Spectrum Trading in Cognitive Radio Networks: An Agent-based Model under Demand Uncertainty

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***Abstract*—In this paper, we propose an *agent-based* spectrum trading model, where an agent can play a third-party role in the spectrum trading process. Providing service to Secondary Users (SUs) with spectrum bought from Primary Users (PUs), the agent can make profits during the process by providing service to secondary users. During each trading period, the agent has to decide how much spectrum it should lease from PUs and what price it should charge SUs. Therefore, the most significant challenge to implement this spectrum trading model is finding the most profitable strategy for agent(s). We address this challenge under two scenarios in which: 1) a single agent and 2) multiple agents. Instead of quantifying SUs' spectrum demand by a deterministic function of price, we take the randomness of secondary users' demand or demand uncertainty into consideration. To the best of our knowledge, this is the first solution to *agent-based* spectrum trading considering *demand uncertainty*.**

***Index Terms*—Cognitive radio, agent, trading, demand uncertainty.**

I. Introduction

To improve the utilization of spectrum, cognitive radio has been considered as one of the most promising technologies [1]. In cognitive radio networks, Primary Users (PUs) who possess licenses can share the spectrum usage with Secondary Users (SUs) who do not. However, whether underlay or overlay spectrum sharing techniques [3] is used, we should design some incentives for primary users to share their spectrum with secondary users. One particular form of trading is *auctions*. For example, in [4], the authors proposed a spectrum auction framework. However, the auction's option maximizes revenue for the PUs but not for the overall system, the broker is non-profit, all the profits come from the SUs. Both PUs and SUs require highly-updated equipment for auction, while still few of the SUs will be satisfied because the highest bidder wins in the auctions. When the system is being used in applications in a commercial society, rare participants want to be non-profit parties. We introduce an agent-based spectrum trading that can provide a good solution to this type of problems. In a real life market, retailers (similar to agents in our model) buy at wholesale with a negotiated price from manufacturers (similar to PUs in our model), and sell goods at a comparably constant price to the consumers (similar to SUs in our model) considering demand uncertainty. Each unit cannot maximize its revenue but the whole market maximizes its revenue, and the trading system has less negotiations than auction.

In addition to traditional spectrum trading system, agent-based model can provide significant advantages. First, the agent different from PUs or SUs, it is an extra entity added to the whole system which makes profit for a third party, this is a big incentive for nodes to serve as agent(s). Second, it does not require high intelligence from PUs or SUs' equipment, because SUs do not need to perform spectrum sensing or price negotiation with PUs by themselves, and PUs' negotiation with the agent is less complex than auctions. Third, agent-based model can reduce the network overhead and control information transmission between multiple PUs and multiple SUs [5]. For example, agents and PUs need only one negotiation to determine the quantity and price for wholesaling spectrums in one process, the selling price to SUs is broadcast by each agent. Usually, the number of agents is much less than the number of PUs and SUs, therefore, we believe that our model reduces the network overhead. Fourth, agents with higher transmission power and greater spectrum sensing coverage can provide SUs with more spectrum trading opportunities. Fifth, it is easy to implement agent's functions on currently existing infrastructures, like BS in 802.22 or AP in 802.11 network (when BS or AP act as agents, they are not considered as SUs in our model, because they do not buy service from themselves, although BS and AP are generally considered as SUs under some other circumstances). All these differences make our agent-based model more applicable to an actual system and can provide more fairness.

In this paper, we keep our model general so that it can be implemented in any future system using CR technology. For example, in an 802.22 system, agents can be built on base stations, which lease (or sense) the spectrum of VHF/UHF TV bands and serve all its associated Consumer Premise Equipments (CPEs) without any harmful interference to TV receivers. This configuration is illustrated in Fig.1.

The objective of our spectrum trading model is to maximize the profit of the agent (the third party of the Cognitive Radio which is not important in the auction model) as well as to enhance the satisfaction of cognitive radio users (SUs). In order to focus on the analysis of the agent's behavior, we assume that the agent(s) wholesale spectrum from primary users, the price paid for the spectrum is not random. It actually follows the basic economic model, supply and demand. For the sake of simplicity, we assume that the PUs and the agents

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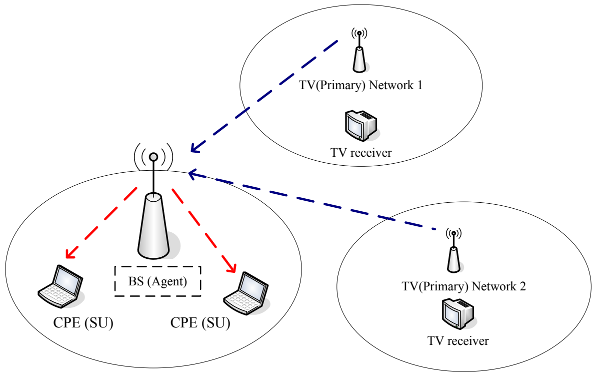

Fig. 1. Exemplary agent deployment in 802.22 system.

have negotiated the fact that PUs provide spectrum to agents by a constant S per unit spectrum (e.g.,MHz). Another reason to exclude the negotiation process between PUs and the agents is to keep the generality of this model. For example, instead of buying spectrum usage from PUs, the agent may perform spectrum sensing by itself, if equipped with specific hardware. So price S can also be viewed as the average cost of spectrum sensing per unit spectrum (e.g., MHz). According to a real market situation, a relatively constant price at a wholesale is achievable. In brief, we just assume that the agent can obtain spectrum usage from PUs with a certain price S and focus on the agent's profit-maximization problem.

Based on microeconomics, we can quantify the spectrum demand of secondary users by a specific demand function of price p, usually denoted as $D(p)$ in the works related to Cogitative Radio Networks([7] [8]). This solution can be found when the market reaches an equilibrium state, which denotes a price for which the spectrum demand is equal to the spectrum supply. Two approaches are the most popular to derive this function. In the first method, SU's satisfaction is expressed by utility, and spectrum demand can be derived by maximizing the utility for a given price [7]. This approach sounds very natural, but it requires the SU to adapt its equipment for varying amount of allocated bandwidth. In the second method, the demand function can be derived by adopting the concept of *Acceptance Probability* $A(W, p)$ which reflects the willingness (probability) of a secondary user to accept the service for a predetermined bandwidth W (we simplified the spectrum quality into a constant bandwidth W) and price p charged by the agent [5]. This approach provides two options only for the SU—-accept or not. But it is easier to be implemented because it does not require too much hardware modification on existing mobile user equipments. In this paper, we will adopt this method.

Based on the above discussion and no matter which method is used to derive $D(p)$, the natural randomness of the network has not been explicitly considered. To be specific, an agent may not be aware of every SU's spectrum efficiency or preference on bandwidth. It is also impossible for an agent to predict the number of SUs in each trading period (such as one hour as specified in [9]). Therefore, the spectrum demand $D(p)$ may be modeled more appropriately by a combination of the deterministic part $d(p)$ and the random part ϵ. In this paper, we mainly consider the case when $D(p)$ can be expressed by a multiplicative form: $D(p) = d(p)\epsilon$.

Given S to be the price or cost of per unit spectrum bought from PUs or sensed by the agent itself and $D(p)$ the spectrum demand of SUs, the agent faces the challenge of deciding the quantity q of the idle spectrum it needs and the price p it charges its customers (i.e., SUs). Therefore, the problem can be summarized by finding the optimal strategy (q^*, p^*), given $D(p)$ and S. Because $D(p)$ cannot be precisely predicted by the agent, we cannot simply let $q = D(p)$ here. In addition, since q^* and p^* depend on each other, they must be determined simultaneously.

To obtain this optimal strategy, analysis of two different situations are necessary. In the first situation, only one agent operates and dominates in the secondary network. In the other situation, more than one agent compete for customers in the secondary network. So, we naturally define them as *noncompeting agents* and *competing agents*, respectively.

