks, probabilistic routing strategy**

I. Introduction

Due to the growing popularity of online social platforms such as MySpace, Facebook and Cyworld, wireless social networks has been increasingly catching the eyes of both the academic and industrial communities intrigued by its affordance and reach. In wireless social networks, source nodes choose to select their destinations according to certain social relationship rather than a common uniform distribution. Tichy *et al.* put forward the original network framework of social relationship in [1], followed by [2] the online social relationship was investigated in comparison with the real-life social networks. Subsequently, the distance model and rank based model which well capture the characteristics of social relationships were established in [3] and [4], respectively. On basis of these works, social characteristics were introduced into wireless networks and their impact on the performance of wireless networks was extensively studied. In [5], [6], Azimdoost *et al.* studied the capacity of wireless social networks where both local and long-range contacts are involved. Fu *et al.* incorporated the rank based model into wireless networks and studied the impact of traffic locality on capacity in [7].

Initiated by Gupta and Kumar's work [8], the study on capacity of wireless networks [9], [10], [11], [12] were constantly being bubbled up. Meanwhile, cognitive radio networks (CRN), among the various kinds of wireless networks, has gained much more attention from the academic circles for its efficient usage of spectrum. Consequently, many researchers began to explore the fundamental performance of CRN. Vu *et al.* considered the throughput of CRN early in [13]. In [14], Jeon *et al.* studied the throughput of CRN under general circumstance. Results showed that the same throughput scaling can be achieved for both primary and secondary networks in order sense and they perform as two stand-alone networks, regardless of a finite outage problem within the secondary network. Yin *et al.* developed transmission protocols to guarantee zero outage probability for secondary networks in [15].

To the best of our knowledge, the research on the combination of the two networks is very limited. It is worth exploring whether the wireless network can still benefit from the CR technique when the social characteristic is taken into account. And we are also wondering if the social characteristic can bring some new discovery, which can be utilized for the better design of wireless networks. To address this problem, we integrate social relationship into CRN and investigate its impact on network performance such as throughput, delay and the corresponding tradeoff [16], [17]. Moreover, we incorporate hybrid network model into our consideration. The throughput and delay of hybrid networks were studied in [18], [19], [20], [21]. It was proved that the base stations (BSs) could enhance the transmissions. Thus, it is also very meaningful to explore how transmissions could benefit from the infrastructures under the new network settings.

In this paper, by employing the rank based model, we give a general analysis of the throughput and delay of cognitive social networks which was rarely referred in previous works. Specifically, we consider two network types: i) ad hoc cognitive social networks, ii) infrastructure-supported cognitive social networks. Noting that one challenge here is that the traffic load as well as the interference between primary and secondary transmissions should be reconsidered under the new network

[1]The following asymptotic notations are used throughout this paper. Given non-negative functions $f(n)$ and $g(n)$:

1) $f(n) = \omega(g(n))$ means that $\lim_{n\to\infty} g(n)/f(n) = 0$.
2) $f(n) = o(g(n))$ means that $g(n) = \omega(f(n))$.
3) $f(n) = O(g(n))$ means that there exists a constant c_1 and integer N such that $f(n) \leq c_1 g(n)$ for $n > N$.
4) $f(n) = \Theta(g(n))$ means that for two constants $0 < c_2 < c_3$, $c_1 g(n) \leq f(n) \leq c_2 g(n)$ for sufficiently large n.
5) $f(n) \sim g(n)$ means that $\lim_{n\to\infty} f(n)/g(n) = 1$.

model. We summarized the main contributions as follows:

• Ad hoc cognitive social networks: We show that the social relationships between nodes do cause the different traffic patterns, thus resulting in the distributed property of throughput and delay within the proposed network. For primary networks, as the range of α (power law exponent which is defined in Section II) changes, the throughput ranges from $\Theta(1/=\sqrt{n\log n})$ to $\Theta(1/\log n)$, n is the density of primary nodes; the corresponding delay ranges from $\Theta(\sqrt{n}/\sqrt{\log n})$ to $\Theta(\log n)$. Based on the reasonable assumption that the secondary users know the locations of the primary transmitters (TXs), we show that both the primary and secondary networks can achieve the same scaling as being a stand-alone network. For secondary networks, the throughput and delay are also altering with the range of α, which are $\Theta(1/\sqrt{m\log m})$ to $\Theta(1/\log m)$ and $\Theta(\sqrt{m}/\sqrt{\log m})$ to $\Theta(\log m)$, respectively, m denotes the density of secondary nodes.

• Infrastructure-supported cognitive social networks: In this setting, $l = o(n)$ base stations are regularly deployed within primary networks. Given the proposed probabilistic routing strategy, we recalculate the throughput of primary networks. Particularly, we reveal that due to the social relationships between nodes, a larger l is required if we expect a significant capacity gain in primary networks compared with the results in [18], [19], [20], [21]. In addition, when l is not large enough to increase the capacity, we provide a compromised solution to guarantee non-negligible capacity gain of a small fraction of primary nodes, which makes our network more applicable in practice. For secondary networks, we prove that it can still achieve the same capacity scaling as in ad hoc cognitive social networks.

The rest of this paper is organized as follows. In Section II, we outline the network model and definitions. We analyze the social impact on S-D distribution in Section III. The throughput and delay of ad hoc cognitive social networks is derived in Section IV. In Section V, we study the throughput of infrastructure-supported cognitive social networks. Our paper is concluded in Section VI.

II. Network Model and Definitions

In this paper, we consider the network extension to be an unit square where primary nodes and secondary nodes coexist.

Ad Hoc Cognitive Social Networks: The primary nodes are distributed according to a Poisson Point Process (P.P.P.) of density n and together form a primary network. We group the primary nodes into one-to-one source and destination (S-D) pairs randomly. The secondary nodes are also distributed according to a P.P.P. of density m and are randomly grouped into one-to-one S-D pairs. n and m are related according to $m = n^{\beta}$, where $\beta > 1$.

Infrastructure-Supported Cognitive Social Networks: The distribution of both primary and secondary nodes are defined the same way as in ad hoc cognitive social networks. In particular, l additional BSs are regularly deployed within the primary network. n and l are related according to $l = n^{\gamma}$, where $0 < \gamma < 1$.

Physical Model: We only consider the path loss issue that the channel power gain $g(d)$ is given by $g(d) = d^{-\delta}$, where d denotes the distance between a transmitter (Tx) and its receiver (Rx), $\delta > 2$ is the path loss exponent.

TABLE I: Notations

P_p^i	Transmit power of the i-th primary pair
P_s^j	Transmit power of the j-th secondary pair
N_0	Thermal noise power
$X_{p,tx}^i$	Tx location of the i-th primary pair
$X_{p,rx}^i$	Rx location of the i-th primary pair
$X_{s,tx}^j$	Tx location of the j-th secondary pair
$X_{s,rx}^j$	Rx location of the j-th secondary pair
I_p^i	Interference power from the primary Txs to the Rx of the i-th primary pair
I_{sp}^i	Interference power from the secondary Txs to the Rx of the i-th primary pair
I_s^j	Interference power from the secondary Txs to the Rx of the j-th secondary pair
I_{ps}^j	Interference power from the primary Txs to the Rx of the j-th secondary pair

The transmission rate is given by the well known formula $R = \log(1+\text{SINR}) \quad bps/Hz$, which can be achieved by deploying proper network schemes. We now characterize the transmitting rate achieved by the primary (or secondary) transmitter and receiver (Tx-Rx) pairs. Suppose that N_p primary Tx-Rx pairs and N_s secondary Tx-Rx pairs transmit simultaneously, the i-th primary Tx-Rx pair can achieve a rate of

$$R_p^i = \log\left(1 + \frac{P_p^i\, g\left(\left\|X_{p,tx}^i - X_{p,rx}^i\right\|\right)}{N_0 + I_p^i + I_{sp}^i}\right), \tag{1}$$

Similarly, the transmission rate of the j-th secondary Tx-Rx pair denotes as

$$R_s^j = \log\left(1 + \frac{P_s^j\, g\left(\left\|X_{s,tx}^j - X_{s,rx}^j\right\|\right)}{N_0 + I_s^j + I_{ps}^j}\right). \tag{2}$$

Rank Based Model: In this paper, we adopt rank based model, which can well capture the social behavior of human beings and is much more general compared to some other related models [3], to generate social relationship between nodes in the network. According to the rank based model, the probability of befriending a particular node is inversely proportional to the α-th power of the number of closer nodes.

