

A Geometry Study on the Capacity of Wireless Networks Via Percolation

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Abstract—We study the effect of various geometries on the capacity of wireless networks via percolation, which was not considered much before. Percolation theory was first applied to derive an achievable rate $\frac{1}{\sqrt{n}}$ in [5] by constructing a highway system, in contrast with the previous result $\Theta(\frac{1}{\sqrt{n \log n}})$ in [6], where n is the number of the nodes. While a highway system that consists of both horizontal and vertical edge-disjoint paths exists in a square network, B. Liu et al. in [1] pointed out that the horizontal paths will disappear if the width of a strip network is increasing more slowly than $\log n$. In this paper, first we take a deeper look at the percolation in a strip network. We discover that when a highway system exists, the capacity is restricted by the maximum length of the sides. Moreover, a sub-highway system is still in presence when the highway system disappears. Secondly, we consider the situations in a triangle network. Conditions that percolation highway exists in it, and the achievable rate for a triangle network are discussed. We find that corner effect can be a bottleneck of the capacity. By combining the achievable rate of the former networks, we attribute the variance between them to their symmetry discrepancies. Finally, we turn to the capacity of three dimensional (3D) networks via percolation. The whole study shows that geometric symmetry plays a significant role in the percolation and the capacity, thereby shedding a light on the network design and the scheduling.

Index Terms—Capacity, geometry, symmetry, percolation theory.

I. INTRODUCTION

A fundamental issue about wireless ad hoc network is that how much information it is capable of conveying, or, the network capacity. In the inaugurating work proposed by Gupta and Kumar [6]), they studied the capacity of such network scaling with the number of nodes in a fixed area. The results of [6] showed that the transport capacity scales as $\Theta(\sqrt{An})$ for arbitrary networks, where A is the area of the region and n is the number of the nodes. Or equivalently, the largest amount of information that can be transferred from each source to destination is $\Theta(\frac{1}{\sqrt{n}})$. Besides, they also showed that the throughput capacity of random networks scales as $\Theta(\frac{1}{\sqrt{n \log n}})$,

which is achieved under a *direct routing strategy* - forwarding the packets along the line connecting the source and the destination. Notice that there is a gap on the capacity between random networks and arbitrary networks. This was believed due to randomness of the node locations and the source-destination selection. However, the upper bound obtained in [18] by information theory permits an attainable rate higher than $\Theta(\frac{1}{\sqrt{n \log n}})$.

To increase the throughput capacity of random networks, a variety of methods have been implemented. For example, node mobility was first exploited to improve the capacity to $\Theta(1)$ in [7], then the induced delay effect was analyzed in [8][9][10] [23]. Another way by adding base stations to the ad hoc networks was studied in [11][12][13] [14][22], and different routing strategies were investigated in [15][16].

While these methods need additional assumptions besides the original random ad hoc networks, M. Franceschetti et al. in [5] showed that a $\frac{1}{\sqrt{n}}$ rate is achievable by importing percolation theory arguments. Based on the results of [5], B. Liu et al. in [1] studied the capacity of an extended wireless network with infrastructure. They considered not only a two-dimensional (2D) square network, but also a one-dimensional (1D) network and a 2D strip network. It turns out to be the capacity of a hybrid network exhibits significantly different scaling laws under different geometries. Specifically, if the width of a strip is at least on the logarithmic order of the size, the scaling law will be the same as a 2D square network; otherwise, it will perform the same as a 1D network. The researchers attributed this transition to the features of the percolation in the corresponding networks.

Since in real situations wireless networks might be deployed at a variety of locations (e.g., cities or towns on a plain area, villages along a river or on the foot of a strait valley, campus domain or among a certain building), these networks are usually not with regular shapes such as squares or discs. Thus, considering the effect of a general geometry on the capacity of wireless networks has both theoretical and practical significance. However, it is hard to take into account all possible geometries with a universal definition. Note that an arbitrary geometry can be decomposed into many basic geometries, we could obtain insight of this problem by considering some basic geometries.

In this paper, we investigate the behavior of the capacity under different geometries, and figure out the essential factor among them. In particular, we are interested in the following questions:

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- In what conditions does the percolation highway exist, and how does it exist in different geometries?
- What is the achievable rate via the percolation in the above settings?

As the starting point, we take a further look at the percolation in a strip network. We prove that when a highway system exists, the achievable rate is $\Omega(\text{Min}\{\frac{1}{m}, \frac{1}{w(m)}\})$, where m is the length and $w(m)$ is the width of the strip¹. If $\text{Max}\{m, w(m)\} = O(\sqrt{n \log n})$, the rate can be between $\Theta(\frac{1}{\sqrt{n \log n}})$ and $\Theta(\frac{1}{\sqrt{n}})$, when we choose a four-phase routing scheme. Furthermore, we find that even if the width is increasing slower than the logarithm of the length, there still remain sub-highways across the strip from left to right. We point out the achievable rate is restricted by malpositions on the sub-highway, and analyze the ultimate capacity. The analysis shows that capacity of a strip network will degenerate to that of a 1D network in a step by step manner.

After that, we consider a triangle network. First, we search the conditions when the highways parallel with an edge exist. The achievable rate via a highway system is examined later. We find that the transmission rate of the nodes in the corner of a triangle can be a bottleneck for the whole throughput.

In the above study, we obtain that the achievable rate will maximize up to $\frac{1}{\sqrt{n}}$, when the width and the length of a strip network, or the edges of a triangle network are $\Theta(\sqrt{n})$. Otherwise, the capacity decreases due to the asymmetry. Thus, we conclude that symmetry is a key factor for the percolation and the capacity.

Finally, we extend the percolation to 3D networks. By a tool of projection decomposition, we prove that the achievable rate is $\frac{1}{\sqrt[3]{n}}$ in a cubic network. Moreover, for a cuboid network, the capacity is determined by the maximum length of the edges. This further confirms the effect of the symmetry.

Our study provides an extensive view of the impact of different geometries on the capacity. Though in the real world, situations will be more complicated as stated above, we can still benefit from the study both in the network design and the scheduling. A potential application is when deploying a network, we should make it as symmetric as possible to obtain a higher capacity. If there are some constraints which induce asymmetry, we may divide it into many symmetric sub-networks and connect them with broadband cables.

The rest of the paper is organized as follows. In section slowromancapii@, we describe the network model and related works. In section slowromancapiii@, we study a strip network. In section slowromancapiv@, we consider a triangle network. Section slowromancapv@ is devoted to 3D networks. Finally, we summarize the paper in Section slowromancapvi@.

II. NETWORK MODEL AND RELATED WORKS

A. Network Model

The basic results acquired in this paper are by letting the number of the nodes tend to infinity. There are two means to fulfill this goal: one is fixing the area (or the volume) of a region (or a space) and letting the node density λ tend to

¹ $\text{Min}\{f(n), g(n)\} = f(n)$, when $f(n) = O(g(n))$; otherwise, $\text{Min}\{f(n), g(n)\} = g(n)$. $\text{Max}\{f(n), g(n)\} = f(n)$, when $f(n) = \Omega(g(n))$; otherwise, $\text{Max}\{f(n), g(n)\} = g(n)$.