At this point, we would like to summarize our main contributions as:

- An agent-based spectrum trading model considering demand uncertainty in cognitive radio network is proposed.
- In a single-agent system, we provide a general solution and algorithm to achieve the optimal strategy.
- In a multiple-agent system, we show that a pure strategy Nash equilibrium can be achieved.

In addition and through network simulation, we achieve the following results:

- Due to demand uncertainty, a gap between agent's maximum achievable profit and realized profit always exists.
- When secondary market's demand becomes more uncertain, an agent prefers higher per sale profit to higher sales.
- Profits of noncompeting agents will be much reduced if they compete for secondary users.

The rest of this paper is organized as follows: related work is reviewed in Section II. In Section III, we formulate the problem for the single-agent system and provide a general solution. An example of uniform distribution and corresponding simulation results are presented. In Section IV, we formulate the competition between agents using the log-supermodular game and prove that a widely preferred Nash equilibrium can be achieved by a distributed algorithm. In Section V, we give out the simulation results for both single-agent model and multi-agent model. Finally, we conclude our work in Section VI.

II. Related Work

A. Spectrum Trading in Cognitive Radio

With Dynamic Spectrum Accessing (DSA), Cognitive Radio Networks (CRNs) are proposed to solve the problem of spectrum congestion [1]. Nevertheless, the implementation of CRN still faces a lot of challenges because of the fluctuating nature of the available spectrum and QoS degradation of primary services. Some appropriate incentives for spectrum sharing must be provided to compensate for the PUs' QoS performance degradation. Spectrum trading, one of the key issues of spectrum management is adopted to address this challenge [9]. It provides PUs incentives (e.g. money) to temporarily lease their spectrum, as well as provides SUs opportunities to access those under-utilized spectra. So, spectrum trading appears to be an attractive tool to promote efficient use of spectrum resources. The main approaches, including microeconomics, classical optimization, noncooperative game and auction, are summarized and illustrated in [7]. Considering spectrum sellers' competing and rational nature, noncooperative game [10] [11] and truthful auction [12] are presently the most popular approaches in CR spectrum trading research.

B. Agent-based Spectrum Trading

Although DSA techniques enable the secondary use of radio spectrum, it puts no restriction on the architecture to implement CRN in practice since its architecture is very flexible [2]. A future network architecture contains two groups: *primary networks* and *secondary networks*. Primary networks operate in the licensed spectrum bands while secondary networks operate in the spectrum bands bought (leased) from primary networks. Instead of trading (leasing) spectrum directly between multiple primary networks (sellers) and secondary networks (buyers), a spectrum broker (agent) may be included to play a role between these sellers and buyers [2]. In such a scheme, the agent buys spectrum from primary networks and resells them to secondary networks (users). An agent-based framework for (secondary) network operators likely to operate under the regulation of a spectrum policy server (SPS) is developed in [5]. The operators dynamically compete for customers as well as portions of available spectrum. This competition is formulated as a noncooperative game and an SPS-based iterative bidding scheme resulting in a Nash equilibrium scheme that is proposed and analyzed. Unlike [5], in our model agents face a greater challenge to maximize their profits, due to the secondary network randomness (e.g., the number of SUs).

III. SPECTRUM TRADING WITH A SINGLE AGENT

A. System Model

In this paper, our model is deployed in the CRN to mediate the secondary users' satisfaction and agents' profit. We assume that the average cost per unit spectrum for the agent is given by S, which may vary with time. SU's satisfaction is reflected by the acceptance probability $A(W,p)$, which decreases with per unit price p but increases with bandwidth W. In our model, the agent is allowed to adapt its price p to achieve an optimal strategy but the bandwidth of the service W is a predetermined constant for easy implementation.

1) Acceptance Probability: As indicated previously, SUs' satisfaction is reflected by accepting a uniform service (W is a constant) from an agent with a corresponding probability. From SU's point of view, the service of the agent is acceptable only if the price and service bandwidth are reasonable. So, we have to design an appropriate function of acceptance probability $A(W,p)$ reflecting SU's rational behavior.

Mathematically, $\forall p > 0$ and $W > 0$ this rationality can be formulated as:

$$\begin{aligned} \frac{\partial A(W,p)}{\partial W} &\geq 0, & \frac{\partial A(W,p)}{\partial p} &\leq 0 \\ \lim_{W\to\infty} A(W,p) &= \alpha, & \lim_{W\to 0} A(W,p) &= 0 \\ \lim_{p\to\infty} A(W,p) &= 0, & \lim_{p\to 0} A(W,p) &= \alpha \end{aligned} \tag{1}$$

In the above restrictions, $\alpha \leq 1$ is used to denote the secondary user's preference on the service or saturating probability to accept the service. Several candidates are available and we choose the exponential expression [5] [6]:

$$A(W,p) = \alpha e^{-pW^{-1}} \tag{2}$$

This function of acceptance probability appropriately reflects that the willingness of secondary users to accept service increases with the allocated bandwidth, saturates at α, and decreases with the price charged.

According to [5], $e^{-pW^{-1}}$ is similar to the Cobb-Douglas demand curves that are used in economics, thus we keep the exponential part unrelated to the difference of SUs. The only variable to differ the SUs is the coefficient α, we will use more variables to differ SUs in our future related work.

2) Demand Uncertainty: After defining the acceptance probability, we can formally derive the *demand function* $D(p)$ of the secondary users as (n denotes the number of SUs):

$$D(p) = \sum_{i=1}^{n} A_i(W,p)W \tag{3}$$

In our model, the randomness of the number of secondary users is assumed to be independent. The agent wholesale spectrum from the PUs that is then broadcast to the SUs, if the SUs want to buy the service but the bandwidth is soldout, they would have to wait for another round. Hence, the process is not continuous. As mentioned earlier, the natural randomness of the secondary network is considered in our model. For example, given the same service (W,p), SUs accept this service with different probabilities according to their own preferences (differed by α). On the other hand, given the same preferences (α), the number of secondary users that exists in the secondary network may vary in different trading periods. So, from the agent's point of view, the spectrum demand of secondary users will be *uncertain*, which means that the demand function $D(p)$ is formed by a deterministic function $d(p)$ as well as a random variable ε.

In the first case, we assume that the preferences of the SU uniformly equal α but the number of SUs n is uncertain, the spectrum demand function can be expressed in a multiplicative form:

$$D(p) = d(p)\varepsilon = \sum_{i=1}^{n} A_i(W,p)W = \alpha e^{-pW^{-1}} Wn \tag{4}$$

Where the deterministic part is $d(p) = \alpha e^{-pW^{-1}} W$ and the random part ε is n (Eq(4)).

In another case, we assume that the number of SUs N is constant, but SUs are heterogenous due to different preferences α_i. The SUs' preferences α_i are randomly distributed in the range of [0,1] (Eq(5)), then the function $D(p)$ can be expressed as:

$$\begin{aligned} D(p) &= \sum_{i=1}^{N} A_i(W,p)W = \sum_{i=1}^{N} \alpha_i e^{-pW^{-1}} W \\ &= e^{-pW^{-1}} W \sum_{i=1}^{N} \alpha_i \end{aligned} \tag{5}$$

Where the deterministic part is $d(p) = e^{-pW^{-1}}W$ and the random part ε is $\sum_{i=1}^{N} \alpha_i$ (Eq(5))

Our following analysis considers only on the form $D(p) = d(p)\varepsilon$, because both of the two different models can be expressed as this form.