Specifically, we consider a single network with n nodes randomly distributed in the given unit square. Randomly select two nodes i and j, define the rank of j with respect to i as

$$Rank_i(j) = \left|\{k : \|X_i - X_k\| < \|X_i - X_j\|\}\right|, \tag{3}$$

where X_i, X_j denotes the location of node i and j in a single network. Then we model the probability that j is a friend of i as $\mathbb{P}(i \to j) \propto \frac{1}{Rank_i^{\alpha}(j)}$, where α is the power law exponent and $\alpha \geq 0$. Denoting for short $H_n = \sum_{j=1}^{n} 1/j^{\alpha}$, the distribution law is

$$\mathbb{P}(i \to j) = \frac{1}{H_n Rank_i^{\alpha}(j)}. \tag{4}$$

TABLE II: The distribution of length of S-D pairs

Range of α	$0 \le \alpha < 1$	$\alpha = 1$	$1 < \alpha < 3/2$	$\alpha = 3/2$	$\alpha > 3/2$
$\mathbb{E}(\|X_i - Y_i\|)$	1	$1/\log n$	$n^{1-\alpha}$	$\log n/\sqrt{n}$	$1/\sqrt{n}$

Capacity: The per-node throughput capacity is defined as the average data rate that each source node can transmit to destination $w.h.p.$ under a particular scheduling and routing scheme.

Delay: In this paper, we use a fluid model to study the delay performance for the primary and secondary networks. Specifically, we divide each time slot into several packet slots, and the packet will scale to arbitrarily small size with respect to the node density in the networks.

III. Social Impact on S-D distribution

In the following, we show how the social relationship between nodes affect the S-D distribution, which distinguishes the traffic pattern from traditional CRN.

***Lemma* 1.** ([7, Lemma 3]) *Supposing there are n+1 nodes in a unit square, the average length of arbitrary S-D pair $\mathbb{E}(\|X_i - Y_i\|)$ is distributed as in Table II.*

It can be seen that the average length of S-D pairs $\mathbb{E}(\|X_i - Y_i\|)$ varies with α and such results will inevitably cause the complexity in our network model. To better understand the impact of such variation on the transmission over the whole network, we use Lemma 2 below to bound the tail probability of random variable $\|X_i - Y_i\|$.

***Lemma* 2.** *The probability that the length of a generic S-D pair i in the network $\|X_i - Y_i\|$ exceeds a range of $\Theta(\mathbb{E}(\|X_i - Y_i\|))$ is zero $w.h.p.$, which means the following equation holds,*

$$\lim_{n\to\infty} \mathbb{P}(\|X_i - Y_i\| \ge \omega(\mathbb{E}(\|X_i - Y_i\|))) \to 0. \tag{5}$$

Proof: The derivation is complicated in some wise and is available in the Appendix. ■

IV. Throughput and Delay of Ad Hoc Cognitive Social Networks

In this section, we mainly focus on the throughput and delay of cognitive social networks operating in pure ad hoc mode.

A. Network Protocols

- We divide the unit square into small-square primary and secondary cells, respectively. Each primary cell has an area of $a_p(n) = \frac{2\log n}{n}$ and for each secondary cell it is $a_s(m) = \frac{2\log m}{m}$. Such choice of cell area ensures that each primary (secondary) cell contains at least one primary (secondary) node, in which the whole network is connected for the data can be transmitted from cell to cell continuously. Also, such choice of cell area can reduce the concurrent interference for both primary and secondary networks, which was proved in [15].
- We employ a widely used time-division multiple access (TDMA) transmission scheme, which was proven to be useful and convenient for the scaling performance analysis in wireless networks. Grouping both primary and secondary cells into clusters that each primary or secondary cluster has 25 cells, and time is partitioned into 25 time slots in each cluster. We note that the duration of each secondary TDMA frame equals to that of one primary time slot to ensure zero outage for the transmission of secondary nodes.
- The routing of packets from source to destination is similar to that of [15], which is first along the horizontal line and then the vertical line. Here we omit the detail about the transmission process of each relay node during its active time slot, and we refer the related content to [15]. Each primary and secondary Tx node transmit with power of $P_p a_p^{\delta/2}$ and $P_s a_s^{\delta/2}$, respectively, where P_p and P_s are constants.
- To limit the interference from secondary nodes to the primary transmission, a *preservation region* is defined. When any one of the primary cells is active, we assume that the active primary cell as well as its surrounding eight primary cells constitute a preservation region. Any secondary user (SU) which is inside the preservation regions can not transmit data or relay packets, only if are outside any present preservation region can the SUs transmit or relay packets. Thus the interference from SUs to the primary transmission can be limited.

B. Throughput Analysis

According to Table II, we can calculate the average number of hops each packet traverse from source to destination. Thus the throughput analysis is divided into two parts: 1) multi-hop case; 2) single-hop case. Particularly, we denote the average distance between primary S-D $\mathrm{E}\left(\left\|X_p^i - Y_p^i\right\|\right)$ as $E_{sd}(n)$ for short. Similarly, the average distance between secondary S-D $\mathrm{E}\left(\left\|X_s^i - Y_s^i\right\|\right)$ is denoted as $E_{sd}(m)$.

1) Multi-Hop Case for $0 \le \alpha \le \frac{3}{2}$: First we will show two lemmas before giving the throughput results.

***Lemma* 3.** ([15, Lemma 6]) *With the primary protocol defined previously, each Tx in a primary cell can support a constant data rate of K_1, where $K_1 > 0$ is independent of n.*

***Lemma* 4.** *The total traffic that each primary cell generates or relays is bounded as $\left(n\, E_{sd}(n) \sqrt{a_p(n)}\right)$ $w.h.p.$, for $0 \le \alpha \le \frac{3}{2}$.*

Proof: As the length of a generic primary S-D pair i is bounded by $\Theta(E_{sd}(n))$ $w.h.p.$, thus it is intuitive that the corresponding HDP (or VDP) has a distance of the same order.

Remark 1: As illustrated in Fig.1, given a primary cell Z, there exists both HDPs and VDPs passing through or

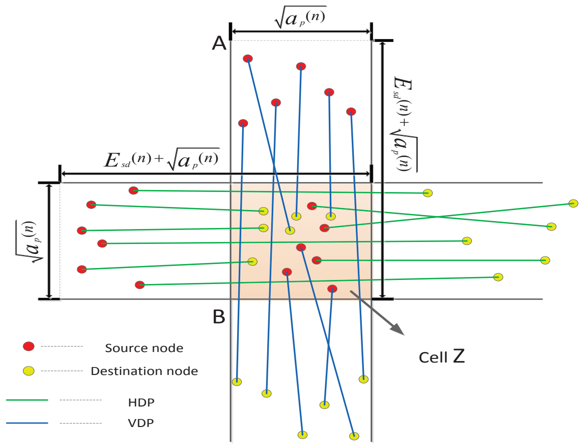


Fig. 1: Illustration of the HDPs and VDPs corresponding to a particular primary cell Z.

originating from it. From a large number of statistic sense, the number of HDPs passing through or originating from each primary cell is equivalent to that of VDPs, and the analysis is similar. Here we will take the HDP for example to show the detailed proof.

According to the predefined network protocols, each primary source transmits data to its destination by first utilizing the HDP. Then, for a given primary cell Z, we have the following three cases (see Fig.2) on the condition that the HDPs pass through or generate from Z:

1) Case 1: The designated destination nodes are located in the region of Z. As the length of S-D pair is bounded, the corresponding source nodes may also be located within Z or anywhere in the region of the dashed area $CDBA$. ($CA = E_{sd}(n)$)
2) Case 2: Both the source nodes and destination nodes are located outside the region of Z. As the length of S-D pair is bounded, the source nodes can only be located in the region of $EFBA$. ($FB = E_{sd}(n) - \sqrt{a_p(n)}$)
3) Case 3: The source nodes are located in the region of Z, and in this case there always exists data flows in Z.

