Coverage and Energy Consumption Control in Mobile Heterogeneous Wireless Sensor Networks

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***Abstract*—In this paper [1], we investigate the coverage and energy consumption control in mobile heterogeneous wireless sensor networks (WSNs). By term heterogeneous, we mean that sensors in the network have various sensing radius, which is an inherent property of many applied WSNs. Two sensor deployment schemes are considered –uniform and Poisson schemes. We study the asymptotic coverage under uniform deployment scheme with i.i.d. and 1-dimensional random walk mobility model, respectively. We propose the equivalent sensing radius (ESR) for both cases and derive the critical ESR correspondingly. Our results show that the network performance largely depends on ESR. By controlling ESR, we can always promise the network achieve full coverage, regardless of the total number of sensors or the sensing radius of a single senor under random mobility patterns, which is a much easier and more general way to operate coverage control. Meanwhile, we can operate a tradeoff control between coverage performance and energy consumption by adjusting ESR. We demonstrate that 1-dimensional random walk mobility can decrease the sensing energy consumption under certain delay tolerance, though requires larger ESR. Also, we characterize the role of heterogeneity in coverage and energy performance of WSNs under these two mobility models, and present the discrepancy of the impact of heterogeneity under different models. Under the Poisson deployment scheme, we investigate dynamic k-coverage of WSNs with 2-dimensional random walk mobility model. We present the relation between network coverage and the sensing range, which indicates how coverage varies according to sensing capability. Both k-coverage at an instant and over a time interval are explored and we derive the expectation of fraction of the whole operational region that is k-covered, which also identifies the coverage improvement brought by mobility.**

***Index Terms*—Coverage control, mobility, heterogeneity, energy consumption control, scaling law.**

I. Introduction

WIRELESS Sensor Networks (WSNs) have inspired a wide range of research, among which investigation on coverage is a fundamental one. The coverage of WSNs is of significance in many applications such as the security surveillance in estates, intrusion detection in battle-field or military restricted zone, etc.

In this paper, we focus on the *blanket coverage* or area coverage, which concentrates on the maximization of detection rate of targets in the sensing field. The operational region is said to be fully blanket covered if every single point in the region is sensed. The past decade has seen a surge of research activities on coverage. Normally, the coverage and energy consumption performance of WSNs are toned by one or several vital network parameter(s), e.g. the sensing range of sensors. By adjusting the sensing range, designer can deploy WSNs with proper coverage capability. Often, quantifiable relation between coverage and the parameter(s) is established for better analysis. We can know accurately from the results how the parameter(s) impact the network performance.

Initially, stationary and flat WSNs received most attention in the coverage study. By the term flat, we mean that sensors in the network have identical sensing radius. In [21], Clouqueur studied the sensor deployment strategy to improve the coverage performance of sensor network and proposed path exposure to measure the performance. Shakkottai [22] took into account the failure probability of sensors and obtained the necessary and sufficient conditions for a sensor grid to achieve asymptotic full coverage. In [23], Liu defined three coverage performance measures and characterized asymptotic behavior of these measures. In [36] and [37], WSNs are also well studied and give us great insights.

The 1-coverage studied in literature mentioned above is not satisfactory in many applications and high degree of coverage is consequently demanded (cf. [9] for the reasons to require k-coverage rather than just 1-coverage). Kumar [9] studied asymptotic k-coverage in a mostly sleeping stationary sensor network and found the coverage highly depend on the value of the function $\frac{np\pi r^2}{\log(np)}$ (n is the number of sensors and p is the probability that a sensor is active). The sufficient value of $\frac{np\pi r^2}{\log(np)}$ to ensure full coverage was derived under three different deployment model. Hence, the WSNs can take proper value of r to achieve full coverage. Another related topic is the approach to guarantee both coverage and connectivity of WSNs. In [10], Bai and Xuan proposed deployment schemes to achieve full coverage and k-connectivity. And a new deployment-polygon based methodology was introduced to prove the optimality of proposed deployment patterns.