3) Profit of the Agent: After deriving the general expression of the spectrum demand function including randomness, we will try to derive the profit of the agent. We first suppose that all spectrum bought (leased) from PUs with amount q and per unit (e.g., MHz) price S are all sold to SUs with price p, then the income will be $(p-S)q$. But due to the uncertainty of needed spectrum, the agent may end up selling all spectrum it bought from the PUs, or end up buying more than it is required. In these cases, the excess of spectrum can be expressed as $E[q - D(p)]^+$. $E[\cdot]$ denotes the mathematical expectation and $E[\cdot]^+ = max\{0, \cdot\}$. Subtract the income of this part spectrum, we can obtain the final expression of the agent's expected profit:

$$\Pi(q,p) = (p-S)q - pE[q - D(p)]^+ \tag{6}$$

Substituting $D(p) = d(p)\varepsilon$ into Eq(6), we can rewrite this equation as:

$$\begin{aligned} \Pi(q,p) &= (p-S)q - pE[q - d(p)\varepsilon]^+ \\ &= (p-S)q - p\int_0^{q/d(p)} [q - d(p)x]g(x)dx \\ &= (p-S)q - pd(p)\int_0^{q/d(p)} G(x)dx \end{aligned} \tag{7}$$

In the above equation, $g(x)$ and $G(x)$ denote the pdf and cdf of the random variable ε, respectively. This equation's concavity in q ensures that the expected profit will saturate with q, and the agent will not lease infinite bandwidth from PUs. It can be checked as follows:

$$\frac{\partial^2 \Pi(q,p)}{\partial q^2} = -Pg[q/d(p)]/d(p) < 0 \tag{8}$$

4) Spectrum Trading Process: This agent-based spectrum trading process is composed of three steps:

- **Step1**: The agent performs spectrum exploitation and obtain the idle spectrum q^* from PUs with average price S.
- **Step2**: The agent broadcasts its service information (W, p^*) in its service area.
- **Step3**: When SU receives the information, it accepts the service with probability $A(W, p^*)$.

B. General Solution

As mentioned earlier, the problem is to find the strategy of the agent to maximize its profit when it faces random spectrum demand. Mathematically, the problem can be expressed as:

$$(q^*, p^*) = \arg\max_{(q,p)} \ \Pi(q,p) \tag{9}$$

This problem can be solved by setting the first order derivative of Eq(6) to be zero. For the sake of analytical simplification, we transform parameter q to z by letting $z = q/d(p)$. Given (z^*, p^*), the optimal strategy (q^*, p^*) is equal to $(z^* d(p^*), p^*)$. So we can rewrite our problem as:

$$(z^*, p^*) = \arg\max_{(z,p)} \ \Pi(z,p) \tag{10}$$

Let $\theta(z) = \int_0^z G(x)dx$, we can also rewrite Eq.(7) and express the expected profit of the agent as finding:

$$\Pi(z,p) = (p-S)zd(p) - pd(p)\theta(z) \tag{11}$$

First, we let the partial derivative $\frac{\partial \Pi(z,p)}{\partial z} = 0$ and obtain the function of z^* in p as follows:

$$z^*(p) = G^{-1}(\frac{p-S}{p}) \tag{12}$$

Similarly, we let the partial derivative $\frac{\partial \Pi(z,p)}{\partial p} = 0$ and obtain the following equation:

$$[z - \theta(z)]d(p) + [(p-S)z - p\theta(z)]d'(p) = 0 \tag{13}$$

Substituting $d(p) = \alpha e^{-pW^{-1}}W$ or $d(p) = e^{-pW^{-1}}W$ and its derivative into the above equation, we obtain the function of p^* in z as follows:

$$p^*(z) = \frac{Sz}{z - \theta(z)} + W \tag{14}$$

Obviously, the closed form of (z^*, p^*) usually cannot be simply obtained by jointly solving Eq(12) and Eq(14). But, in general we can obtain this optimal strategy (z^*, p^*) by searching for the intersection points of Eq(12) and Eq(14).

First, we can find that both Eq(12) and Eq(14) are increasing functions by checking:

$$\frac{\partial p^*(z)}{\partial z} = \frac{S[zG(z) - \theta(z)]}{[z - \theta(z)]^2} \geq 0 \tag{15}$$

$$\frac{\partial z^*(p)}{\partial p} = \frac{1 - G(z)}{Pg(z)} = \frac{1}{Pr(z)} \geq 0 \tag{16}$$

Note that $\theta(z) \leq zG(z)$ and $0 \leq G(z) \leq 1$ naturally prove the above results and $r(z) = g(z)/[1 - G(z)]$. Furthermore, we can find the concavity of these two equations by checking:

$$\frac{\partial^2 p^*(z)}{\partial z^2} = \frac{S[\alpha'(z)\beta(z) - \alpha(z)\beta'(z)]}{\beta^2(z)} \tag{17}$$

$$\frac{\partial^2 z^*(p)}{\partial p^2} = \frac{-\frac{dr(z)}{dz}\frac{dz^*(p)}{dp}p - r(z)}{r^2(z)p^2} = \frac{-\frac{dr(z)}{r(z)dz} - r(z)}{r^2(z)p^2} \tag{18}$$

In Eq(17), we have $\alpha(z) = zG(z) - \theta(z)$ and $\beta(z) = z - \theta(z)$. Furthermore, Eq(18) implies that the concavity of $z^*(p)$ depends on the hazard rate function $r(z)$ of ε. In particular, for uniform, normal, logistic, chi-squared and exponential distributions, Eq(18) is less than zero, which shows that $z^*(p)$ is concave in p [15]. However, whether the optimal strategy is unique depends on the status of the random part ε. In the next section, we give two examples to prove the unique optimal strategy under the circumstance of uniform and normal distribution for ε.

C. Examples of the Uniform Distribution and Normal Distribution

In our model, we simplify the service to a constant bandwidth W. Thus, we first assume that secondary users are identical by using the uniform distribution. Then, we assume that secondary users are heterogenous by different preference α_i, which is normally distributed. In our future work, we plan to extend the heterogenous case with more different variables and forms.

1) Solution for Uniform Distribution: In the previous subsection, we derived a series of results depending on the distribution of the random part ε, i.e., the number of SUs n. In fact, the distribution of ε (or n) may be uniform, normal, or other distributions. But in this subsection, we only solve the problem of uniform distribution for illustration and reserve other cases for future work. We assume that the number of SUs is random but preferences of SUs are uniform. According to Eq(4), the random variable ε is exactly the number of SUs, i.e., n. Therefore we assume that n is uniformly distributed on $[A, B]$, where both A and B are positive integers. Thus, we can derive the following expressions in this case:

$$g(z) = \frac{1}{B-A}, G(z) = \frac{z-A}{B-A} \text{ and } \theta(z) = \frac{(z-A)^2}{2(B-A)} \quad (19)$$

Substituting Eq(19) into Eq(12) and Eq(14), we can find the optimal strategy of the agent:

$$\begin{cases} p^*(z) = \dfrac{2Sz(B-A)}{2zB - z^2 - A^2} + W, \quad A \le z \le B \\ z^*(p) = B - \dfrac{B-A}{p} S, \quad S + W \le p \le \dfrac{2SB}{B+A} + W \end{cases} \quad (20)$$

As indicated by Eq(16) and Eq(15), these two equations are increasing in Z and P, respectively. For further analysis, we unify the variables in Eq(20) to be z, and rewrite them as:

$$\begin{cases} p^*(z) = \dfrac{2Sz(B-A)}{2zB - z^2 - A^2} + W \\ p(z^*) = \dfrac{B-A}{B-z^*} S \end{cases} \quad (21)$$

So the convexity of the above two equations in z can be found by:

$$\begin{cases} \dfrac{\partial^2 p^*(z)}{\partial z^2} = \dfrac{4S(B-A)[z(z^2-A^2) + 2A^2(B-z)]}{(2zB - z^2 - A^2)^3} \ge 0 \\ \dfrac{\partial^2 p(z^*)}{\partial (z^*)^2} = \dfrac{2S(B-A)(B-z)}{(B-z)^4} \ge 0 \end{cases} \quad (22)$$

Therefore, we can plot these two curves as shown in Fig.2. It is obvious that at least one intersection exists in the feasible area defined by Eq(20). But, in fact we can prove that this optimal strategy is unique and it can be summarized as a theorem.