It is worth noting that the value of $E_{sd}(n)$ is larger than that of $\sqrt{a_p(n)}$ for $0 \leq \alpha \leq \frac{3}{2}$. And we assume that the packets is forwarded from left to right.

Based on these three cases, we affirm that all the primary sources of HDPs that pass through or originate from a particular primary cell should be located in an area of $\left(E_{sd}(n) + \sqrt{a_p(n)}\right)\sqrt{a_p(n)}$ $w.h.p.$ which is shown in Fig 2. Since $\sqrt{a_p(n)} = o\left(E_{sd}(n)\right)$, we have $\left(E_{sd}(n) + \sqrt{a_p(n)}\right)\sqrt{a_p(n)}$ equals to $\Theta\left(E_{sd}(n)\sqrt{a_p(n)}\right)$. Suppose $\Theta\left(E_{sd}(n)\sqrt{a_p(n)}\right)$ is bounded by $c_1 E_{sd}(n)\sqrt{a_p(n)}$ where c_1 is a constant independent of n, we proceed the proof as follows:

Let n_h denote the number of HDPs passing through or originating from each primary cell. The upper bound on n_h follows Poisson distribution ($\lambda = n c_1 E_{sd}(n)\sqrt{a_p(n)}$) by supposing that all primary nodes are sources. The same result can be derived for VDPs (the number of VDPs is denoted as n_v). Using the well known inequality $\mathbb{P}(X \geq x) \leq \frac{e^{-\lambda}(e\lambda)^x}{x^x}$, for $x > \lambda$ in [11] and union bound, we obtain

$$\mathbb{P}(n_h + n_v \geq 4\lambda) \leq 2\left(\frac{e}{4}\right)^{c_1 n \mathbb{E}_{sd}(n)\sqrt{a_p(n)}}. \tag{6}$$

As there are n cells within the network at most, we can easily bound the probability that there is at least one cell supporting more than 4λ data paths as $2n(\frac{e}{4})^{c_1 n \mathbb{E}_{sd}(n)\sqrt{a_p(n)}}$. Also, It is proved that $2n(\frac{e}{4})^{c_1 n \mathbb{E}_{sd}(n)\sqrt{a_p(n)}} \to 0$ $w.h.p.$ however $\mathbb{E}_{sd}(n)$ ranges. ■

Combine Lemmas 3 and 4, we give the following Theorem.

Theorem 1. *With the predefined model and network protocols, the primary network can achieve the following per-node throughput $w.h.p.$:*

$$\lambda(n) = \Theta\left(\frac{1}{nE_{sd}(n)\sqrt{a_p(n)}}\right). \tag{7}$$

Proof: Since a given Tx node can support a transmission rate of K_1 according to Lemma 3, each primary S-D pair can achieve a rate of at least K_1 divided by the maximum number of data paths $\Theta\left(nE_{sd}(n)\sqrt{a_p(n)}\right)$ handled by a particular primary cell which is obtained in Lemma 4 $w.h.p.$. This completes the proof. ■

Next we consider the per-node throughput of secondary networks.

Since the secondary users opportunistically access the spectrum and share a portion of secondary time slot with primary nodes, the secondary network can still operate as being a stand-alone network at the presence of primary networks, which was already analyzed in [15] and is still holds in spit of the social relationships between nodes in this paper. In the following, we directly show the results of secondary networks as the analysis is similar to that of primary networks.

***Lemma* 5.** *With the network protocols defined previously, each Tx in a secondary cell can support a constant data rate of K_2, where $K_2 > 0$ is independent of n.*

The proof is similar to that of Lemma 3 which we will not repeat here. The following lemma regarding the total traffic within a particular secondary cell is also straightforward based on Lemma 4.

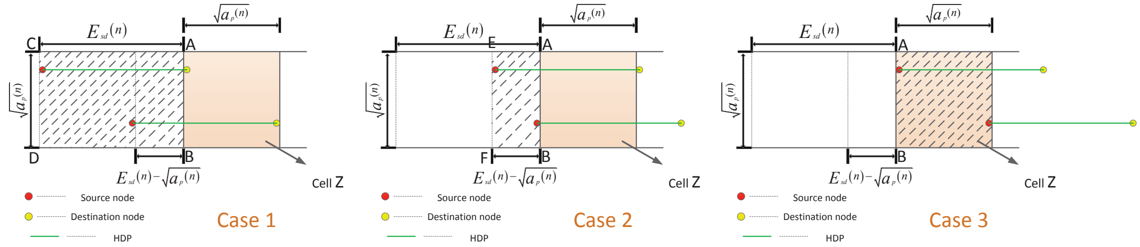
***Lemma* 6.** *The total traffic that each secondary cell generates or relays is bounded as $\Theta\left(m\,E_{sd}(m)\sqrt{a_s(m)}\right)$ $w.h.p.$, for $0 \leq \alpha \leq \frac{3}{2}$.*

Combine Lemmas 5 and 6 we have the following theorem.

Theorem 2. *With the predefined model and transmission protocols, the secondary network can achieve the following per-node throughput $w.h.p.$:*

$$\lambda(m) = \Theta\left(\frac{1}{m\,E_{sd}(m)\sqrt{a_s(m)}}\right). \tag{8}$$

2) Single-Hop Case for $\alpha > \frac{3}{2}$: First we use the following lemma to show the transmission scenario under this condition.



(a) (b) (c)

Fig. 2: The three cases of HDPs corresponding to a particular primary cell Z.

Lemma 7. *When $\alpha > \frac{3}{2}$, it holds $w.h.p.$ for every cell in a single network:*

$$\mathrm{P}\left(\mathrm{E}\left(\|X_i - Y_i\|\right) > \sqrt{\frac{\log n}{n}}\right) \to 0. \tag{9}$$

Proof: The proof is in the Appendix. ∎

Since we remain the TDMA scheduling scheme unchanged, Lemma 3 and Lemma 5 still hold in this case, where constant rate can be achieved for both primary and secondary Tx. In addition, the source and destination are located in the same cell and the transmitting power can support a single-hop transmission. Thus, the source node within a particular cell just needs to take turns to utilize the spectrum.

Theorem 3. *In our proposed single-hop case, the primary and secondary network can achieve the following per-node throughput $w.h.p.$:*

$$\lambda\left(n\right) = \Theta\left(\frac{1}{\log n}\right), \lambda\left(m\right) = \Theta\left(\frac{1}{\log m}\right). \tag{10}$$

C. Delay Analysis

Similar to the throughput analysis, we divide this section into two parts. First we consider the situation when $0 \le \alpha \le \frac{3}{2}$. Then we focus on the single-hop case, where $\alpha > \frac{3}{2}$.

1) Multi-Hop Case for $0 \le \alpha \le \frac{3}{2}$: The delay performance of the primary network is given by the following theorem.

Theorem 4. *Based on the predefined network protocol, the delay is given by*

$$D\left(n\right) = \Theta\left(\frac{E_{sd}\left(n\right)}{\sqrt{a_p\left(n\right)}}\right), 0 \le \alpha \le \frac{3}{2}. \tag{11}$$

Proof: According to Lemma 1 and the area of each primary cell, we can easily get the average number of hops that each primary packet takes to reach the destination along the primary S-D path as $\mathbb{E}_h^p \sim \frac{\mathbb{E}_{sd}(n)}{\sqrt{a_p(n)}}$. In addition, it will cost each primary S-D pair a constant time for one hop transmission. Thus, based on the former analysis and fluid model, we can easily prove the result. ∎

As for the secondary network, we first present the following lemma to show that even the secondary nodes should share a portion of time slots with primary nodes, it still costs each secondary S-D pair a constant time for one hop transmission.