One common feature of the above papers is that they all study static WSNs. Sensor mobility is actually a concern in coverage study. Plenty of related works concentrate on refining algorithm to reposition and control sensors to improve coverage [11][20][30]. In this sense, mobility is exploited to reconfigure the topology of the network. One commonly used approach for coverage control is developing Voronoi-based algorithm. In [32], Wang *et al.* proposed three movement protocols for sensors to fill the coverage hole based on the Voronoi approach. In [28], the authors optimized coverage performance with sensors of limited mobility. In [35], the authors considered

[1]The early version of this paper appeared in the Proceedings of IEEE ICDCS'11 [3].

coverage control for mobile sensing networks under controlled deployment. Other three extensions concerning the location optimization of heterogeneous sensors were developed in [29], based on a framework for optimized quantization derived in In [35]. On the other hand, in [4], Liu studied dynamic coverage which considered the coverage of sensors during their movement. The work demonstrated how to maintain various fractions of covered area by adjusting sensing range and sensor velocity in the movement. Coverage over a time interval was also explored, which distinguished the work.

These previous works on coverage control mainly focus on the controlled movements. From our perspective, however, random mobility has its own advantage and practical meaning. Under random mobility patterns, we can operate the coverage and sensing energy consumption control in a simpler and more general way. We only need to control the equivalent sensing radius (ESR) of the network to promise the full coverage performance. We can also carry out a flexible tradeoff control between coverage performance and sensing energy consumption, by changing ESR, regardless of the total number of sensors or the sensing radius of a single senor. In this sense, random mobility patterns provide us a way to achieve coverage and sensing energy consumption control with no need of communication or cooperation between sensors, which is needed for controlled movements and results in much overhead. Since random mobility patterns can save the energy used to collect and deal with information, it can relatively prolong the lifetime of the whole network and requires less complex equipments as well. Furthermore, in some cases, the sensors or their mobile hosts cannot communicate with each other due to nature or human factors, making it difficult to collect information from neighborhood and decide where to move, or their purpose is simply to monitor the area without any specific target (i.e. they have no destinations in their mind). As a result, the controlled motion as well as the corresponding methods can't be applied. Another reason for considering random model is that studying random mobility can provide basic guidelines for other more complicated models, including controlled ones, as it can give us an outlook of the impact of mobility for coverage in a relatively clear way. By calculating the energy consumption under random walk mobility, we have demonstrated the benefits for coverage performance brought by mobility, promising the worthiness of developing controlled algorithms for sensor movements.

In this paper, we investigate the coverage as well as the sensing energy consumption property in WSNs that are both mobile and heterogeneous. Although the sensing energy consumption is much less than the energy consumption due to communication among sensors, the former should be a concern in energy-efficient WSN design, since continuous sensing is usually the case in applications while the connection between sensors is not required all the time. In literature, mobility has been proved to enhance various aspects of network performance [14], [15], [17], and many applied WSNs are actually inherently mobile [6]. Meanwhile, it has been found that WSNs achieve better balance between performance and cost of sensors if opportune degree of heterogeneity is incorporated into the network by employing high-end and low-end sensors with different sensing capabilities [18]. Actually, due to various underlying reasons, sensors in WSNs are more likely to have different sensing radii. One scenario is that sensors in WSNs are products coming from different manufacturers and there is no uniform standard on the sensing range. Plus, sensors deployed in different times may also lead to heterogeneity of the network. And as the service lifetime passes by, degradation of sensing capability may be inevitable. Hence, heterogeneity is an inherent property of many WSNs, which deserves our attention.

We partition sensors into groups based on the sensing radius. The *equivalent sensing radius* (ESR) of the mobile heterogeneous WSNs will be defined to assist the analysis. The results demonstrate that full coverage of the operational region largely depends on the value of ESR, and so does the energy consumption. By controlling ESR, WSNs may achieve full coverage. In the study, we concentrate on how mobility and heterogeneity together influence the WSN performance. We derive the critical (necessary and sufficient) ESR in stationary flat WSNs and mobile heterogeneous WSNs, respectively. The value of critical ESR can help evaluate the overhead for the WSN to achieve full coverage. The advantages and drawbacks brought by mobility and heterogeneity are analyzed based on the results. The trade-offs between coverage and delay, sensing energy consumption and cost of sensors will be presented to provide insights on WSNs design.