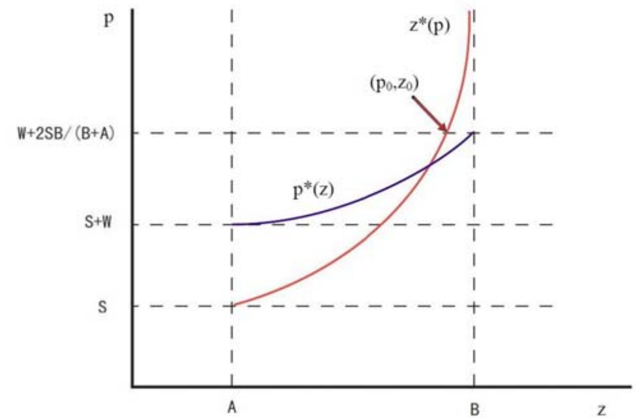

Fig. 2. Intersection Point of $p^*(z)$ and $z^*(p)$ is Optimal Strategy (z^*, p^*) of uniform distribution.

Algorithm 1: Search for an intersection point

Input: p_0 and δ
Output: p^* and z^*
$k = 0;\ p_k = p_0;$
while do
  k++;
  Calculate $z_k = z^*(p_{k-1})$ by equation (12);
  Calculate $p_k = z^*(p_{k-1})$ by equation (14);
  Calculate $\Pi_k = \Pi(p_k, z_k)$ by equation (11);
  if $k > 1$ && $|\Pi_k - \Pi_{k-1}| < \delta$ then
    break;
  end
end
return (p_k, z_k);

Fig. 3. Algorithm 1 to search for an intersection point.

***Theorem** 1: If the demand function of secondary users can be expressed as $D(p) = d(p)\varepsilon$, where ε is a random variable uniformly distributed on [A,B]. Then the agent has a unique optimal strategy $(q^*, p^*) = (z^* d(p^*), p^*)$ defined by the intersection point of Eq(12) and Eq(14).*

Proof: Because we have: $(2zB - z^2 - A^2)^2 = [2(B - z) + zB - A^2]^2 \ge [z(B-z) + zB - z^2]^2 = 4z^2(B-z)^2 \ge 2(z^2 - A^2)(B-z)^2$, we can show that:

$$\frac{\partial p(z^*)}{\partial z^*} = \frac{(B-A)S}{(B-z)^2} \ge \frac{\partial p^*(z)}{\partial z} = \frac{2S(B-A)(z^2-A^2)}{(2zB - z^2 - A^2)^2} \quad (23)$$

Considering the fact that $p^*(z)$ connects p^0 and p^1, this inequality ensures that there is a unique intersection point of $p^*(z)$ and $z^*(p)$. ■

2) Algorithm: We have shown that there is a unique optimal solution $(q^*, p^*) = (z^* d(p^*), p^*)$. In practice, we provide an iterative algorithm for each agent to find this optimal strategy. This algorithm is shown in pseudo-code in Fig.3., where δ represents the precision used as a stopping criterion and k is the iteration times. For this algorithm, we show its validity by Theorem 2 and its proof as discussed below:

***Theorem** 2: Starting from $p_0 = p^*(B) = W + 2SB/(B + A)$, Algorithm (Fig.3.) converges to an optimal solution*

(z^*, p^*).

Proof: First, we observe the sequences of p_k and z_k:

$$p_0 = p^*(B),\ p_1 = p^*(z_0),\ p_2 = p^*(z_1),\ p_3 = p^*(z_2)...$$
$$z_0 = z^*(p_0),\ z_1 = z^*(p_1),\ z_2 = z^*(p_2),\ z_3 = z^*(p_3)...$$

It is shown in Fig.2. that $p_0 \geq p^*$ and $z^*(p)$ is an increasing function. Because $z^* = z^*(p^*)$ we can obtain that $B \geq z_0 \geq z^*$. Since $p^*(z)$ is also an increasing function, we can similarly obtain that $p_0 \geq p_1 \geq p^*$. Continuing this process, we will have:

$$p_0 \geq p_1 \geq p_2 \geq ... \geq p^* \text{ and } z_0 \geq z_1 \geq z_2 \geq ... \geq z^*$$

This result indicates that both p_k and z_k are decreasing and lower bounded, thus they must finally converge to (z^*, p^*). ■

3) Solution for Normal Distribution: In the normal distribution example, we assume that the number of SUs N is constant but the preferences of SUs α_i are random. We assume that α_i is normally distributed on [0,1]. Then, we have the random variable ε be $\sum_{i=1}^{N} \alpha_i$, which is normally distributed on [0,N], according to Eq (5)). Therefore, we can derive the following expressions in this case:

$$\begin{cases} g(z) = \dfrac{1}{\sqrt{2\pi\sigma^2}} e^{\frac{-(z-\mu)^2}{2\sigma^2}} \\ G(z) = \dfrac{1}{2}[1 + erf\dfrac{z-\mu}{\sqrt{2\sigma^2}}] \\ \theta(z) = \displaystyle\int_0^z G(x)dx \end{cases} \tag{24}$$

Substituting Eq(24) into Eq(12) and Eq(14)) and unify the variables as the former case, we can find the optimal strategy of the agent:

$$\begin{cases} p^*(z) = \dfrac{Sz}{z - \frac{1}{2}\int_0^z [1 + erf(\frac{x-\mu}{\sqrt{2\sigma^2}})]dx} + W, \quad 0 \leq z \leq N \\ p(z^*) = \dfrac{2S}{1 - erf(\frac{z-\mu}{\sqrt{2\sigma^2}})}, \quad S + W \leq p \leq \dfrac{2S}{1 - erf(\frac{N-\mu}{\sqrt{2\sigma^2}})} \end{cases} \tag{25}$$

From the two curves shown in Fig.4., it is obvious that at least one intersection exits in the feasible area defined by Eq (25). In fact, this optimal strategy is unique and it can be summarized as a theorem.

***Theorem** 3: If the demand function of secondary users can be expressed as $D(p) = d(p)\varepsilon$, where ε is a random variable normally distributed on [0,N]. Then the agent has a unique optimal strategy $(q^*, p^*) = (z^*d(p^*), p^*)$ defined by the intersection point of Eq(12) and Eq(14) if the agent has optimal strategies within the range.*

Proof: Because $\frac{d}{dz}erf(z) = \frac{2}{\sqrt{\pi}}e^{-z^2} > 0$, the error function here is an increasing function. We can show that:

$$\frac{\partial p(z^*)}{\partial z^*} = \frac{4S}{\sqrt{2\pi\sigma^2}(1 - erf(\frac{x-\mu}{\sqrt{2\sigma^2}}))^2} e^{-(\frac{x-\mu}{\sqrt{2\sigma^2}})^2} > 0 \tag{26}$$

Thus, $p(z^*)$ is an increasing function. Then we can prove that $p^*(z)$ is an increasing function as well.

We rewrite $p^*(z)$ into:

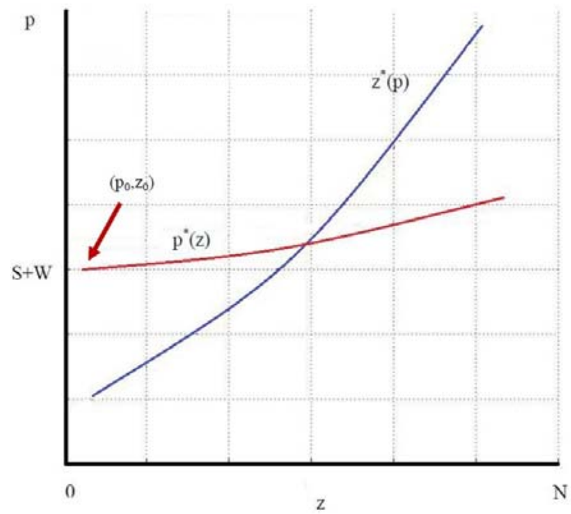

Fig. 4. Intersection Point of $p^*(z)$ and $z^*(p)$ is Optimal Strategy (z^*, p^*) of Normal Distribution.

$$p^*(z) = \frac{S}{1 - \frac{1}{2z}\int_0^z [1 + erf(\frac{x-\mu}{\sqrt{2\sigma^2}})]dx} + W \tag{27}$$

Let $\phi(z) = \frac{1}{z}\int_0^z [1 + erf(\frac{x-\mu}{\sqrt{2\sigma^2}})]dx$, if $\phi(z)$ is an increasing function, then $p^*(z)$ is an increasing function.