Lemma 8. *The average packet delay of each secondary hop is $\Theta(1)$.*

Proof: We denote $D_h^i(m)$ as the packet delay for a specific hop of secondary S-D pair i. A secondary cell could be active once during each primary time slot without the preservation regions. However, due to the effect of preservation regions in this paper, in the worst case where the minimum opportunistic factor $\eta_{\min} = \frac{9}{25}$, $D_h^i(m) = \frac{1}{\eta_{\min}} t_p$, where the opportunistic factor is defined as the probability with which the secondary users can successfully access the licensed spectrum, t_p is the duration of each primary time slot. Similarly, for the best case $D_h^i(m) = \frac{1}{\eta_{\max}} t_p$, where $\eta_{\max} = \frac{16}{25}$. Therefore, for a general case, $\frac{1}{\eta_{\max}} t_p < D_h^i(m) < \frac{1}{\eta_{\min}} t_p$, i.e., $\mathbb{E}(D_h^i(m)) = \Theta(1)$, which completes the proof. ∎

Based on Lemma 8, we follow the same logic as that in previous analysis regarding delay for the primary network and obtain the delay performance of secondary networks below.

Theorem 5. *According to the proposed network protocol and Lemma 8, the packet delay is given by*

$$D\left(m\right) = \Theta\left(\frac{E_{sd}\left(m\right)}{\sqrt{a_s\left(m\right)}}\right), 0 \le \alpha \le \frac{3}{2}. \tag{12}$$

Proof: Compared with the primary network, each hop of the secondary network consumes more time. However, based on Lemma 8, it is clear that this difference will not influence the delay of secondary networks in order sense. Thus we have Theorem 5 proved. ∎

2) Single-Hop Case for $\alpha > \frac{3}{2}$: We provide the delay performance of primary and secondary networks through the following theorem.

Theorem 6. *The primary and secondary networks can achieve the following delays,*

$$D\left(n\right) = \Theta\left(\log n\right), D\left(m\right) = \Theta\left(\log m\right), \alpha > \frac{3}{2}. \tag{13}$$

Proof: According to Lemma 7, each S-D pair is located in a particular cell $w.h.p.$ in this case and the source node just needs to communicate with the destination through one hop transmission. It is clear that there exists $\Theta\left(\log n\right)$ primary nodes in a primary cell at most. So in any given time slot, we model the probability of each secondary S-D being chosen to transmit as $\Theta\left(\frac{1}{\log n}\right)$. Thus it is easy to get the delay result of $\Theta(\log(n))$ for primary networks. The delay of secondary

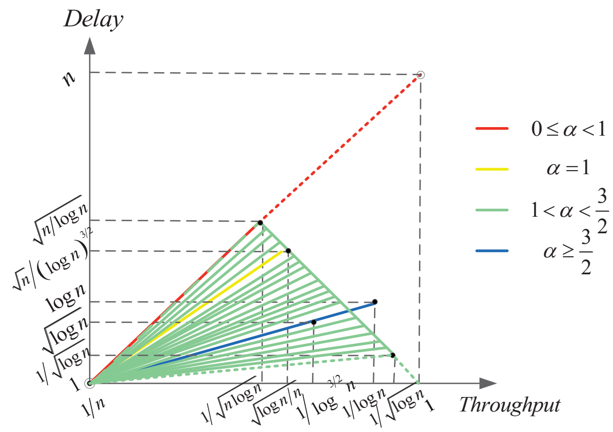


Fig. 3: Delay-throughput tradeoff of primary networks for different ranges of α.

networks can be similarly derived, which is $\Theta(\log(m))$. ■

D. Discussion On Results

As illustrated in Table IV, under our network model and transmission scheme, both throughput and delay of primary or secondary networks are affected by the social relationships between nodes, which are altering in accordance with the varying α. This result demonstrates that the social relationships between nodes do cause the different traffic patterns, thus resulting in the distributed property of throughput and delay. We note that when $0 \leq \alpha < 1$, our network model is reduced into that in [14] and [15], which proves the inclusiveness of our network model. As α increases, varying degrees of traffic locality occurs and both the throughput and delay are improved. The result provides us a strong motivation of considering more about the social relationships between people when designing the wireless network in reality. For example, we should not just be wrapped up in laying more infrastructures to establish the long-range communications, which is really a huge cost. On the contrary, it is of great significance to figure out better ways of allocating the resources such as the high definition movies. If the resources can be appropriately deployed locally, then the performance of capacity and delay can be both enhanced. In Fig.3, we can also find that as α increases, a better tradeoff of throughput and delay is achieved, which is also an irradiative result for the design of wireless networks in practice.

V. Throughput of Infrastructure-Supported Cognitive Social Networks

In this section, we investigate the throughput of infrastructure-supported cognitive social networks where $l = o(n)$ additional BSs are regularly deployed within primary networks. First, we reanalyze the traffic load as we incorporate social relations into the hybrid network.

A. Traffic Load Reanalysis

In the hybrid network setting, there exists $l = o(n)$ BSs in primary networks and the given unit square is regularly divided into BS cells, we denote $a_b = \frac{1}{l}$ as the area of each BS cell. It is necessary for the primary network that the source node and its corresponding destination should be located in different BS cells. Otherwise, adding BSs to primary networks is trivial. According to Lemma 1, we have:

- *Case* 1: $E_{sd}(n) = o(\frac{1}{n^{\gamma}}) = o(\sqrt{a_b})$, for i) $\alpha > \frac{3}{2}$, ii) $\alpha = \frac{3}{2}$ and $0 < \gamma < 1$.
- *Case* 2: $E_{sd}(n) = \omega(\frac{1}{n^{\gamma}}) = \omega(\sqrt{a_b})$, for i) $1 < \alpha < \frac{3}{2}$, ii) $\alpha = 1$, iii) $0 \leq \alpha < 1$ and $0 < \gamma < 1$.

In summary, each primary S-D pair tends to be within the area of a particular BS cell in case 1. Therefore, the wired infrastructure could not help to improve the throughput capacity of primary networks. In Section V, we focus on case 2 where $0 \leq \alpha < \frac{3}{2}$ for primary networks and remove this restriction when considering the secondary network.

B. Network Protocols

The network protocols of infrastructures-supported cognitive social networks are similar to that of ad hoc cognitive social networks including the node density, S-D data path (HDP or VDP), transmitting power and so on. In the following, we just highlight the analysis of probabilistic routing strategy and modified TDMA transmission scheme which are newly added to the primary network.

1) Probabilistic Routing Strategy: We propose a probabilistic routing strategy similar to that in [18], i.e., with probability p the primary nodes choose to transmit data in ad hoc mode while the other $1-p$ fraction of nodes choose to transmit data in infrastructure mode. The detailed routing process is showed as follows:

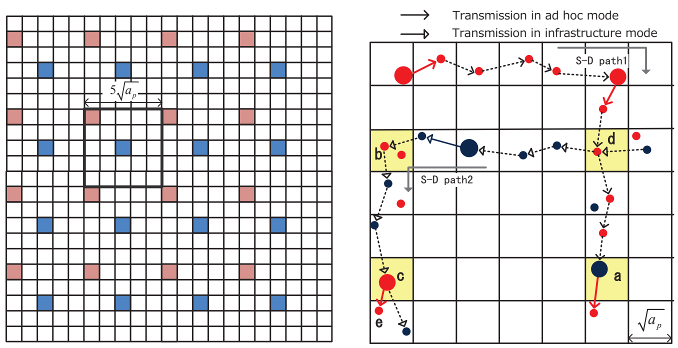
- *W.h.p.*, np primary nodes transmit data in ad hoc mode through the cell-partitioned network during each time slot.
- *W.h.p.*, $n(1-p)$ primary nodes transmit data in infrastructure mode through the BSs during each time slot. The transmission consists of three phases including *uplink phase*, *infrastructure relay phase* and *downlink phase*. Each primary source node transmits data to the closest BS during the uplink phase in a multi-hop fashion, then the data are delivered through high-bandwidth BS-to-BS links to the BS closest to the corresponding destination. The downlink phase is similar to the uplink phase in spit of the opposite direction of data flow.