Our main contributions are presented as below:

- Under the uniform deployment scheme, we study asymptotic coverage of heterogeneous WSNs with the i.i.d mobility model and 1-dimensional random walk mobility model. We define ESR of WSNs and analyze network performance using this metric. We obtain the critical value of ESR for the WSNs to achieve asymptotic full coverage of the operational region. We demonstrate that 1-dimensional random walk mobility reduces the energy consumption by the order $\Theta\left(\frac{\log n+\log\log n}{n}\right)$ [2] at the expense of $\Theta(1)$ delay;
- Under the uniform deployment scheme, heterogeneity is shown to impose no impact on energy consumption in stationary WSNs or WSNs with i.i.d. mobility model but to *'slightly'* increase sensing energy consumption in WSNs with the 1-dimensional random walk model;
- We study the WSNs under Poisson deployment strategy with the 2-dimensional random walk mobility model and derive the expectation of the fraction of operational region that is k-covered by the heterogeneous WSNs at an instant

[2]The following asymptotic notations are used throughout this paper. Given non-negative functions $f(n)$ and $g(n)$:

1) $f(n)=\Theta(g(n))$ means that for two constants $0<c_1<c_2$, $c_1 g(n)\leq f(n)\leq c_2 g(n)$ for sufficiently large n.
2) $f(n)\sim g(n)$ means that $\lim_{n\to+\infty}\frac{f(n)}{g(n)}=1$.

as well as over a time interval. The results hold for a general number of sensors except for the asymptotic case, and can provide guidelines on designing WSNs of high degree coverage. Plus, the detection time in the sensing process is investigated.

The rest of the paper is organized as follows. In Section II, the system model and several performance measures are defined. In Section III, we present our main results. The asymptotic coverage problem in mobile heterogeneous WSNs is studied in Section IV and we discuss the impact of mobility and heterogeneity based on the results. In Section V, we study k-coverage under Poisson deployment model. In Section VI, the detection delay is investigated. Finally, we conclude our work in Section VII.

II. System Model and Performance Measures

In this section, we describe the system model regarding sensing, deployment and mobility pattern, respectively and present several measures to assess the coverage performance of mobile heterogeneous WSNs.

A. Deployment Scheme

Let the operational region of the sensor network $\mathcal{A}$ be an unit square and this square is assumed to be a torus. This is to eliminate the boundary effect and equalize all points in the region for convenience of analysis. We consider two models according to which the sensors are deployed.

- Uniform Deployment Model — n sensors are randomly and uniformly deployed in the operational region, independent of each other.
- Poisson Deployment Model — Sensors are deployed according to a 2-dimensional Poisson point process with density parameter $\lambda = n$.

These random deployment strategies are favored in the situation that the geographical region to be sensed is hostile and inimical. Under such circumstance, wireless sensors might be sprinkled from aircraft, delivered by artillery shell, rocket, missile or thrown from a ship, instead of being placed by human or programmed robots. Specifically, uniform deployment model is commonly employed when the priori knowledge of the target area is unavailable, and as a most simple scheme, it provides insights for exploring more complex deployment strategies. Poisson deployment is a widely used method in literature to model the location of randomly-dropped sensors due to its memoryless and annexable property [23].

B. Sensing Strategy

Basically, we employ the *binary disc sensing model* in this study, where we assume that a sensor is capable to sense perfectly within the disc of radius r centered at the sensor. Beyond this sensing area, the sensor cannot sense. Here, r denotes the *sensing radius* of a sensor.