We have:

$$\begin{aligned} \frac{\partial \phi(z)}{\partial z} &= \frac{[1 + erf(\frac{z-\mu}{\sqrt{2\sigma^2}})]z - \int_0^z [1 + erf(\frac{x-\mu}{\sqrt{2\sigma^2}})]dx}{z^2} \\ &= \frac{z \times erf(\frac{z-\mu}{\sqrt{2\sigma^2}}) - \int_0^z erf(\frac{z-\mu}{\sqrt{2\sigma^2}})dx}{z^2} \end{aligned} \tag{28}$$

Let $\xi(z) = z \times erf(\frac{z-\mu}{\sqrt{2\sigma^2}}) - \int_0^z erf(\frac{z-\mu}{\sqrt{2\sigma^2}})dx$, if $\xi(z) > 0$, then $\frac{\partial \phi(z)}{\partial z} > 0$.

$$\begin{aligned} \frac{\partial \xi(z)}{\partial z} &= erf(\frac{z-\mu}{\sqrt{2\sigma^2}}) + \frac{2z}{\sqrt{2\pi\sigma^2}} e^{-(\frac{x-\mu}{\sqrt{2\sigma^2}})^2} - erf(\frac{z-\mu}{\sqrt{2\sigma^2}}) \\ &= \frac{2z}{\sqrt{2\pi\sigma^2}} e^{-(\frac{x-\mu}{\sqrt{2\sigma^2}})^2} > 0 \end{aligned} \tag{29}$$

So, $\xi(z)$ is an increasing function. Obviously, when z approaches 0, $\xi(z)$ approaches 0 as well. Thus, $\xi(z) > 0$, then we have $\frac{\partial \phi(z)}{\partial z} > 0$. We then have $\phi(z)$ is an increasing function, and therefore, $p^*(z)$ is an increasing function.

We rewrite $p^*(z)$ as:

$$\begin{aligned} p^*(z) &= \frac{s}{1 - \frac{1}{2z}\int_0^z [1 + erf(\frac{x-\mu}{\sqrt{2\sigma^2}})]dx} + W \\ &= \frac{2sz}{z - \int_0^z erf(\frac{x-\mu}{\sqrt{2\sigma^2}})dx} + W \end{aligned} \tag{30}$$

Then we do the subtraction as follows:

$$
\begin{aligned}
p(z^*) - p^*(z) &= \frac{2s}{1 - erf(\frac{x-\mu}{\sqrt{2\sigma^2}})} - \frac{2sz}{z - \int_0^z erf(\frac{x-\mu}{\sqrt{2\sigma^2}})dx} - W \\
&= \frac{z \times erf(\frac{x-\mu}{\sqrt{2\sigma^2}}) - \int_0^z erf(\frac{x-\mu}{\sqrt{2\sigma^2}})dx}{[1 - erf(\frac{x-\mu}{\sqrt{2\sigma^2}})][z - \int_0^z erf(\frac{x-\mu}{\sqrt{2\sigma^2}})dx]} - W \\
&= \frac{\int_0^z \frac{2x}{\sqrt{\pi}} e^{-x^2} dx}{[1 - erf(\frac{x-\mu}{\sqrt{2\sigma^2}})][z - \int_0^z erf(\frac{x-\mu}{\sqrt{2\sigma^2}})dx]} - W
\end{aligned} \tag{31}
$$

Obviously, when z approaches 0, Eq (31)) approaches $-W < 0$, when z approaches to positive infinity, Eq (31)) approaches to positive infinity as well, thus, Eq (31)) has one zero point.

Considering the fact that $p^*(z)$ and $z^*(p)$ are both increasing functions, Eq (31)) ensures that there is a unique intersection point of $p^*(z)$ and $z^*(p)$ if the intersection can be approached. ■

4) Algorithm: The iterative algorithm for each agent to find this optimal strategy in this normal distributed model is the same as the one in the uniform distributed model. This algorithm is shown in pseudo-code in Fig.3., where δ represents the precision used as a stopping criterion and k is the iteration times.

Similar to theorem 2, we present another theorem for this model.

***Theorem** 4: Starting from $p_0 = p^*(0) = S+W$, Algorithm (Fig.3.) converges to an optimal solution (z^*, p^*).*

Proof: First, we observe the sequences of p_k and z_k:

$$p_0 = p^*(0),\ p_1 = p^*(z_0),\ p_2 = p^*(z_1),\ p_3 = p^*(z_2)...$$

$$z_0 = z^*(p_0),\ z_1 = z^*(p_1),\ z_2 = z^*(p_2),\ z_3 = z^*(p_3)...$$

It is shown in Fig.4. that $p_0 \leq p^*$ and $z^*(p)$ is an increasing function. Because $z^* = z^*(p^*)$ we can obtain that $0 \leq z_0 \leq z^*$. Since $p^*(z)$ is also an increasing function, we can similarly obtain that $p_0 \leq p_1 \leq p^*$. Continuing this process, we will have:

$$p_0 \leq p_1 \leq p_2 \leq ... \leq p^* \text{ and } z_0 \leq z_1 \leq z_2 \leq ... \leq z^*$$

This result indicates that both p_k and z_k are increasing and z_k increases faster than p_k, thus they must finally converge to (z^*, p^*). ■

D. Summary

At the end of this section, we conclude that the demand function of secondary users can be expressed by $D(p) = d(p)\varepsilon$, where ε represents the agent's uncertainty in its customers' demand. Given the probability distribution function $g(\cdot)$, general solutions are given in Eq(12) and Eq(14).

Next, we consider the case of spectrum trading with multiple agents.

IV. Spectrum Trading with Multiple Agents

A. System Model

In this section, we consider a system in which more than one agent are competing for customers (SUs). We assume

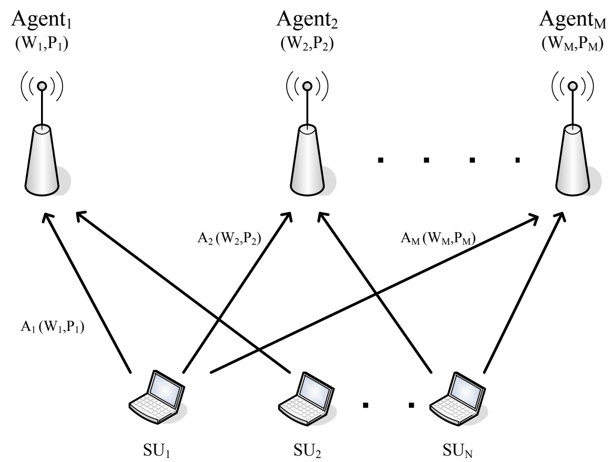

Fig. 5. General Model of Multiple Agents.

that M agents are providing services to n SUs, where M is constant while n is uncertain. Receiving a service offer (w_i, p_i), SU's willingness to accept this offer is still modeled by the *Acceptance Probability* $A_i(w_i, p_i)$. This concept is naturally extended from the previous section where a single-agent system is considered, except for the subscript i denoting different agents. The general model of this multi-agent system is illustrated in Fig.5. Open markets enable SUs to make options among different agents, but definitely complicates our problem. Indeed, the spectrum demand of agent$_i$'s customers is determined by not only its own service (w_i, p_i), but also other agents' services: (w_j, p_j), $\forall j \neq i$.