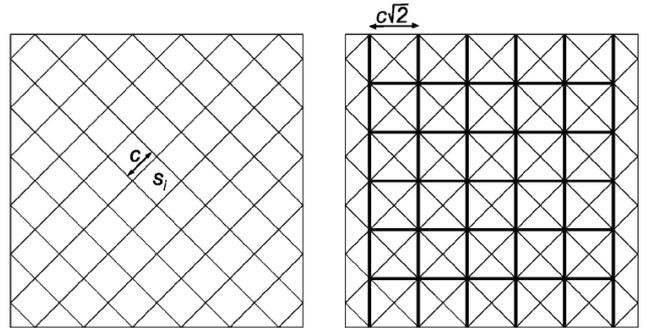


Fig. 1. Map the tessellation to an edge-percolation model, cited from [5]

infinity, which is called *dense network*; the other is fixing the node density λ and letting the area (or the volume) of the region (or the space) tend to infinity, which is called *extended network*. We focus on the extended network model, whereas the results are ready to be extended to a dense network.

Assume nodes are distributed according to a Poisson point process of unit density within a 2D region or a 3D space. Denote n the area of the region or the volume of the space. Therefore, the expected number of the nodes is n for each of the network models.

The source-destination pairs are picked uniformly at random, so that each node is the destination of exactly one source. Suppose each node i transmits at a same power P , and the power of the receiving signal at node j is $Pl(i, j)$, where $l(i, j)$ represents the path loss from node i to node j . Let the Euclidean distance between node i and j be denoted by d_{ij} . For simple approximation, we assume $l(i, j) = \min\{1, d_{ij}^{-\alpha}\}$, with $\alpha > 2$ for 2D case and $\alpha > 3$ for 3D case.

We treat all the interferences at the receiver simply as noise, since we are interested only about a lower bound on the achievable rate. For a direct link between any two nodes with an unit bandwidth, data rate is given by the channel capacity

$$R_{ij} = \log \left(1 + \frac{Pl(i, j)}{N_0 + \sum_{k \neq i} Pl(k, j)} \right) \text{bit/s} \quad (1)$$

where N_0 is the power of the Gaussian white noise at the receiver. Note that this assumption is more realistic compared with the Protocol Model and the Physical Model. And, the results of this paper can also be generalized to random networks under the Protocol Model when different transmission ranges are allowed, or under the Physical Model using different powers for transmission.

B. Related Works

Percolation theory characterizes the behavior of connected clusters in a random graph. A representative example in physics is described in [5], and [2][3][4] provide more information about this field.

In [5], M. Franceschetti et al. tessellate the network by sub-squares s_i of a constant side length c large enough. As shown in Fig. 1, they declare that each square on the left side of the picture *open* if there is at least a node inside it, and *closed* otherwise. Via mapping this tessellation model into an edge-percolation model, they declare an edge traversing diagonally

in each square *open* or *closed*, if the corresponding square is open or closed, as shown on the right side of the figure.

A percolation highway is a path across the boundary of a network with the length of each hop bounded by a constant. Thus, it forms when there exists an open path which consists of open edges crossing the network. For a square network, they proved that there exist a number of edge-disjoint open paths across the square both horizontally and vertically. Data packets generated by an upper bounded number of nodes can be transmitted over short distance along the paths, which ensures a constant rate as a whole. Therefore, these paths are referred to as a *highway system*. Afterwards, routing scheme is divided into four phases. In a first phase, nodes send their packets to the *entry point*, i.e., the nearest point of a horizontal highway. In a second and a third phase, packets are passed along the horizontal and the vertical highway successively. In a last phase, the destination nodes receive packets from the *exit point*, i.e., the nearest point of a vertical highway. TDMA scheme is adopted during the transmissions, and the authors indicated that the achievable rate in a first and a last phase is higher than that of a second and a third phase. This is due to less burden on packets relay, even if the transmission distance is larger in these two phases.

Moreover, B. Liu *et al.* in [1] found a condition that the highway does not exist in the direction parallel with the length of a strip network. Thus, we call it a *supercritical scenario* when highway emerges in both directions parallel with the width and the length, and a *subcritical scenario* otherwise.

III. STRIP NETWORK: SUPERCRITICAL SCENARIO AND SUBCRITICAL SCENARIO

We construct an extended network in an area of a strip with sides length $m \times w(m)$. Nodes are placed in the network according to a Poisson point process with unit density. Thus we can denote the total number of nodes in the extended network as $n = m \times w(m)$, which also denotes the area of the network. The number of nodes n in the extended network goes to infinity with length $m \rightarrow \infty$ and $w(m) \rightarrow \infty$.

In this section, we focus on the percolation and the capacity of a strip network. We define the supercritical scenarios and subcritical scenario in terms of whether a highway system exists as in Section II-B. Achievable rate is presented by continuously varying the scale of the strip.

A. Supercritical scenario in a strip network

In this subsection, we study the supercritical scenario. For completeness, here we draw a conclusion on the necessary and sufficient condition for the percolation highway in a strip network. Consider a strip network with length $m \rightarrow \infty$ and width $w(m) \rightarrow \infty$, we have

Theorem 3.1: Partition the width of a strip network evenly on the logarithmic scale of the length, as shown in Fig. 2, then there exists a number of $\Theta(\log m)$ edge-disjoint highways across each sub-rectangle R_m^i from left to right w.h.p., if and only if $\lim_{m \rightarrow \infty} \frac{w(m)}{\log m} > 0$.

The proof is enclosed in Appendix slowromancapi@. Theorem 3.1 indicates the equivalent condition for the presence of the highway backbone in the direction parallel with the length

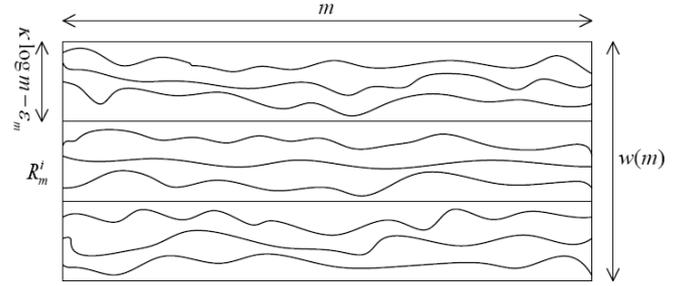


Fig. 2. Horizontal highways in a strip network. The strip is partitioned into sub-rectangles R_m^i with sides length $m \times (\kappa \log m - \varepsilon_m)$ with ε_m vanishingly small so that the side length of each rectangle is an integer.

of a strip. Observe the symmetry of the width and the length, we have the equivalent condition that the highway emerges in the direction parallel with the width is $\lim_{m \rightarrow \infty} \frac{m}{\log w(m)} > 0$. Moreover, the following corollary is immediate by Theorem 3.1.

Corollary 3.1: If $\lim_{m \rightarrow \infty} \frac{w(m)}{\log m} > 0$, then there are a number of $\Theta(w(m))$ edge-disjoint highways across from left to right in a strip network.