2) Modified TDMA Transmission Scheme: We propose a modified cellular TDMA transmission scheme that within each primary cluster, two cells are active during one time slot for transmission of ad hoc and infrastructure modes, respectively. Since each transmission is either along HDP or VDP, neither of the Rxs will receive data from both of the two active cells.

Remark 2: In general, we assume that there exists source nodes for both transmission modes in each primary cell thus the connectivity is achieved over the primary network. When connectivity is not satisfied, we can slightly adjust the transmission protocol which is illustrated in Figure 4 (b). For S-D path 1, data are transmitted in ad hoc mode. It is easy to discover that along this path there is a cell containing only one node a, which is of infrastructure mode. Therefore node a relays data for path 1 during its active slots for transmission

TABLE III: Throughput and Delay of Ad Hoc Cognitive Social Networks

Ranges of α	PU throughput	PU delay	SU throughput	SU delay
$\alpha > \frac{3}{2}$	$\Theta(1/\log n)$	$\Theta(\log n)$	$\Theta(1/\log m)$	$\Theta(\log m)$
$\alpha = \frac{3}{2}$	$\Theta\left(1\Big/\log^{3/2} n\right)$	$\Theta\left(\sqrt{\log n}\right)$	$\Theta\left(1\Big/\log^{3/2} m\right)$	$\Theta\left(\sqrt{\log m}\right)$
$1 < \alpha < \frac{3}{2}$	$\Theta\left(n^{\alpha-3/2}\Big/\sqrt{\log n}\right)$	$\Theta\left(n^{3/2-\alpha}\Big/\sqrt{\log n}\right)$	$\Theta\left(m^{\alpha-3/2}\Big/\sqrt{\log m}\right)$	$\Theta\left(m^{3/2-\alpha}\Big/\sqrt{\log m}\right)$
$\alpha = 1$	$\Theta\left(\sqrt{\log n/n}\right)$	$\Theta\left(\sqrt{n}\Big/\log^{3/2} n\right)$	$\Theta\left(\sqrt{\log m/m}\right)$	$\Theta\left(\sqrt{m}\Big/\log^{3/2} m\right)$
$0 \le \alpha < 1$	$\Theta\left(1/\sqrt{n \log n}\right)$	$\Theta\left(\sqrt{n/\log n}\right)$	$\Theta\left(1/\sqrt{m \log m}\right)$	$\Theta\left(\sqrt{m/\log m}\right)$



(a) (b)

Fig. 4: (a) shows the 25-TDMA transmission pattern. (b) shows two specific S-D paths: red nodes transmit data in ad hoc mode while blue nodes transmit data in infrastructure mode. The bigger nodes are currently active. Full lines represent currently active transmissions and dotted lines represent inactive transmissions.

in ad hoc mode. Similarly, nodes b, c, d will relay data for S-D pair 2 during their active slots for transmission in infrastructure mode.

C. Throughput Analysis

1) Primary Networks: For the primary network, we will begin with the lemma which proves that the interference between ad hoc mode and infrastructure mode is small enough for achieving constant data rate within primary networks.

***Lemma* 9.** *With the transmission protocols defined previously, each primary Tx node of ad hoc or infrastructure mode can achieve a constant data rate of K_3, where $K_3 > 0$ is a constant.*

Proof: We denote $I^k_{ia,p}$ as the interference power from the Txs of infrastructure mode to the Rx of the k-th ad hoc pair, and $I^k_{ai,p}$ is the interference power from the Txs of ad hoc mode to the Rx of the k-th infrastructure pair. $R^k_{a,p}$ and $R^k_{i,p}$ are denoted as the achievable data rate of the k-th Tx in ad hoc mode and infrastructure mode, respectively.

Consider the interference model in Figure 2 (a), we have

$$\begin{aligned} I^k_{ia,p} &= \sum_{j=1}^{N_{i,p}} P_i^j \, g\left(\left\| X^j_{i,p,tx} - X^k_{a,p,rx} \right\|\right) \\ &\le P_p a_p^{\frac{\delta}{2}} \sum_{t=1}^{\infty} (8t-4)[(4t-3)\sqrt{a_p}]^{-\delta} \\ &= I'_p, \end{aligned} \tag{14}$$

where I'_p is a constant independent of n due to the fact that $\sum_{t=1}^{\infty}(8t-4)(4t-3)^{-\delta}$ converges to a constant for $\delta > 2$. For a particular Tx node of ad hoc mode, we have

$$\begin{aligned} R^k_{a.p} &\ge \frac{1}{25}\log\left(1 + \frac{P_p\Big/\left(\sqrt{5}\right)^{\delta}}{N_0 + I^k_{a,p} + I^k_{ia,p} + I^k_{sp}}\right) \\ &\ge \frac{1}{25}\log\left(1 + \frac{P_p\Big/\left(\sqrt{5}\right)^{\delta}}{N_0 + I^k_{a,p} + I'_p + I^k_{sp}}\right), \end{aligned} \tag{15}$$

According to Lemma 3 and definition, we have $K_1 \le \frac{1}{25}\log(1 + \frac{P_p/(\sqrt{5})^{\delta}}{N_0 + I^k_p + I^k_{sp}})$. Combine Equation (19), we obtain

$$\begin{aligned} \frac{R^k_{a,p}}{K_1} &\ge \frac{\log\left(1 + \frac{P_p\big/\left(\sqrt{5}\right)^{\delta}}{N_0 + I^k_{a,p} + I'_p + I^k_{sp}}\right)}{\log\left(1 + \frac{P_p\big/\left(\sqrt{5}\right)^{\delta}}{N_0 + I^k_p + I^k_{sp}}\right)} \\ &\ge \frac{\log\left(1 + \frac{P_p\big/\left(\sqrt{5}\right)^{\delta}}{N_0 + I'_p}\right)}{\log\left(1 + \frac{P_p\big/\left(\sqrt{5}\right)^{\delta}}{N_0}\right)} = c, \end{aligned} \tag{16}$$

where we use the fact that $I^k_{a,p} = I^k_p$. By setting $K_3 = cK_1$, and according to symmetry of the interference model, we complete the proof. ■

Before we proceed, it is necessary to consider the impact of the probabilistic routing strategy on homogeneity of the traffic load over the network. In an extreme case where p (or (1-p)) asymptotically converges to zero, the sparse distribution of np (or n(1-p)) nodes may lead to concern that the traffic load of each cell (or BS cell) is disproportionate. The following lemma shows that when p converges to zero or one slower than a particular threshold, every cell including the BS cell will carry a certain amount of traffic which guarantees homogeneity of the traffic load over the network.

***Lemma* 10.** *The following two facts hold $w.h.p.$:*

(a) each primary cell serves at least one S-D pair of ad hoc mode when p converges to zero at an asymptotic rate of: i) $\omega\left(\frac{\sqrt{n \log n}}{n}\right) \& o(1)$, for $0 \leq \alpha < 1$; ii) $\omega\left(\frac{\sqrt{n}\log^{3/2} n}{n}\right) \& o(1)$, for $\alpha = 1$; iii) $\omega\left(\frac{n^{\frac{1}{4-2\alpha}}\sqrt{\log n}}{n}\right) \& o(1)$, for $1 < \alpha < 3/2$. & is defined as follow: if p equals to $a\&b$, then p can be any value between a and b.

(b) each BS cell holds at least one node of infrastructure mode when $1-p$ converges to zero at an asymptotic rate of $(1-p) \overset{\to 0}{=} \omega(n^{\gamma-1})$, for $0 \leq \alpha < 3/2$.

Proof: The proof is available in the Appendix. ■

In this work, we consider the probabilistic routing scheme satisfying Lemma 10 to achieve the homogeneity of traffic load over primary networks when p (or (1-p)) asymptotically converges to zero. Based on this, we first calculate the the number of primary S-D paths passing through or originating from a particular primary cell in ad hoc mode and infrastructure mode, respectively.