1) Acceptance Probability: In order to quantify the spectrum demand, again we have to define the *acceptance probability*. Unlike the single-agent system, we have to consider M agents this time. So, we denote the prices charged and bandwidth provided by two vectors: $\mathbb{P} = \langle p_1, p_2 ... p_M \rangle$ and $\mathbb{W} = \langle w_1, w_2 ... w_M \rangle$. Then, the probability of SU$_j$ to accept the service from agent$_i$ can be defined as:

$$A_i^j(\mathbb{W}, \mathbb{P}) = \alpha_j e^{-p_i/w_i} \prod_{k \neq i}^{M} e^{-w_k/p_k} \tag{32}$$

In Eq(32), $\alpha_j \leq 1$ is used to denote the preference of SU$_j$ to access the spectrum via an agent. We adopt this expression in multi-agent model, because the first part $\alpha_j e^{-p_i/w_i}$ satisfies the economic properties defined in Eq(1) and represents SU$_j$'s rationality in spectrum decisions. The second part, $\prod_{k \neq i}^{M} e^{-w_k/p_k}$ represents the influences of agent$_i$'s competitors' presences on SU$_j$'s decision. Note that this influence diminishes when all competing services' price-bandwidth ratios $\phi_k = p_k/w_k$ become infinitely high, i.e., $\prod_{k \neq i}^{M} e^{-w_k/p_k}$ approaches one and Eq(32) becomes Eq(2). The acceptance probability of each SU to access the spectrum via agent$_i$ decreases with agent$_i$'s price-bandwidth ratio ϕ_i and increases with agent$_k$'s price-bandwidth ratio ϕ_k for $k \neq i$. These properties can be expressed as:

$$\frac{\partial A_i^j(\mathbb{W}, \mathbb{P})}{\partial p_i} \leq 0, \quad \frac{\partial A_i^j(\mathbb{W}, \mathbb{P})}{\partial p_k} \geq 0 \tag{33}$$

2) Spectrum Demand: After defining secondary users' willingness to accept the service, we can derive the spectrum demand function for each agent. In order to simplify the problem, we assume that agent$_i$ provides service with constant bandwidth w_i, and with variable price p_i to attract customers (SUs). If we assume that there is a total of n secondary users in the network, the total spectrum demand for agent$_i$ can be expressed as:

$$D_i(\mathbb{P}) = \sum_{j}^{n} A_i^j(\mathbb{P}) w_i \tag{34}$$

In this paper, we consider secondary users with a uniform saturation acceptance probability, i.e., $\alpha_j = \alpha$ and $A_i^j(\cdot) = A_i(\cdot)$ for all j. So the spectrum demand can be simplified as:

$$D_i(\mathbb{P}) = d_i(\mathbb{P}) n = A_i(\mathbb{P}) w_i n \tag{35}$$

3) Profit of the Agent: In this multiple agents' system, we still consider the randomness of n. To keep our expressions general enough, we denote this random number of SUs as ε instead of n. In order to obtain the optimal strategies of the agents, we have to derive the profit of the agent again. Unlike the simpler case where only one agent dominates the secondary market, the profits of agents are no longer independent. Similar to the single-agent system, the strategies of each agent still consist of two elements: price p and spectrum quantity q. However, agent$_i$'s profit is only influenced by its competitors' strategy of price but not the strategy of spectrum quantity. So the formal definition of the agent$_i$'s profit can be expressed as:

$$\Pi_i(q_i, \mathbb{P}) = (p_i - s_i) q_i - p_i E[q_i - D_i(\mathbb{P})]^+ \tag{36}$$

4) Nash Equilibrium of Agent Strategies: Unlike a system of *noncompeting agents*, we address the problem of finding the Nash equilibrium of strategies if each agent$_i$ has obtained the distribution of ε based on its observations. As mentioned earlier, the expected profit of agent$_i$ does not depend on its competitors' decisions on q, which is the quantity of spectrum bought from PUs. So, we let $\Pi_i(q_i, \mathbb{P})$'s partial derivative on q_i equal to 0 and get the optimal q_i under a given price vector as:

$$q_i^*(\mathbb{P}) = d_i(\mathbb{P}) G^{-1}(r_i) \tag{37}$$

where $r_i = (p_i - s_i)/p_i$. Eq(37) shows that for a given optimal price vector $\mathbb{P}^*$, we can obtain the optimal bandwidth quantity q_i^* agent$_i$ should buy from the PUs. Thus, we can reduce the *original problem* of finding (q_i^*, p_i^*) to a *reduced problem* of finding p_i^*. So the Nash equilibrium of strategies of this game where agents are competing for secondary users can be defined as finding the price vector $\mathbb{P}^*$ such that for each agent$_i$:

$$\Pi_i(\imath^*) = \Pi_i(p_i^*, p_{-i}^*) \geq \Pi_i(p_i, p_{-i}^*) \tag{38}$$

where p_{-i} represents the price vector of all agent$_i$'s competitors. Indeed, substituting Eq(37) into Eq(36), we can simplify the expression of the expected profit as:

$$\begin{aligned} \Pi_i(\imath) &= (p_i - s_i) d_i(\imath) G^{-1}(r_i) \\ &\quad - p_i E[d_i(\imath) G^{-1}(r_i) - d_i(\mathbb{P}) N]^+ \\ &= (p_i - s_i) d_i(\imath) \frac{1}{r_i} \int_0^{G^{-1}(r_i)} x g(x) dx \\ &= \Pi_i^{det}(\imath) * L_i^{rnd}(r_i) \end{aligned} \tag{39}$$

In Eq(39), the first term: $\Pi_i^{det}(\imath) = (p_i - s_i) d_i(\imath)$ is independent of the random variable n and represents the deterministic part of the profit. The second term: $L_i^{rnd}(r_i) = \frac{1}{r_i} \int_0^{G^{-1}(r_i)} x g(x) dx \leq E[n]$ represents the random part of the profit due to the uncertainty of n.

B. Existence of the Nash Equilibrium

In order to find a Nash equilibrium of *competing agents'* strategies (see Eq.38), we first introduce the concept of *Supermodular Game* [15][16], which is better known as games with strategic complementarities. Compared to the Bayesain non-cooperative game, the supermodular game needs no more assumptions in our model, which will render the system more applicable. Moreover, supermodular reduces the complexity of the mathematical work to find the NE convergence since we consider only the increasing or reducing of the variable (prices charged by agents). Moreover, by using the supermodular game we avoid multiple-equilibrium case (see Section IV, Part C). To be specific, when we try to optimize multiple endogenous variable (prices charged by agents), then all of them are complements if their increase are mutually reinforced.

Increasing difference: Function $f(x,t) : X \times T \to \mathbb{R}$ has an *increasing difference* in (x,t), if $f(x',t') - f(x,t') \geq f(x',t) - f(x,t)$ or $\frac{\partial^2 f(x,t)}{\partial x \partial t} \geq 0$, for all $x' \geq x$ and $t' \geq t$.

Supermodular Game: Each player i and its competitors' strategy set p_i and $p_{-i} \in \mathbb{R}$, and payoff $\Pi_i(p_i, p_{-i})$ has an *increasing difference* in (p_i, p_{-i}), i.e., incremental payoff to choose a higher strategy is increasing with competitors' strategies.

In fact, our reduced game is not supermodular but we can prove that it is log-supermodular, that is, the log payoff is supermodular [16]. Although log-supermodularity is essentially weaker than supermodularity, it is enough to ensure the existence of Nash equilibrium in our reduced game. Before presenting the theorem, we first introduce two conditions:

Condition A: *For each i=1,...,M, the function* $\log_2 d_i(\mathbb{P})$ *has increasing differences in* (p_i, p_k) *for all* $k \neq i$.)

Condition B: *Each agent chooses its price* p_i *from a closed interval* $[p_i^{min}, p_i^{max}]$.