Consider the relationship between $w(m)$ and m in Fig. 2. Provided that $\lim_{m \rightarrow \infty} \frac{w(m)}{\log m} > 0$ and $\lim_{m \rightarrow \infty} \frac{m}{\log w(m)} > 0$, by Theorem 3.1, we declare that highway exists in both directions parallel with the width and the length. Now, if we adopt the four-phase routing strategy as in [5], it will follow that

Theorem 3.2: In the supercritical scenario of a strip network, the throughput capacity is $\Omega(\text{Min}\{\frac{1}{m}, \frac{1}{w(m)}\})$, where $n = m \times w(m)$ via using a four-phase routing strategy. Supplementary conditions $\lim_{m \rightarrow \infty} \frac{w(m)}{\log m} > 0$ and $\lim_{m \rightarrow \infty} \frac{m}{\log w(m)} > 0$ should be added to ensure that it is a supercritical network.

To prove this, we need present the following lemmas in the first place.

Lemma 3.1: For any integer $d > 0$, there exists an $R(d) > 0$, such that in each square s_i of Fig. 1 there is a node that can transmit w.h.p. at rate $R(d)$ to any destination located within distance d . Furthermore, as d tends to infinity, we have $R(d) = \Omega(d^{-\alpha-2})$.

Lemma 3.2: If u is a constant, partition the strip into an integer number $\frac{w(m)}{u}$ of rectangles with sides length $m \times u$, then there are w.h.p. no more than $3 \max\{um, \log \frac{w(m)}{u}\}$ nodes inside each rectangle.

Lemma 3.3: If u is a constant, partition the strip into an integer number $\frac{m}{u}$ of rectangles with sides length $u \times w(m)$, then there are w.h.p. no more than $3 \max\{uw(m), \log \frac{m}{u}\}$ nodes inside each rectangle.

Remark 3.1: First, Lemma 3.1 is obtained by letting $\gamma = 0$ in Theorem 3 of [5]. Secondly, regard each rectangle $m \times u$ as a “cell” in [1], then Lemma 3.2 yields as an adaptation of Lemma 3 in [1] by letting $b = \frac{w(m)}{u}$. In addition, Lemma 3.2 is attained by letting $b = \frac{m}{u}$.

Then we prove Theorem 3.2.

Proof: We slice the strip into horizontal slabs of width $w_R = (\kappa \log m - \varepsilon_m)$ such that there are as many highways inside each R_m^i , as shown in Fig. 2, which denotes the i -th $m \times (\kappa \log m - \varepsilon_m)$ slab. Then, we can impose nodes inside the

i -th slab transmit directly to the entry point of the horizontal highways in the i -th slab. As for the destination nodes, we divided the network into vertical slabs of constant width, and define a same mapping between slabs and vertical highways. Then, each destination can receive information from the exit point of the corresponding highway.

Since the maximum length of the hop for a node in each square s_i , similar to that shown in Fig. 1, to reach the corresponding horizontal highway is $\kappa \log m$, according to Lemma 3.1, we have a rate $\frac{1}{(\kappa \log m)^{\alpha+2}}$ is achievable². Notice that w.h.p. the maximum number of nodes within s_i is $\log \sqrt{n}$ (Lemma 1 of [5]), we have the achievable rate in a first phase is $\frac{1}{(\kappa \log m)^{\alpha+2} \log \sqrt{n}}$. Analogously, the achievable rate in a last phase is $\frac{1}{(\kappa' \log w(m))^{\alpha+2} \log \sqrt{n}}$, where constant κ' is defined similarly as κ .

Then, we consider the traffic load on a horizontal highway. Refer to Lemma 3.2, we have the maximum number of nodes whose packets need to be relayed along the horizontal highway is $3 \max\{um, \log \frac{w(m)}{u}\}$ w.h.p., where u is a constant. Observe that $\lim_{m \rightarrow \infty} \frac{m}{\log w(m)} > 0$, the number of nodes is w.h.p. $O(m)$. Since the length of each hop along the highway is bounded by a constant, by Lemma 3.1, the capacity of the highway is $\Omega(1)$. Thus, the achievable rate for each node is $\Omega(\frac{1}{m})$ in a second phase. Likewise, we have the achievable rate in a third phase is $\Omega(\frac{1}{w(m)})$ by using Lemma 3.3. Having aware that the rate is determined by the minimum rate among each routing phase, we declare that the achievable rate is $\Omega(\text{Min}\{\frac{1}{m}, \frac{1}{w(m)}\})$. ■

Here we have a discussion of the minimal rate's dependence on n . According to Theorem 3.2, without losing generality, assume $w(m) = O(m)$, we have $T_{\text{highway}} = \Omega(\frac{1}{m})$. Notice that $T_{\text{highway}} = \Omega(\frac{1}{\sqrt{n}})$ is maximum, when $m = w(m)$; and is minimum when $w(m) = \Omega(\log m)$. Recall that the attainable rate by adopting a direct routing scheme in [6] is $T_{\text{direct}} = \Theta(\frac{1}{\sqrt{n \log n}})$. Then, if $m = O(\sqrt{n \log n})$, or equivalently $\frac{m}{w(m)} = O(\log n)$, we have $T_{\text{direct}} \leq T_{\text{highway}}$; otherwise, if $m = \Omega(\sqrt{n \log n})$, or equivalently $\frac{m}{w(m)} = \Omega(\log n)$, we have $T_{\text{direct}} \geq T_{\text{highway}}$.

B. Subcritical scenario in a strip network

In the next place, we derive an achievable rate in the subcritical scenario via constructing a sub-highway system.

To start with, consider a scenario $\lim_{m \rightarrow \infty} \frac{w(m)}{\log m} = 0$, but $\lim_{m \rightarrow \infty} \frac{w(m)}{\log \log m} > 0$, the sub-highway is constructed as follows. In Fig. 3, partition the length on the scale of $\log m$. More precisely, for a given κ_1 , we divide the strip into rectangles R_i of sides length $(\kappa_1 \log m - \varepsilon_1) \times w(m)$. ε_1 is selected as the smallest value such that the number of the rectangles $\frac{m}{\kappa_1 \log m - \varepsilon_1}$ is an integer.

Then we divide each rectangle R_i into slices R_i^j of sides $(\kappa_1 \log m - \varepsilon_1) \times (\kappa_2 \log \log m - \varepsilon_2)$, among which κ_2 is a given constant and ε_2 is chosen similarly as ε_1 . Applying the same arguments of sufficient condition in Theorem 3.1 to each slice, we have that there are a number of $\delta_1 \log \log m$ paths inside R_i^j , where $\delta_1 = \delta_1(\kappa_2, p)$ is a constant.

²Rigorously speaking, the maximum length of the hop is $\kappa \log m + \sqrt{2}c$. While it has no affect if we omit the constant $\sqrt{2}c$.