***Lemma* 11.** *For each primary cell, $w.h.p.$, the number of primary S-D path passing through or originating from it in ad hoc mode and infrastructure mode is upper bounded by $npE_{sd}(np)\sqrt{a_p(n)}$ and $\Theta(\frac{1}{l}n(1-p))$, respectively.*

Proof: We first consider transmission of ad hoc mode. According to the predefined primary transmission protocol, np primary nodes will choose to transmit data to their destinations via multiple hopping over the cell-partitioned network during each time slot. It is worth noting that the only difference between transmission in this case and that in pure primary networks is that the source nodes are randomly chosen with probability p. Even though some of the np nodes may serve as relay nodes for HDPs or VDPs of infrastructure mode, such extra load has no impact on transmissions of ad hoc mode since we separate transmissions of the two modes in different active cells. Thus following the same logic in the proof of Lemma 4, the number of primary S-D path passing through or originating from it in ad hoc mode are bounded by $c_1 E_{sd}(np)\sqrt{a_p(n)}$. We define n_h' and n_v' as the number of HDPs and VDPs respectively, thus n_h' (or n_v') follows poisson distribution with density of $\lambda = c_2 npE_{sd}(np)\sqrt{a_p(n)}$. To prove this, we have the following inequality corresponding to Inequality (8):

$$\mathbb{P}(n_h' + n_v' \geq 4\lambda) \leq 2\frac{e^{-\lambda}(e\lambda)^x}{x^x}\,|_{x=2\lambda} = 2\left(\frac{e}{4}\right)^{c_2\, np\, E_{sd}(np)\sqrt{a_p(n)}}, \tag{17}$$

Therefore the probability that at least one cell supporting more than 4λ data paths is upper bounded by $2n\left(\frac{e}{4}\right)^{c_1\, np\, E_{sd}(np)\sqrt{a_p(n)}}$, which approaches zero as n tends to infinity, the detailed proof is similar to that in the proof of Lemma 4. As to the transmission of infrastructure mode, we note that within each BS cell, the centering BS serves as a fixed destination during the uplink phase and a fixed data source during the downlink phase. In addition, transmission paths within each BS cell follow the same pattern as ad hoc mode. Since there are at most $2\frac{1}{l}n(1-p)$ source nodes of infrastructure mode within each BS cell, the upper bound could be easily obtained. Thus we complete the proof. ■

Theorem 7. *Based on Lemma 11 and the predefined primary transmission protocol, the primary network can achieve the following per-node throughput $w.h.p.$:*

$$\lambda_a(n,l,p) = \Theta\left(\frac{1}{np\,E_{sd}(np)\sqrt{a_p(n)}}\right)$$
$$\lambda_i(n,l,p) = \Theta\left(\frac{l}{(1-p)\,n}\right), \tag{18}$$

for $0 \leq \alpha < \frac{3}{2}$ and p satisfying Lemma 10. Here, $\lambda_a(n,l,p)$ and $\lambda_i(n,l,p)$ denotes the per-node throughput of transmission in ad hoc mode and infrastructure mode, respectively.

The proof follows the same logic as that of Theorem 1 which is omitted here.

2) Secondary Networks: First, we show in the following lemma that the accessing opportunistic factor for secondary networks is larger than a constant and therefore guaranteeing zero transmission outage in the network. Then we derive the throughput of secondary networks.

***Lemma* 12.** *With the proposed network model and transmission protocols, the opportunistic factor for a secondary cell is $\frac{2}{25} \leq \eta \leq \frac{8}{25}$. Therefore, each secondary node has a finite opportunity to transmit data with zero transmission outage, $w.h.p.$.*

Proof: The preservation region of the primary network is shown in Figure 5 as the red and blue shaded area. The best and worst places for secondary transmission are also given in the figure. For the best places, secondary cells within these cells have a opportunistic factor of $\eta_{max} = \frac{8}{25}$. For the worst places, the corresponding opportunistic factor is $\eta_{min} = \frac{2}{25}$ which could be obtained according to the lower two subpictures in Figure 5. ■

Interference from the primary network should also be considered since there are more active primary cells during each primary slot. We notice that due to the preservation region, a minimum distance of $\sqrt{a_s}$ is guaranteed between active secondary cells and active primary cells in infrastructure mode. Therefore a constant data rate is achievable for secondary transmissions.

***Lemma* 13.** *With the proposed secondary transmission protocol , each Tx node in a secondary cell can achieve a constant data rate of K_4, where $K_4 > 0$ is a constant.*

Proof: The proof is in the Appendix. ■

According to Lemma 13 and the underlying secondary transmission protocol, the per-node throughput of secondary networks is the same as that in Theorem 2.

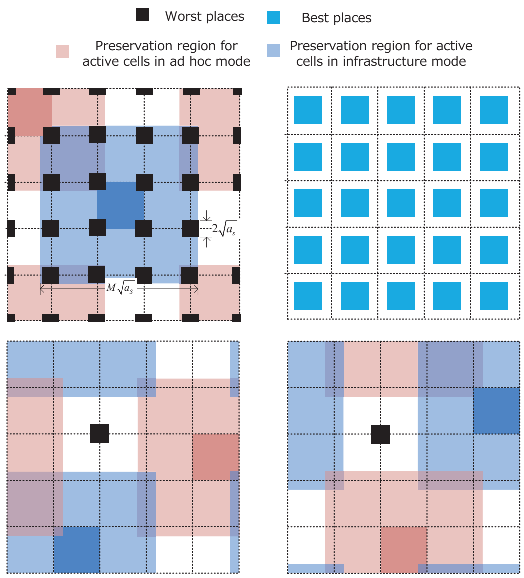


Fig. 5: Preservation regions of the primary network, and places with the maximum (best places) and minimum (worst places) opportunistic factor. The dark red cell represents the active cell for ad hoc transmission, and the dark blue cell represents the active cell for infrastructure transmission.

D. Discussion of Results and Implications

Apparently, the throughput of primary networks not only depends on the length of primary S-D pair, but also depends on p and l. Thus in the following, we further explored the result of primary networks.

1) Probabilistic Routing Strategy with Constant P: In this case, p is a constant, we have $\lambda_a(n,l,p)=\Theta(\lambda(n))$. Before proceeding, we provide the following lemma:

***Lemma* 14.** *To guarantee that adding BSs could bring capacity gain of the primary network, i.e. $\lambda_i(n,l,p)=\omega(\lambda(n))$, γ should satisfy: i) $\frac{1}{2}\leq\gamma<1$, for $0\leq\alpha<1$; ii) $\frac{1}{2}<\gamma<1$, for $\alpha=1$; iii) $\alpha-\frac{1}{2}\leq\gamma<1$, for $1<\alpha<3/2$.*

Proof: Taking the case of $0\leq\alpha<1$ for example as other cases can be proved similarly. Based on Theorem 1 and Theorem 7, we have $\lambda_i(n,l,p)/\lambda(n)=\Theta\left(\frac{n^{\gamma-1/2}\sqrt{\log n}}{1-p}\right)$. It is obvious that $1/2\leq\gamma<1$ is needed to make $\frac{n^{\gamma-1/2}\sqrt{\log n}}{1-p}$ approaches infinity thus guaranteeing $\lambda_i(n,l,p)=\omega(\lambda(n))$, which completes the proof. ■

Social Impact on Threshold of l: From above analysis, we can find that the aggregate throughput of primary networks is larger than that of pure ad hoc network. However, we discover that the threshold of l is improved compared with the results in [18] and [19] under our new network setting. Due to the social impact, the minimum number of BSs required to enhance primary transmission increases from $\Theta(n^{\frac{1}{2}})$ to $\Theta(n^{\alpha-\frac{1}{2}})$, for $0\leq\alpha<3/2$.

2) Probabilistic Routing Strategy with $P\to1$: In this case, we consider $(1-p)\overset{\to0}{=}\omega(n^{\gamma-1})$. The per-node throughput of $\lambda_a(n,l,p)\sim\lambda(n)$ is achieved as $np\to n$. For the transmission in infrastructure mode, we have the following lemma:

***Lemma* 15.** *It holds $w.h.p.$ that $\lambda_i(n,l,p)=\omega(\lambda(n))$, for γ, p satisfying:*

$$(1-p)\overset{\to0}{=}\begin{cases}\omega(n^{\gamma-1})\&O(n^{\gamma-\frac{1}{2}}), & 0\leq\alpha<1\\ \omega(n^{\gamma-1})\&o(n^{\gamma-\frac{1}{2}}), & \alpha=1\\ \omega(n^{\gamma-1})\&O(n^{\gamma+\frac{1}{2}-\alpha}), & 1<\alpha<3/2.\end{cases}\quad(19)$$

The proof is similar to that of Lemma 14.