In fact, it is easily verified that Condition A is satisfied and it is essentially weaker than the requirement that $d_i(\mathbb{P})$ has an increasing difference in (p_i, p_k), for $k \neq i$. Condition B is used to bound the intervals where the algorithm searches for an equilibrium. Giving these two conditions, we can present the following theorem ensuring the existence of the Nash equilibrium:

Theorem 5: *If Conditions A and B are satisfied, then we have the following results:*

(a) Nash equilibrium $\mathbb{P}^*$ *for the reduced game exists, and its corresponding strategy* $(\mathbb{P}^*, q(\mathbb{P}^*))$ *is a Nash equilibrium for the original game.*

(b) If multiple Nash equilibria exist, there is a smallest and largest equilibrium $\overline{\mathbb{P}^*}$ *and* $\underline{\mathbb{P}^*}$*, respectively.*

(c) The equilibrium $\overline{\mathbb{P}^*}$ *is preferred by all agents.*

Proof: To prove parts (a) and (b), we only need to show that the reduced game is a log-supermodular that is $\log \Pi_i(p_i, p_{-i})$ has an increasing difference in (p_i, p_{-i}) (see Milgrom and Roberts [16]). Since $\Pi_i(\mathbb{P}) = \Pi_i^{det}(\mathbb{P}) L_i^{rnd}(r_i)$, $\log \Pi_i(\mathbb{P}) = \log d_i(\mathbb{P}) + \log(p_i - s_i) + \log L_i^{rnd}(r_i)$, the required log-supermodularity follows from Condition A and the fact that the second and third terms only depend on p_i. To prove (c), we should notice that $\Pi_i(p_i, p_{-i})$ increases with p_{-i}, then $\forall \mathbb{P}^* \leq \overline{\mathbb{P}^*}$, $\Pi_i(p_i, p_{-i}) \leq \Pi_i(p_i, \overline{p_{-i}})$. Consider the fact that $\overline{\mathbb{P}^*}$ is a Nash equilibrium, we obtain that $\Pi_i(p_i, \overline{p_{-i}}) \leq \Pi_i(\overline{p_i}, \overline{p_{-i}})$. Thus, $\Pi_i(p_i, p_{-i}) \leq \Pi_i(\overline{p_i}, \overline{p_{-i}})$, that is, the largest equilibrium makes more profits for agents than other equilibria. ■

C. Algorithm

As proved earlier, the price strategies of competing agents have log-supermodular properties so that $\overline{\mathbb{P}^*}$ can be computed easily by a well-known scheme [17]: Starting with an arbitrary price vector $\mathbb{P}^0 \geq \overline{\mathbb{P}^*}$, e.g, $\mathbb{P}^0 = \mathbb{P}^{max} = (p_1^{max}, ..., p_M^{max})$. In the k_{th} iteration $\mathbb{P}^k$ is obtained from $\mathbb{P}^{k-1}$ by $p_i^k = \arg\max_{p_i} \Pi_i(p_i, p_{-i}^{k-1})$, then the sequence $\{\mathbb{P}^k, k > 0\}$ converges to $\overline{\mathbb{P}^*}$. Based on this log-supermodular game model, fortunately we can avoid multiple-equilibrium case by setting the initial price vector to $\mathbb{P}^{max}$, and it converges to the greatest Nash equilibrium $\overline{\mathbb{P}^*}$ which is preferred by all agents.

V. Simulation Results

In this section, we first use network simulations to evaluate the performance of our algorithm, and study the impact of demand uncertainty in our single-agent model. After which, we carry out the simulations to evaluate the performance of our spectrum trading model with more than one agent.

A. Simulation Results of Single-Agent Systems

1) Simulation Results of Uniform Distribution: First, we implement the iterative algorithm to find the optimal strategy of a single agent. Simulation parameters are set as: $S = 10$ \$/MHz, $W = 10$ MHz, $\delta = 0.01$, and n is uniformly distributed in the range of $[10, 90]$. We assume that the SUs' have uniform preference $\alpha = 1$.

We simulate our algorithm to find the optimal strategy (q^*, p^*) of the agent, and show its converging path with expected profit contour in Fig. 6. The results show that the optimal price $p^* = 23.029$ \$/MHz and the optimal quantity of the idle spectrum agent should lease $q^* = 55.245$ MHz bandwidth. The expected profit of this strategy $\Pi(q^*, p^*) = 425.010$ \$ and the iteration time is $k = 8$.

Although we have shown that this algorithm can achieve an optimal expected profit, the realized profit is usually less than the maximum achievable profit. According to microeconomics theory, the maximum achievable profit can be achieved if the agent can exactly predict the number of SUs in each trading period. We define the ratio of realized profit over the maximum achievable profit as "optimality" and the gap between them as "optimality gap".

We quantify the uncertainty by the variance of the uniform distribution $g(x)$. So we can study its impact by simulation.

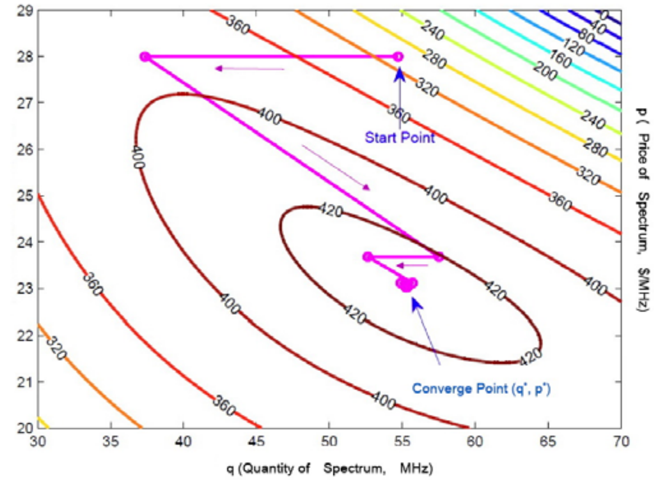

Fig. 6. Path Convergence of Algorithm 1 and Expected Profit Contour.

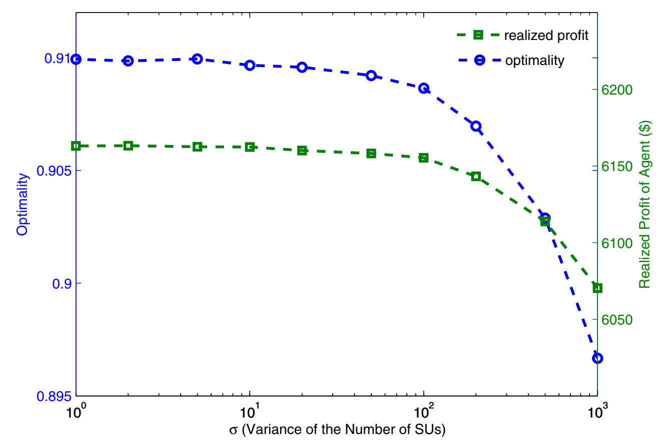

Fig. 7. Impact of Demand Uncertainty.

For the fairness, we fix the mean value of n as $n_{AVR} = 500$ and simulate using variance σ from 1 to 1000, and each point is averaged by 10^6 rounds.

Fig.7. shows the impact of demand uncertainty on the strategies' optimality and realized profit. Hence, we come up with three interesting results:

- The optimality and realized profit drop with the uncertainty of spectrum demand (variance of σ).
- Even in the most uncertain case (variance equals 10^3), the optimality is still above 89%.
- The optimality and realized profit saturate when the variance $\sigma < 10$.

In order to study the impact of this uncertainty further, we present the optimal strategies as a function of variance σ in Fig. 8. The network setup is similar to that shown in Fig. 7., and we can find an interesting result: the optimal price of spectrum increases when the number of SUs is more unpredictable, while the optimal quantity of spectrum decreases. Intuitively, it shows that when the market becomes unpredictable, the agent prefers higher profit per unit product (bandwidth) to higher sales.

In Fig.9., we present the realized profit of the agent as a function of the average number of secondary users (n_{AVR}).

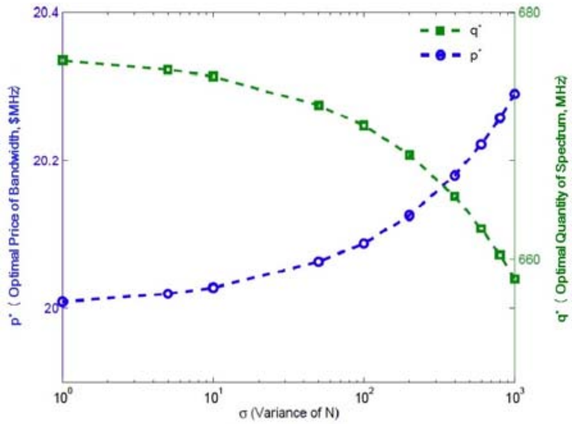

Fig. 8. Impact of Demand Uncertainty.