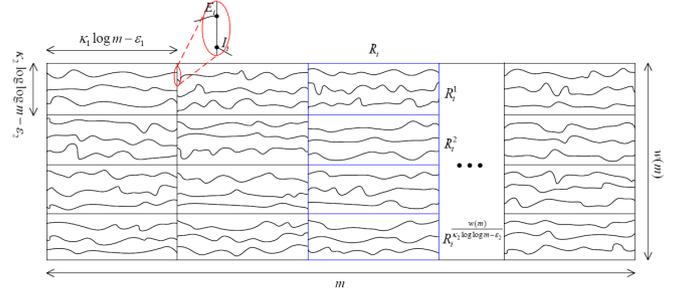


Fig. 3. Sub-highway system in a strip network. The strip is partitioned into rectangles R_i at first, and a sample rectangle is denoted by the blue box.

Then each R_i is divided into slices $R_i^j \sim R_i^{\kappa_2 \log \log m - \varepsilon_2}$. The extreme right point of the i th highway in R_i^1 is E_i , and the extreme left point of the i th highway in R_i^2 is I_i . A malposition between E_i and I_i can be seen more clearly in the red ellipse above.

These paths are percolation highways within each slice, so they cross the slice from left to right continuously. But now that $\lim_{m \rightarrow \infty} \frac{w(m)}{\log m} = 0$, they are probably not able to connect with the highways in a neighboring slice to form an integral highway. In Fig. 3, a malposition of the paths between two adjacent slices is illustrated.

While if we number the highways inside R_i^j from 1 to $\lceil \delta_1 \log \log m \rceil$, the malposition of the corresponding number of highways between two slices is upper bounded by the width of the slice $\kappa_2 \log \log m - \varepsilon_2$. Data packets can be carried through the paths consisted of highways with the same mark number. We call these paths *sub-highways*. Notice that percolation highway exists in the vertical direction. Thus, to exploit the horizontal sub-highway and the vertical highway, we use a four-phase routing scheme as well, and present the following theorem to make the achievable rate precise.

Theorem 3.3: In the subcritical scenario of a strip network, if $\lim_{m \rightarrow \infty} \frac{w(m)}{\log m} = 0$, but $\lim_{m \rightarrow \infty} \frac{w(m)}{\log \log m} > 0$, the achievable rate is $\Omega(\frac{1}{(\log \log m)^{\alpha+2} m})$.

Proof: In accordance with the routing scheme, each node transmits data to the entry point of a corresponding sub-highway. The maximum length of the hop is less than $\kappa_2 \log \log m$, thus by Lemma 3.1, we have the achievable rate is $\Omega(\frac{1}{(\log \log m)^{\alpha+2}})$. Moreover, since the number of nodes inside each square s_i is w.h.p. no more than $\log \sqrt{n}$, the bit rate per node is $\Omega(\frac{1}{(\log \log m)^{\alpha+2} \log \sqrt{n}})$. Notice that $\Theta(\log \sqrt{n}) = \Theta(\log m \cdot w(m)) = \Theta(\log m)$, then the achievable rate is $\Omega(\frac{1}{(\log \log m)^{\alpha+2} \log m})$ in a first phase.

In a second phase, packets are delivered along the sub-highway. Although there is not a continuous highway going through out the strip from the right to the left, the sub-highways with the same mark number can be made to compose a highway with breaking points. However, the breaking points would lower the rate along the deemed highway since the distance between two breaking points is no longer bounded by a constant. Observe that the length of each hop on the highway inside each R_i^j is uniformly bounded by a constant, so the achievable rate is restricted by the transmission rate over the hop between two neighboring highways. According to Lemma 3.1, this rate can achieve $\Omega(\frac{1}{(\log \log m)^{\alpha+2}})$, which is slightly worse than a continuous highway, whose rate can be

a constant. On the other side, since each sub-highway need only relay the packets generated by the nodes inside a slab with sides length $m \times u$, where u is a certain constant, and from Lemma 3.2 the number of the nodes is w.h.p. no more than $3 \max\{um, \log \frac{w(m)}{u}\} = 3um$, thus the achievable rate along the sub-highway is $\Omega(\frac{1}{(\log \log m)^{\alpha+2m}})$.

Next, we consider the rate of a third phase. Since $\lim_{m \rightarrow \infty} \frac{m}{\log w(m)} = \infty$, there are $\Theta(\log w(m))$ vertical percolation highways in each rectangle with sides length $w(m) \times \log w(m)$. Hence, with reference to Lemma 3.3, packets generated by at most $3 \log \frac{m}{u}$ w.h.p. need to be relayed along a vertical highway. Then, we have the achievable rate in a third phase is $\Omega(\frac{1}{\log m})$.

In a last phase, the length of the hop that a destination node receives information from the corresponding vertical highway is bounded by $\log w(m)$, thus according to Lemma 3.1 the achievable rate over the hop is $\Omega(\frac{1}{(\log w(m))^{\alpha+2}})$. Besides, the maximum number of nodes inside each square s_i is $\log m$ w.h.p., therefore the achievable rate for each node is $\Omega(\frac{1}{(\log w(m))^{\alpha+2} \log m})$.

Taking the minimum rate in the four phases into account, the achievable rate is determined by that of a second phase, which concludes the theorem. \blacksquare

In a more general scenario, assume $f(m, k) = \underbrace{\log \log \dots \log m}_k$, then if $\lim_{m \rightarrow \infty} \frac{w(m)}{f(m, k-1)} = 0$, but $\lim_{m \rightarrow \infty} \frac{w(m)}{f(m, k)} > 0$, availing the same procedures as the proof in Theorem 3.3, we have that the achievable rate $T_{subcritical} = \Omega(\frac{1}{(f(m, k))^{\alpha+2m}})$. Observe that with k increases, $f(m, k)$ and m will increase. To be extreme, if $w(m) = \omega$, where ω is a constant, then it follows $k \rightarrow \infty$, thus we have $T_{subcritical} \rightarrow \Omega(\frac{1}{n})$, which means that the throughput capacity of a strip network will equal to that of a 1D network.

IV. TRIANGLE NETWORK

In this section, we consider a triangle network. For a triangle network, necessary conditions that percolation highway exists in a certain direction are presented. Then, we study the capacity via using the highway system. Moreover, by combining the former networks, we can see the impact of the geometry on the capacity more clearly.

A. Percolation highway in a triangle network

Triangle network is more complex than a strip network, since a triangle should be determined by three edges. Similarly, we construct an extended network in a triangle area with a total number of n nodes placed in the area according to Poisson point process with unit density. In this section, we assume that the length of each edge tends to infinity when the area of the triangle n goes to infinity, and they are all no more than n . To make the problem easier, we consider the highway in the direction parallel with the edges.