Compromised Solution: As p asymptotically converges to one, we note that a possible benefit brought by such network is that, when the number of BSs l is not large enough to improve the throughput over most of the primary network, transmissions among a small fraction of nodes could still be aided by the BSs. From Lemma 15, we can see that at least n^γ primary nodes benefit from the BSs. Therefore, probabilistic routing strategy with $P\to1$ could be considered a compromised solution to make up for the deficiency of BSs in practice. Due to the social impact, the maximum number of nodes that can use BSs to improve capacity decreases from $\Theta(n^{\gamma+\frac{1}{2}})$ to $\Theta(n^{\gamma+\frac{3}{2}-\alpha})$, for $0\leq\alpha<3/2$.

3) Probabilistic Routing Strategy with $P\to0$: In this case, we consider $p\to0$ according to Equation (10). It holds $w.h.p.$ that $\lambda_a(n,l,p)=\omega(\lambda(n))\&o(1/\log n)$ for $0\leq\alpha<3/2$. To obtain this, we may substitute the lower bound of p in Equation (10) for p in Theorem 1.

As for $\lambda_i(n,l,p)$, the per-node throughput of $\Theta(\frac{l}{n})$ is achievable in this case on condition that l exceeds the thresholds in Lemma 14. Compared with the situation where p is a constant, throughput of primary networks is further improved as $n(1-p)\to n$ primary nodes utilize the high-bandwidth BS-to-BS links.

Remark 3: The main results are summarized in Table V. When l is large enough (satisfy Lemma 14), the probabilistic routing strategy with $P\to0$ can be used to further increase capacity. When l is not large enough, we could also utilize the probabilistic routing strategy with $P\to1$ to guarantee non-negligible capacity gain of a small fraction of primary nodes. Considering the actual wireless network, it is still a tough problem in deploying a large number of base stations in the crowded area such as the commercial district where lots of people are localized. Maybe we should make efforts to develop the ad hoc technique to guarantee the communication quality, which is supported by our results.

VI. Conclusion

This paper studies the throughput and delay of wireless cognitive social networks. By incorporating the rank based model into our network, the average length of S-D pairs within CRN is investigated. For the ad hoc cognitive social network, we analyzed the throughput and delay in detail and proved that both primary and secondary networks can achieve the same scaling laws as being a stand-alone network in spite of the introduction of social relations. For the infrastructure-supported cognitive social network, as we proposed a novel transmission scheme where transmissions of ad hoc mode and infrastructure mode are allocated according to a probabilistic

TABLE IV: Throughput Capacity of Primary Networks with Convergent P

Varying Degrees of Traffic Locality	The Number of BSs	Probabilistic Routing Strategy with p	Throughput of Primary Networks $\lambda_a(n,l,p)$	Throughput of Primary Networks $\lambda_i(n,l,p)$
$0 \le \alpha < 1$	$n^{\frac{1}{2}} \le l < n$	$p \overset{\to 0}{=} \omega(\frac{\sqrt{n \log n}}{n})\&o(1)$	$\omega(\frac{1}{\sqrt{n \log n}})\&o(\frac{1}{\log n})$	$\omega(\frac{1}{\sqrt{n \log n}})$
	$l < n^{\frac{1}{2}}$	$1-p \overset{\to 0}{=} \omega(n^{\gamma-1})\&O(n^{\gamma-\frac{1}{2}})$	$\Theta(\frac{1}{\sqrt{n \log n}})$	
$\alpha = 1$	$n^{\frac{1}{2}} < l < n$	$p \overset{\to 0}{=} \omega(\frac{\sqrt{n} \log^{3/2} n}{n})\&o(1)$	$\omega(\frac{\sqrt{\log n}}{\sqrt{n}})\&o(\frac{1}{\log n})$	$\omega(\frac{\sqrt{\log n}}{\sqrt{n}})$
	$l \le n^{\frac{1}{2}}$	$1-p \overset{\to 0}{=} \omega(n^{\gamma-1})\&o(n^{\gamma-\frac{1}{2}})$	$\Theta(\frac{\sqrt{\log n}}{\sqrt{n}})$	
$1 < \alpha < \frac{3}{2}$	$n^{\alpha-\frac{1}{2}} < l < n$	$p \overset{\to 0}{=} \omega(\frac{n^{\frac{1}{2(2-\alpha)}} \sqrt{\log n}}{n})\&o(1)$	$\omega(\frac{n^{\alpha-\frac{3}{2}}}{\sqrt{\log n}})\&o(\frac{1}{\log n})$	$\omega(\frac{n^{\alpha-\frac{3}{2}}}{\sqrt{\log n}})$
	$l \le n^{\alpha-\frac{1}{2}}$	$1-p \overset{\to 0}{=} \omega(n^{\gamma-1})\&O(n^{\gamma+\frac{1}{2}-\alpha})$	$\Theta(\frac{n^{\alpha-\frac{3}{2}}}{\sqrt{\log n}})$	

routing strategy, the social impact on throughput of both primary and secondary networks are also well investigated. An interesting thing is that we revealed the improvement of threshold of l due to the social issues. Meanwhile, we put forward a compromised solution to cope with the situation when lacking enough BSs, which makes our network model more applicable in practical scenarios.

APPENDIX

Proof of Lemma 2:

Proof: First we will show that the conditional PDF of $\|X_i - Y_i\|$ could be expressed in term of the PDF of the r-th order statistic.

$$
\begin{aligned}
f_{r:n}^{(\|X_i-Y_i\|)}(d) &= \mathbb{P}\{\|X_i - Y_i\|_{r:n} \le d\} \\
&= \int_0^d \frac{n!}{(r-1)!(n-r)!} 2\pi^r t^{2r-1}(1-\pi t^2)^{n-r} dt \\
&= \int_0^{\sqrt{\pi} d} \frac{n!}{(r-1)!(n-r)!} \frac{2}{\sqrt{\pi}} t^{2r-1}(1-t^2)^{n-r} dt.
\end{aligned}
$$

Therefore we have $f_{r:n}^{(\|X_i-Y_i\|)}(d) \sim f_{r:n}(d)$ according to Example 2.2.2 in [22]. Here we use a well know approximation $\sqrt{n}(\frac{r}{n} - \lambda) \to t$, where $-\infty < t < \infty, 0 < \lambda < 1$ [23] to bound the tail probability of order statistic. Then $w.h.p.$ $1 - F_{r:n}(x) = Q\left(\frac{\sqrt{n}(F(x)-r/n)}{\sqrt{r/n(1-r/n)}}\right)$ where $F_{r:n}(x)$ denotes cumulative distribution function (CDF) of the r-th order statistic, and $Q(\cdot)$ denotes Q-function, the tail probability of the standard normal distribution. For $F(x)$, we take it as x^2 according to our network model since it denotes the CDF of distance between two random nodes distributed in a unit square independently. Now we bound the tail probability of the length of a generic S-D pair i.

$$
\begin{aligned}
&\mathbb{P}(\,\|X_i - Y_i\| \ge \omega(\mathbb{E}(\|X_i - Y_i\|))) \\
&\sim \frac{1}{H_n}\sum_{r=1}^{n}\frac{1}{r^\alpha}\int_{\omega(\sqrt{r/n})}^{1/\sqrt{\pi}} f_{r:n}^{(\|X_i-Y_i\|)}(t)dt \\
&\sim \frac{1}{H_n}\sum_{r=1}^{n}\frac{1}{r^\alpha}\int_{\omega(\sqrt{r/n})}^{1} f_{r:n}(t)dt \\
&= \frac{1}{H_n}\sum_{r=1}^{n}\frac{1}{r^\alpha}\frac{1}{H_n}\sum_{r=1}^{n}\frac{1}{r^\alpha}\left(1 - F_{r:n}\left(\omega\left(\sqrt{r/n}\right)\right)\right) \\
&= \frac{1}{H_n}\sum_{r=1}^{n}\frac{1}{r^\alpha} Q\left(\sqrt{n}\frac{\omega\left(r/n\right)}{\sqrt{r/n\left(1-r/n\right)}}\right),
\end{aligned}
$$