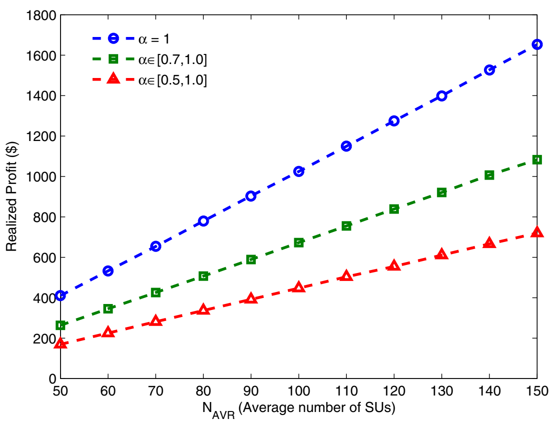

Fig. 9. Impact of Demand Uncertainty.

We plot three curves with α=1, α uniformly selected from [0.7,1.0] and [0.5,1.0], respectively. As expected, the agent's profit increases linearly with the mean value of n. We can also see that the randomness of α again decreases the profit of the agent.

2) Simulation Results of Normal Distribution: Second, we implement the iterative algorithm to find the optimal strategy of a single agent when the random part is normally distributed. Simulation parameters are set as: $S = 10$ \$/MHz, $W = 10$ MHz, $\delta = 0.001$, and α_i is normally distributed in the range of $[0, 1]$. We assume that the number of SUs is N.

We simulate our algorithm to find the optimal strategy (q^*, p^*) of the agent, the results show that the optimal price $p^* = 20.239$ \$/MHz and the optimal quantity of the idle spectrum agent should lease $q^* = 72.738$ MHz bandwidth. The expected profit of this strategy $\Pi(q^*, p^*) = 710.400$ \$ and the iteration time is $k = 3$.

B. Simulation Results of Multi-Agent System

In Fig.10., we simulate the spectrum trading process with two agents, i.e., $M = 2$. We assume that the number of secondary users is uniformly distributed in the range of [100, 200], and their preferences are all set to be one unit. To distinguish between the services of two agents, we assume that agent$_1$ provides bandwidth w_1=4 MHz with per unit cost

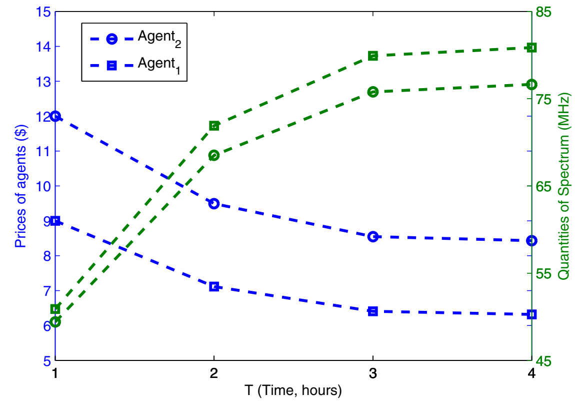

Fig. 10. Optimal Strategies of Two *Competing* Agents.

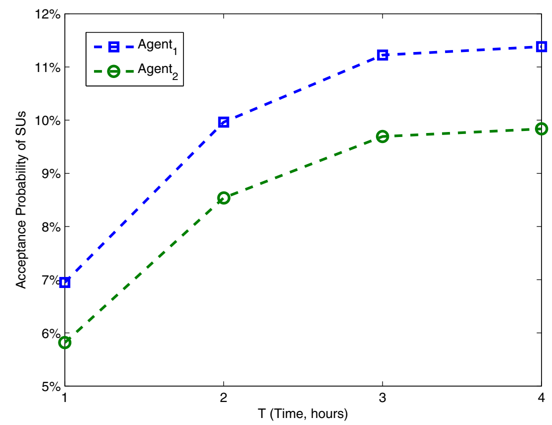

Fig. 11. Acceptance Probabilities of Two Agents.

2 \$/MHz while agent$_2$ provides bandwidth w$_2$=5 MHz with per unit cost 3 \$/MHz. To search for the optimal strategies, we start the algorithm with an initial price vector $\mathbb{P}$=[9 \$/MHz, 12 \$/MHz].

Results shown in Fig.10. indicate that strategies of two agents converge to different sets. In equilibrium, agent$_1$ provides service with lower price but it leases more spectrum from PUs. This result is due to the fact that its lower price attracts more secondary users to select service from: in equilibrium the acceptance probability of secondary users for agent$_1$ is 11.38% while that for agent$_2$ is only 9.84%, which are shown in Fig.11. In this figure, we can also observe that the acceptance probabilities of secondary users are always increasing, which implies that strategies of two agents are all converging to optimal.

In order to study the impact of the competition, we compare the realized profits of two agents with competition and without competition. The number of secondary users is still uniformly generated in [100, 200], and their preferences are all set to be one unit. Agent$_1$ provides bandwidth w$_d$1=4 MHz with per unit cost 2 \$/MHz while agent$_2$ provides bandwidth w$_2$=5 MHz with per unit cost 3 \$/MHz. In the competing scenario, we still start the algorithm with an initial price vector $\mathbb{P}$=[9 \$/MHz, 12 \$/MHz]. Then, we simulate each of these two different scenarios for 10^4 spectrum trading periods (each

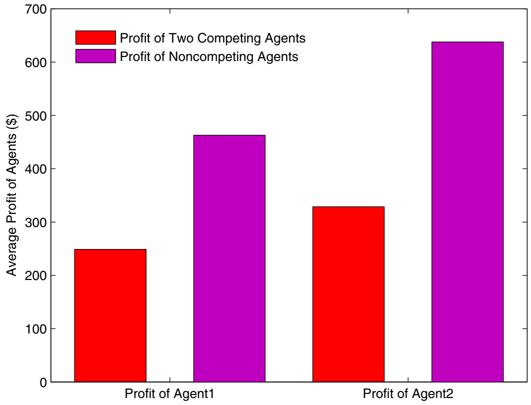

Fig. 12. Profit Reduction due to Competition.

period is one hour), and plot the average profits of $agent_1$ and $agent_2$ in Fig.12. We observe that profits made by two competing agents are reduced compared to the profits made if they are operating independently.

We separate single-agent and multi-agent because the single-agent model scenario may exist for a small group in a restricted area in which one agent will be enough to sustain the system. In such a case, multiple agents may reduce the profits but not provide better service than the single-agent case. It may also undermine the incentives for someone to volunteer serving as the agent(s) in the system. However and under some certain circumstances (such as when one agent cannot sustain the system), multiple agents model can then be considered.

VI. Conclusion and Future Work

In this paper, we proposed an agent-based model for spectrum trading in a cognitive radio network. The optimal strategy of the agent is obtained under conditions where the spectrum demand of secondary users is "uncertain".

For a single-agent system, our solution is proved optimal and demand uncertainty decreases the agent's profit. It is interesting that when the market becomes more unpredictable, the agent prefers higher profit of per unit product (bandwidth W) to higher sales. On the other hand when multiple agents operate, we can obtain the equilibrium of agents' strategies. However, profits made by two competing agents are reduced compared to the profits made when they are operating independently.

We also tried to derive the optimal solutions for agents in an uncertain market. We gave general solutions, but detailed analysis was given only under the Uniform Distribution. We are in the process of carrying similar analysis under other distributions that will be reported in the future. We also plan to investigate cases where we will consider the price paid for the spectrum from a primary user and the channel quality, where either one or both could be random. We believe that our model may be further studied in the framework of IEEE 802.22 network.

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