In Fig. 4, suppose AD is the height associated with edge BC , and BD is the projection of the triangle along BC . Denote $a = |BC|$, $b = |AC|$, $c = |AB|$, $m = |BE|$, $w(m) = |AD|$. We use a covering scheme to cover a

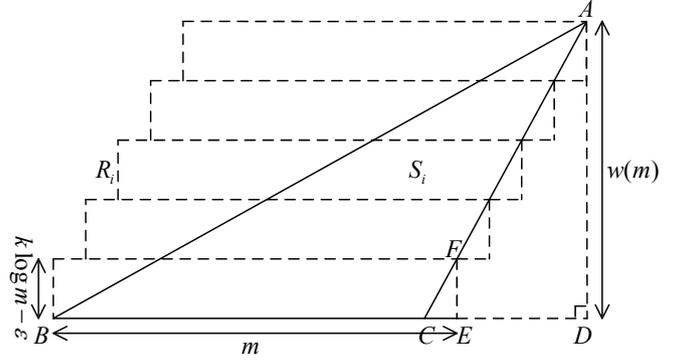


Fig. 4. Sufficient condition for percolation highway in a triangle network.

triangle by rectangles R_i with sides length $m \times (\kappa \log m - \varepsilon)$. Accordingly, the triangle is divided into slices S_i . Then we have:

Theorem 4.1: Consider the situation $\angle ABC$ or $\angle ACB$ is obtuse, if $\frac{m}{\log m} = \Omega(a)$ and $\lim_{m \rightarrow \infty} \frac{w(m)}{\log m} > 0$, then there are $\Theta(\log m)$ percolation highways across each slice S_i from left to right.

Proof: It is easy to see that if the covering scheme exists, there will be $\Theta(\log m)$ percolation highways across each slice S_i . First, we have $\lim_{m \rightarrow \infty} \frac{w(m)}{\log m} > 0$. Next, we prove that $\lim_{m \rightarrow \infty} \frac{m}{\log m} = \Omega(a)$.

In Fig. 4, since $\triangle FCE \sim \triangle ACD$, we have $\frac{|CF|}{|CA|} = \frac{|EF|}{|DA|}$, i.e. $|CF| = \frac{b}{w(m)}(\kappa \log m - \varepsilon)$. Thus, in $\triangle FEC$, $|CE| = \sqrt{|CF|^2 - |EF|^2} = (\kappa \log m - \varepsilon) \sqrt{(\frac{b}{w(m)})^2 - 1}$. If $\lim_{m \rightarrow \infty} \frac{b}{w(m)} = \infty$, then $|CE| = \frac{b}{w(m)}(\kappa \log m - \varepsilon)$; else, $|CE| = \kappa \log m - \varepsilon$.

Observe $|BE| = |BC| + |CE|$, hence in the first case, we have $|BE| = a + \frac{b}{w(m)}(\kappa \log m - \varepsilon) = a + \frac{ab}{2n}(\kappa \log m - \varepsilon)$. Now that $b \leq n$, $|BE| \leq a + a\kappa \log m$. Thus, if $m = \Omega(a\kappa \log m)$, then the covering scheme exists. It yields that $\lim_{m \rightarrow \infty} \frac{m}{\log m} = \Omega(a)$. In the second case, we have $|BE| = a + (\kappa \log m - \varepsilon)$, which means $m = \Omega(a)$. Combine these two cases together, we conclude the theorem. \blacksquare

Next, we analyze the achievable rate when we utilize the highway system. In Fig. 5, for simplicity, suppose $a = \Theta(n^{a_1})$, $b = \Theta(n^{b_1})$, $c = \Theta(n^{c_1})$, where a_1, b_1, c_1 are constants. We provide the following theorem.

Theorem 4.2: If the sides length of a triangle network are given above, the maximum achievable rate is $\Omega(\frac{1}{\sqrt{n}})$, when $a_1 = b_1 = c_1 = \frac{1}{2}$.

Proof: Without losing generality, we assume that $a_1 \geq b_1 \geq c_1$. Then, according to the triangle inequality, we have $a \leq b + c$, which yields that $b_1 = a_1$.

Firstly, consider the percolation highway in the direction parallel with AB , as illustrated in the left side of Fig. 5.

If $\angle CAB$ and $\angle CBA$ are acute, then since $\lim_{n \rightarrow \infty} \frac{|CG|}{\log |AB|} = \lim_{n \rightarrow \infty} \frac{2n^{1-c_1}}{\log n^{c_1}} = \infty$, there are $\Theta(\log n^{c_1}) = \Theta(\log n)$ highways across the triangle from left to right. If $\angle CAB$ or $\angle CBA$ is obtuse, see Fig. 4, in Theorem 4.1, let $m = c^2$, notice that $\frac{m}{\log m} = \frac{c^2}{2 \log c} = \Omega(c)$, and $\lim_{m \rightarrow \infty} \frac{|CG|}{\log m} = \lim_{n \rightarrow \infty} \frac{2n^{1-c_1}}{\log n^{2c_1}} = \infty$, then there are $\Theta(\log m) = \Theta(\log n^{2c_1}) = \Theta(\log n)$ highways

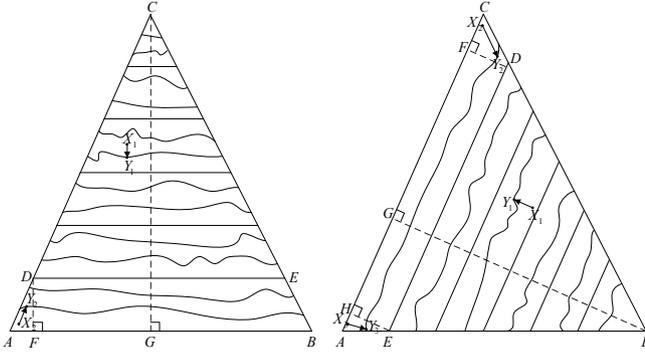


Fig. 5. Achievable rate in a triangle via using the percolation highway. Percolation highway in the direction parallel with edge AB and AC are shown in the left side and the right side of the figure respectively.

across the triangle from left to right. Moreover, similarly, we have percolation highway exists in the direction parallel with edge AC as well. Hence, we can also use a four-phase routing scheme to deliver the packets.

According to the routing scheme, in a first phase, each source node sends packets to the entry point on the highway. Commonly, the highway nearest to the source node is assigned to it for convenience. For example, for node X_1 in Fig. 5, its entry point is denoted by Y_1 on the highway under it. Thus, the maximum length of the hop to reach the highway is on the scale of $\log m$. While there are some exception nodes in the corner of the triangle, like X_2 , they can not find a highway nearest to them in the vertical direction. Thus, they must deliver their packets to a nearest highway in the inclined direction. We refer to this as *corner effect*.

In this case, the maximum hop length will be on the scale of $|AD|$ or $|BE|$, which is larger than that of the normal case. Notice that $\triangle ADF \sim \triangle ACG$, we have $|AD| = \frac{|DF|}{|CG|} \cdot |AC| = \Theta\left(\frac{\log n^{2c_1}}{2n^{1-c_1}} n^{a_1}\right) = \Theta\left(\frac{\log n}{n^{1-a_1-c_1}}\right)$. Then, by Lemma 3.1 and consider the maximum number of nodes in a sub-square s_i , the achievable rate in a first phase is $\Omega\left(\frac{1}{\left(\frac{\log n}{n^{1-a_1-c_1}}\right)^{\alpha+2} \log n}\right) = \Omega\left(\frac{1}{n^{(a_1+c_1-1)(\alpha+2)(\log n)^{\alpha+3}}}\right)$. Similarly, we have $|BE| = \frac{|DF|}{|CG|} \cdot |CB| = \Theta\left(\frac{\log n^{2c_1}}{2n^{1-c_1}} n^{a_1}\right) = \Theta\left(\frac{\log n}{n^{1-a_1-c_1}}\right)$. From this, we can get that $|BE|$ and $|AD|$ are on the same scale.