By using a bounds for Q-function $Q(x) < \frac{1}{x} \cdot \frac{1}{\sqrt{2\pi}} e^{-x^2/2}$, we continue the proof,

$$
\begin{aligned}
\mathbb{P}(\|X_i - Y_i\| \ge \omega(\mathbb{E}(\|X_i - Y_i\|))) &< \frac{1}{H_n}\sum_{r=1}^{n}\frac{1}{r^\alpha} Q(\omega(\sqrt{r})) \\
&< \frac{1}{H_n}\sum_{r=1}^{n}\frac{1}{r^\alpha}\cdot\frac{1}{\sqrt{r}} e^{-\frac{\omega(r)}{2}},
\end{aligned}
$$

By applying approximate algorithm and Cauchy-Schwarz inequality, we get:

$$
\begin{aligned}
\frac{1}{H_n}\sum_{r=1}^{n}\frac{1}{r^\alpha}\cdot\frac{1}{\sqrt{r}} e^{-\frac{\omega(r)}{2}} &\sim \frac{1}{H_n}\sum_{r=1}^{n}\frac{1}{r^\alpha}\cdot\frac{1}{\sqrt{r}} 2^{-\omega(r)} \\
&< \frac{1}{H_n}\sqrt{\sum_{r=1}^{n}(\frac{1}{r^{\alpha+1/2}})^2 \sum_{r=1}^{n} 4^{-\omega(r)}} \\
&\overset{(d)}{<} o(1),
\end{aligned}
$$

which completes the proof of Lemma 3. ■

Proof of Lemma 7:

Proof: We will prove this lemma in a similar way as that of Lemma 2. When considering the event that $\mathbb{E}(\|X_i - Y_i\|) > \sqrt{\frac{\log n}{n}}$, it follows that $\mathbb{E}(\|X_i - Y_i\| \,|r:n) > \sqrt{\frac{r \log n}{n}}$ for each $1 \le r \le n$, since $\mathbb{E}(\|X_i - Y_i\|) \sim \frac{1}{\sqrt{n}}$and

$\mathbb{E}(\|X_i - Y_i\| \, |r:n) \sim \sqrt{\frac{r}{n}}$, then we have

$$P\left(E\left(\|X_i - Y_i\|\right) > \sqrt{\frac{\log n}{n}}\right)$$
$$\sim \frac{1}{H_n}\sum_{r=1}^{n}\frac{1}{r^\alpha}P\left(E\left(\|X_i - Y_i\| \, |r:n\right) > \sqrt{\frac{r\log n}{n}}\right)$$
$$= \frac{1}{H_n}\sum_{r=1}^{n}\frac{1}{r^\alpha}\left(1 - F_{r:n}\left(\sqrt{r\log n/n}\right)\right)$$
$$= \frac{1}{H_n}\sum_{r=1}^{n}\frac{1}{r^\alpha}Q\left(\sqrt{n}\frac{r\log n/n}{\sqrt{r/n\,(1-r/n)}}\right),$$

for

$$Q\left(\sqrt{n}\frac{r\log n/n}{\sqrt{r/n\,(1-r/n)}}\right) < Q\left(\sqrt{r}\log n\right)$$
$$< \frac{1}{\sqrt{r}}e^{-\frac{r\log^2 n}{2}}$$
$$< \frac{1}{\sqrt{r}}n^{-\frac{\log n}{2}},$$

thus, we have

$$P\left(E\left(\|X_i - Y_i\|\right) > \sqrt{\frac{\log n}{n}}\right) < \frac{1}{H_n}\sum_{r=1}^{n}\frac{1}{r^\alpha}\cdot\frac{1}{\sqrt{r}}\,n^{-\frac{\log n}{2}}$$
$$< n^{-\frac{\log n}{2}},$$

by using the union bound we obtain $w.h.p.$

$$\mathbb{P}\Big(\bigcup_{i=1}^{n}[E\{\|X_{tx}^i - X_{rx}^i\|\} > \sqrt{\frac{\log n}{n}}]\Big) < n \cdot n^{-\frac{\log n}{2}} \to 0.$$

■

Proof of Lemma 10:

Proof: First we consider the case where p converges to zero. In this case, due to the sparsity of np randomly distributed nodes, it is possible that there exist some primary cells with no S-D pairs of ad hoc mode originating from or passing through them while other primary cells carry a certain amount of traffic. But when p scales slower than a particular threshold, we prove that such heterogeneity phenomenon will not happen over the hybrid primary network.

We assume that primary nodes of ad hoc mode are randomly distributed according to P.P.P. with density np. We denote the probability that a particular primary cell carries no traffic of ad hoc mode as $\mathbb{P}_{zero}$, then $\mathbb{P}_{zero} = \frac{\lambda^k e^{-\lambda}}{k!}\Big|_{k=0,\lambda=E_{sd}(np)\sqrt{a_p}np} = e^{-E_{sd}(np)\sqrt{a_p}np} \overset{(a)}{<} n^{-\sqrt{2}}$, where (a) could be obtained by using Lemma 1 and three cases of converging rate in (a). We omit the detailed derivation here since it is not such complicated.

Thus we have $\mathbb{P}$(some cells carry no traffic of ad hoc mode) $< n \cdot n^{-\sqrt{2}} \to 0$.

Now we consider the case where p converges to one, i.e., most transmissions are of ad hoc mode, left $n(1-p)$ nodes transmit data through the wired BS network. In this case we focus on the number of nodes held by each BS square. We assume that primary nodes of infrastructure mode are randomly distributed according to P.P.P. with density $n(1-p)$, and each BS square has an area of a'_p. We denote $\mathbb{P}'_{zero}$ as the probability that some BS squares hold no nodes of infrastructure mode, then $\mathbb{P}'_{zero} < n \cdot e^{-\lambda}\Big|_{\lambda=a'_p n(1-p)} = n \cdot e^{-n^{1-\gamma}(1-p)} \to 0$, where we use the fact that $(1-p) > n^{\gamma-1}$ according to the converging rate in (b). Thus we complete the proof of fact (b). ■

Proof of Lemma 13:

Proof: Since in each cluster there are two active primary cells at a time, then we get

$$R_s^j = \log\left(1 + \frac{P_s^j\, g\left(\left\|X_{s,tx}^j - X_{s,rx}^j\right\|\right)}{N_0 + I_s^j + I_{ps}^j}\right)$$
$$\geq \frac{1}{25}\eta_{min}\log\left(1 + \frac{P_s\Big/\left(\sqrt{5}\right)^\delta}{N_0 + I_s^j + 2I_{ps}^j}\right)$$
$$\geq \frac{1}{25}\eta_{min}\log\left(1 + \frac{1}{2}\frac{P_s\Big/\left(\sqrt{5}\right)^\delta}{N_0 + I_s^j + 2I_{ps}^j}\right),$$

Using the inequality $K_2 \leq \log\left(1 + \frac{P_s/\left(\sqrt{5}\right)^\delta}{N_0 + I_s^j + I_{ps}^j}\right)$ based on Lemma 5, the following inequality holds:

$$R_s^j \geq \frac{1}{25}\eta_{\min}\log\left(1 + \frac{e^{k_2} - 1}{2}\right),$$

By setting $K_4 = \frac{1}{25}\eta_{\min}\log\left(1 + \frac{e^{k_2} - 1}{2}\right)$, we complete the proof. ■

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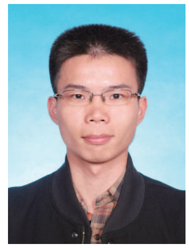

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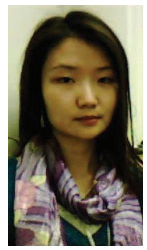

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