In a second phase, the packets are relayed along the highway. As shown in Fig. 6, if we slice the triangle into slices S_i with a constant height c , and assign the nodes inside the i th slice to send their packets to the i th highway, then the number of the nodes whose packets a highway needs to relay is upper bounded by the number of the nodes inside S_i . Next, we consider a rectangle $BCC'B'$ with $|BB'|$ equals to the height of the triangle, and suppose there are also nodes distributed according to a Poisson point process inside it. Note that the size of the rectangle is twice of the triangle, implicating the number of the nodes inside them are on the same order. We have that the number of nodes inside S_i is w.h.p. no more than that inside each sub-rectangle R_i with sides length $|BC| \times c$. Since the latter quantity can be bounded by $O(n^{c_1})$ using Lemma 3.2, we obtain that the number of nodes draining data to a highway is $O(n^{c_1})$. Thus, we have the achievable rate is $\Omega\left(\frac{1}{n^{c_1}}\right)$.

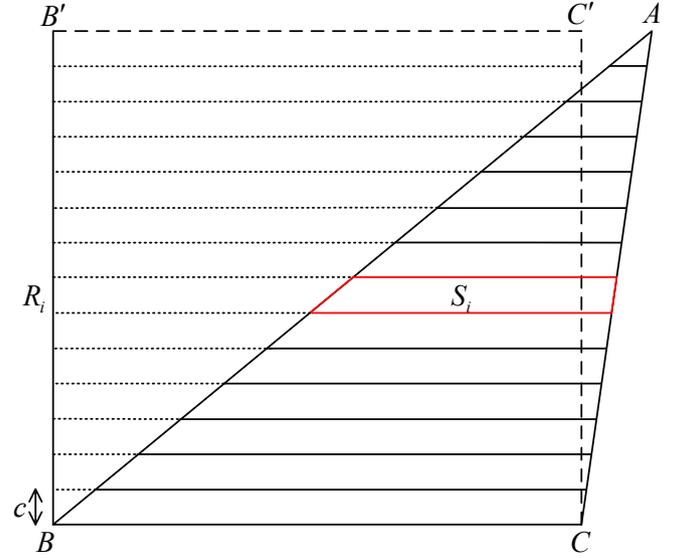


Fig. 6. Bound the number of nodes inside each slice S_i .

Afterward, we look at the highway in the direction parallel with $|AC|$. Similar to a second phase, we have that the achievable rate is $\Omega\left(\frac{1}{n^{b_1}}\right) = \Omega\left(\frac{1}{n^{a_1}}\right)$.

In a last phase, destination nodes receive the packets from the highway. Due to the corner effect, the whole all rate is restricted by the transmission rate in the corner of $\angle ACB$ and $\angle BAC$. Thus, we have the achievable rate is $\Omega\left(\text{Min}\left\{\frac{1}{|CD|^{\alpha+2} \log n}, \frac{1}{|AE|^{\alpha+2} \log n}\right\}\right) = \Omega\left(\text{Min}\left\{\frac{1}{\left(\frac{1}{n^{2a_1-1} \log n}\right)^{\alpha+2} \log n}, \frac{1}{\left(\frac{1}{n^{a_1+c_1-1} \log n}\right)^{\alpha+2} \log n}\right\}\right) = \Omega\left(\text{Min}\left\{\frac{1}{n^{(2a_1-1)(\alpha+2)(\log n)^{\alpha+3}}}, \frac{1}{n^{(a_1+c_1-1)(\alpha+2)(\log n)^{\alpha+3}}}\right\}\right)$.

Taking the minimum rate in the above discussion into account, we have the achievable rate in fact is $\Omega\left(\text{Min}\left\{\frac{1}{n^{(2a_1-1)(\alpha+2)(\log n)^{\alpha+3}}}, \frac{1}{n^{a_1}}\right\}\right)$. Observe that $|AC| \geq |CG|$ in the left figure, hence $a_1 \geq 1 - c_1$. Combining with $a_1 \geq c_1$, we have $a_1 \geq \frac{1}{2}$. Thus, the maximum achievable rate is $\Omega\left(\frac{1}{\sqrt{n}}\right)$, which is obtained when $a_1 = c_1 = \frac{1}{2}$. ■

B. Comparison and Discussion

In the above analysis, we have studied the capacity of a strip network, a triangle network from a point of view of the percolation. We find that the geometric symmetry is a key factor for the capacity of the network.

Specifically, for a strip network, the achievable rate is maximum when the order of the width equals to that of the length. Otherwise, the capacity will decrease, if the gap between the two sides increases. For a triangle network, the achievable rate is maximum when the orders of the edges are equal. Otherwise, the capacity may be restricted by the corner effect.

A reasonable interpretation about these phenomenon can be obtained by considering the minimum sparsity cut in an arbitrary geometry network. This cut restricts the possible number of paths that can percolate³ across it. Thus, the

³Here “percolate” means relaying packets over a hop with the length bounded by a constant.

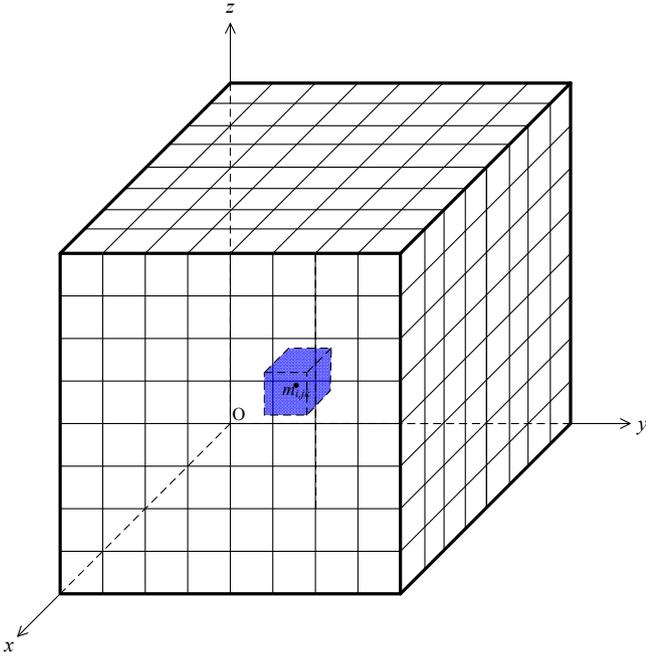


Fig. 7. The tessellation of a cubic network.

throughput capacity of the whole network is bottlenecked. Observe that the minimum length of the sparsity cuts will decrease when the region becomes asymmetry, we have that the capacity will also degenerate. Moreover, if the network is asymmetry, nodes in the corner need a longer hop to reach the highway, which also bottlenecks the capacity.

V. CUBIC NETWORK AND CUBOID NETWORK

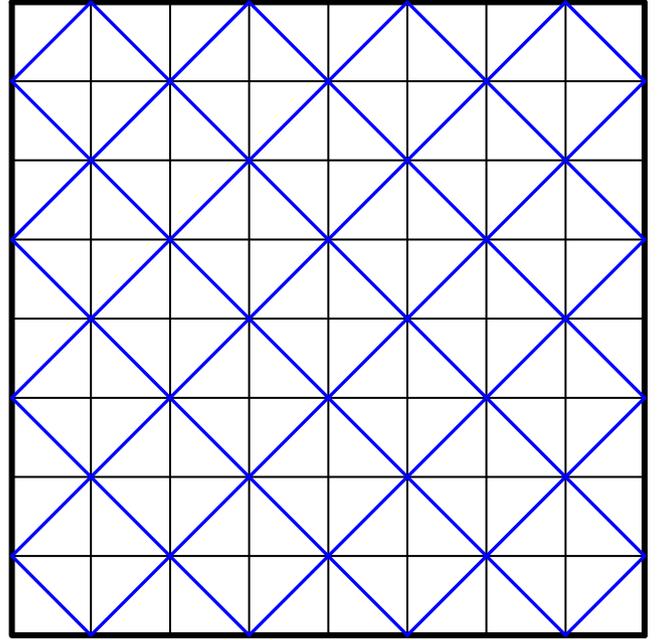
In the former study, we have investigated 2D geometries. Now we study 3D geometries to gain more insight about the impact of dimension on the capacity. The 3D case has been considered in [19], [20] and [21]. In this section, we mainly consider a cubic network by percolation theory, then spend a few words on a cuboid network.

A. Percolation highway in a cubic network

In Fig. 7, the side length of the cubic network is $\sqrt[3]{n}$. For convenience, we setup a coordinate system fixing the origin O at an vertex of the cube. To begin our construction, we tessellate the cube by micro-cubes of constant edge length c . We call such a micro-cube m -cube for brevity. Denote a m -cube by $m_{i,j,k}$, if the coordinate of the center of the cube is $(\frac{2i-1}{2}c, \frac{2j-1}{2}c, \frac{2k-1}{2}c)$. Consider the number of the nodes $N(m_{i,j,k})$ inside $m_{i,j,k}$. For all i, j, k , we have the probability that there is at least one node in it, or, the m -cube is filled in, is

$$p \equiv P(N(m_{i,j,k}) \geq 1) = 1 - P(N(m_{i,j,k}) = 0) = 1 - e^{-c^3}. \quad (2)$$

We expect there exists a highway system composed of percolation highways in x, y, z directions, and the data can be delivered towards any direction such as in the 2D scenario. Suppose i is an integer, we select a vertical section V from the cube, where all the points share an x -coordinate between


 Fig. 8. The tessellation of a vertical section. The blue edges are open if there is at least a node inside the corresponding cube of the m -square.

$(i-1)c$ and ic . Then, the section cuts a number of $\frac{\sqrt[3]{n}}{c} \times \frac{\sqrt[3]{n}}{c}$ m -cubes, and the section will be tessellated by micro-squares (or m -squares) formed by projecting these m -cubes onto V , as Fig. 8 shows. We call a m -cube is *corresponding cube* to a m -square, or vice versa, if the square is the projection of this cube.

Now, we connect a diagonal of each m -square in the way depicted in Fig. 8. Then, a 45° -angled square lattice of side length $\sqrt{2}c$ emerges. If the corresponding cube of a m -square is filled in, then we call the associated edge of the square lattice is *open*; otherwise, it is called *closed*. The edges in the lattice can be open, with probability p , or closed with probability $1 - p$. And, the edges are open or closed independent of each other. Though here the square lattice is 45° -angled, it is not harmful for us to implement the edge-percolation model to prove the existence of the percolation highway inside V . The only difference is that we need change the coefficient $\frac{4}{3}$ in (15) of [5] to $\frac{2}{3}$, since there are only two choices for the direction of the edge at the initial point. Thus, if we select p large enough, percolation highway will emerge in the direction of y and z . Be Aware the symmetry of the x, y, z coordinates, we know that the highway exists in the direction of x as well.

To determine the achievable rate for a cubic network, we extend the following lemmas.

Lemma 5.1: For any integer $d > 0$, there exist an $R(d) > 0$, such that in each m -cube $m_{i,j,k}$ there is a node that can transmit w.h.p. at rate $R(d)$ to any destination located within distance d . Furthermore, as d tends to infinity, we have

$$R(d) = \Omega(d^{-\alpha-3}). \quad (3)$$

Lemma 5.2: There are w.h.p. at most $2c^2 \sqrt[3]{n}$ nodes inside a cuboid of sides length $c \times c \times \sqrt[3]{n}$, or $c \times \sqrt[3]{n} \times c$, or $\sqrt[3]{n} \times c \times c$.

Remark 5.1: Lemma 5.1 is obtained by proposing a TDMA scheme for the transmissions, and bounding the interferences and the signal at the receiver respectively. The proof is given in Appendix slowromancapii@. Besides, Lemma 5.2 is straightforward by the Chernoff bound adopted in Lemma 2 of [5].

Since there are three orthogonal directions in a cubic network, we need a five-phase routing strategy to deliver the packets. In a first phase, a source transmits data to the entry point of the highway. In the next three phases, packets are relayed along the highway in the three directions successively. And in a last phase, the destination receives data from the exit point of the highway. Like a square network, the bottleneck of the achievable rate is that along the highway. Observe the length of the hop is bounded by a constant, thus by Lemma 5.1, we have the rate over each hop is $\Omega(1)$. Furthermore, according to Lemma 5.2, the number of nodes draining their packets to a highway in any direction is upper bounded by $2c^2 \sqrt[3]{n}$. Hence, we obtain that the achievable rate for each node is $\Omega(\frac{1}{\sqrt[3]{n}})$, which proves the following theorem.

Theorem 5.1: The achievable rate in a cubic network is $\Omega(\frac{1}{\sqrt[3]{n}})$.

B. Cuboid network

Consider a more complex case, say a cuboid network, where the length of the edges could be unequal. Assume the scale of the cuboid is $a(n) \times b(n) \times c(n)$, if $\lim_{n \rightarrow \infty} \frac{b(n)}{\log a(n)} = 0$ and $\lim_{n \rightarrow \infty} \frac{c(n)}{\log a(n)} = 0$, then we declare the absence of the percolation highway in the direction of x ; otherwise, the highway exists. If highway exists in the x, y, z directions, the achievable rate could be $\Omega(\text{Min}\{\frac{1}{a(n)}, \frac{1}{b(n)}, \frac{1}{c(n)}\})$, which is maximum as $\Omega(\frac{1}{\sqrt[3]{n}})$, when all the edges are $\Theta(\sqrt[3]{n})$.

In this section, we find that the capacity of a higher dimensional network is larger due to the percolation highway emerges towards more orthogonal directions, which will reduce the burden on the packets relay. Moreover, from a cuboid network, we find that the symmetry of the scale is still an important factor for the capacity.

VI. CONCLUSIONS

In this paper, we investigate the impact of the geometry on the percolation and the capacity among different network models. In a strip network, the achievable rate is restricted by the maximum length of the sides in the supercritical scenario. And in the subcritical scenario, we figure out that the capacity will degenerate to that of a 1D network in a step by step manner. These results are basically due to the asymmetry of the strip, while a square network has an achievable rate $\frac{1}{\sqrt{n}}$ owing to its symmetry.

Similar phenomena are found in a triangle network and 3D networks as well. The transmission rate of nodes in the corner of a triangle network can be a bottleneck for the whole network. In a cuboid network, percolation highway disappears when the length of the other edges are ‘‘too small’’ compared with the maximum length of the edges. If we regard corner effect as the asymmetry of the locations of the nodes relative to the highway system, then these can also be attributed to an effect of asymmetry.

In summary, results from the above networks suggest that geometry applies a sharp impact on the capacity. The disproportion of the orders of the edges and the inducing corner effect will harm the achievable rate, while a higher dimension can increase the capacity. To obtain a higher capacity, we should pay attention to the design of the network geometry and properly choose the routing scheme.

VII. ACKNOWLEDGMENT

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APPENDIX SLOWROMANCAPI@

NECESSARY AND SUFFICIENT CONDITION FOR THE PERCOLATION HIGHWAY IN A STRIP NETWORK

Proof: If $\lim_{m \rightarrow \infty} \frac{w(m)}{\log m} = 0$, then $\lim_{n \rightarrow \infty} \frac{w(m)}{\log n} = \lim_{m \rightarrow \infty} \frac{w(m)}{\log m} \cdot \frac{\log m}{\log n} < \lim_{m \rightarrow \infty} \frac{w(m)}{\log m} \cdot 1 = 0$. From Appendix B in [1], we know the absence of highway backbone parallel with the width, which points out the necessary condition in Theorem 3.1. On the other hand, if $\lim_{m \rightarrow \infty} \frac{w(m)}{\log m} > 0$, then by following the same proof procedures of Theorem 5 in [5], we have

$$\begin{aligned} P_p(N_m \leq \delta \log m) &= (P_p(C_m^i \leq \delta \log m))^{\frac{w(m)}{\log m - \varepsilon_m}} \\ &\leq \left(\frac{4}{3}(m+1)m^{\delta \log \frac{p}{1-p} + \kappa \log 6(1-p)}\right) \\ &\quad \times (6(1-p))^{-\varepsilon_m} \end{aligned} \quad (4)$$

where C_m^i is the maximal number of edge-disjoint left to right crossings inside rectangle R_m^i , $N_m = \min_i C_m^i$, and p is the probability that an edge is open in the edge-percolation model of Fig. 1. Notice that δ , κ and ε_m are constants, we have the exponent $\frac{w(m)}{\log m - \varepsilon_m}$ in (4) will be greater than a certain positive constant m_0 , when $m \rightarrow \infty$. Thus if we let

$$\delta \log \frac{p}{1-p} + \kappa \log 6(1-p) < -1, \quad (5)$$

and by choosing $\delta(\kappa, p)$ small enough so that (5) is satisfied, we will obtain

$$\lim_{m \rightarrow \infty} P_p(N_m \leq \delta \log m) = 0, \quad (6)$$

which proves that there are w.h.p. at least $\delta \log m$ highways inside each rectangle R_m^i . Besides, notice that there are at most $\frac{\log m}{\sqrt{2c}}$ edge-disjoint horizontal highways inside it, we have the number of the highway is $\Theta(\log m)$. ■

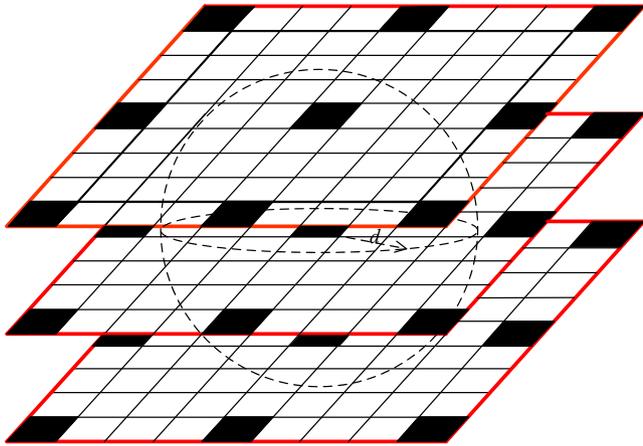


Fig. 9. A TDMA schedule. Nodes inside the cubes corresponding the gray squares can transmit simultaneously.

APPENDIX SLOWROMANCAPII@

ACHIEVABLE RATE OF THE TRANSMISSIONS WITHIN A CERTAIN DISTANCE IN 3D NETWORKS

Proof: We divide time into a sequence of k^3 successive slots, with $k = 2(d+1)$. Then, we consider the disjoint set of m -cubes $m_{i,j,k}$ that are allowed to transmit at the same time, as illustrated in Fig. 9. Suppose a transmitter in a given m -cube transmits toward a destination located in a cube at distance no more than d m -cubes away. First, we find an upper bound on the interferences at the receiver. Notice that the transmitters in the $3^3 - 1$ closest m -cubes are located at Euclidean distance at least $c(d+1)$ from the receiver. The $5^3 - 3^3$ next closest m -cubes are at Euclidean distance at least $c(3d+3)$, and so on. Thus, we can bound the interferences by

$$\begin{aligned} I(d) &\leq \sum_{i=1}^{\infty} ((2i+1)^3 - (2i-1)^3) P(c(2i-1)(d+1)) \\ &= P(c(d+1))^{-\alpha} \times \sum_{i=1}^{\infty} 2(12i^2+1)(2i-1)^{-\alpha} \quad (7) \end{aligned}$$

notice that this sum will converge if $\alpha > 3$.

Next, we observe that the distance between the transmitter and the receiver is at most d . Hence, the signal at the receiver can be bounded by

$$S(d) \geq P(d) = P \min\{1, d^{-\alpha}\} \quad (8)$$

Combining (7) and (8), we obtain a lower bound on the rate

$$R(d) = \log \left(1 + \frac{S(d)}{N_0 + I(d)} \right) \quad (9)$$

which does not depend on n , and hence the first part of the theorem immediately follows. We now let $d \rightarrow \infty$, then after some computations it follows easily that $R(d) = \Omega(d^{-\alpha})$. Finally, accounting for the time division into $k^3 = 8(d+1)^3$ time slots, the rate available in each m -cube is actually $\Omega(d^{-\alpha-3})$. ■

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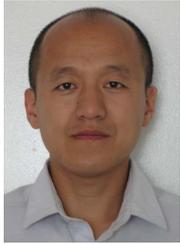
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