Further, our study takes into account the general case that sensors in the network have different sensing radii. We assume that there are u groups $G_1, G_2, \cdots, G_u$ in this heterogeneous network, where u is a positive constant. For $y = 1, 2, \cdots, u$, group G_y consists of $n_y = c_y n$ sensors, where n is the total number of sensors in the network and $c_y (y = 1, 2, \cdots, u)$ is called the *grouping index* which is a positive constant invariant of n (i.e., $c_y = \Theta(1)$) and $\sum_{y=1}^{u} c_y = 1$. For a given group G_y, sensors in this group possess identical sensing radius r_y. When $n_y = 1$, $u = n$, it is the special case where each sensor has different sensing radius from each other.

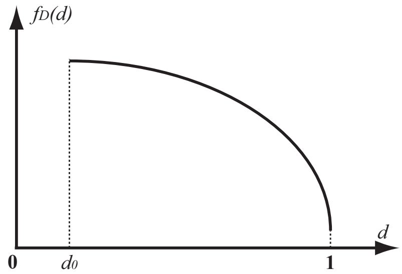

Fig. 1. A Typical Distribution of D.

C. Mobility Pattern

Sensors move according to certain mobility patterns.

- I.I.D. Mobility Model — The sensing process is partitioned into time slots with unit length. At the beginning of each time slot, each sensor will randomly and uniformly choose a position within the operational region and remain stationary in the rest of the time slot.
- 1-Dimensional Random Walk Mobility Model — Sensors in each group are classified into two types of equal quantity, H-nodes and V-nodes. And sensors of each type move horizontally and vertically, respectively. The sensing process is also divided into time slots with unit length. At the very beginning of each time slot, each sensor will randomly and uniformly choose a direction along its moving dimension and travel in the selected direction a certain distance D which follows the distribution function $f_D(d)$. To make the model non-trivial, $f_D(d)$ satisfies such requirement: $f_D(d) = 0$ when $d < d_0$ or $d > 1$, where d_0 is an arbitrary constant and $0 < d_0 < 1$. This requirement implies that the distance a sensor travel can be neither too short [3] nor too long [4]. A typical distribution is illustrated in Fig 1. We do not set requirements on the velocity of sensor during its movement, but sensors must reach destination within the time slot.
- 2-Dimensional Random Walk Mobility Model — Each sensor in group $G_y (y = 1, 2, \cdots, u)$ randomly and independently chooses a direction $\theta \in [0, 2\pi)$ according

[3] Short range travel approximates remaining stationary or i.i.d. model and may fail to gain benefits from movements.

[4] Long range travel is energy-consuming due the movement. And if the sensor can travel beyond the dimension of the operational area (i.e. $d > 1$), it can always cover the area along its moving dimension.

to probability distribution function (p.d.f.) $f_{\Theta}^{(y)}(\theta)$, and selects a velocity $v \in [0, v_{\max})$ according to p.d.f. $f_{V}^{(y)}(v)$.

The i.i.d. mobility pattern is widely used since it can provide some intuitions and characterize the upper or lower bound. We will present the main approach to asymptotic coverage problem under this model. The 1-dimensional mobility model is motivated by certain networks that nodes move along determined tracks such as the networks employed in streets, systems consisted of satellites moving in fixed orbits and so forth. The 2-dimensional random walk model can highly exploit the randomness of the motion of nodes and is suitable to depict realistic situation that the statistics about the habit of moving platforms is unknown.

D. Performance Measures

To assess the coverage performance of the wireless sensor networks, we propose four measures.

- ASYMPTOTIC COVERAGE — We define the equivalent sensing radius (ESR) to evaluate the overall performance of sensors with different radii.

  Definition 2.1: The ESR of the mobile heterogeneous WSNs with i.i.d. mobility model is $r_\star = \sqrt{\sum_{y=1}^{u} c_y r_y^2}$, and the ESR for 1-dimensional random walk mobility model is $r_\diamond = \sum_{y=1}^{u} c_y r_y$.

  Intuitively, the coverage capability of the network is positively correlated with ESR. The ESR needed when the network exactly achieves asymptotic coverage is called critical ESR. This direct connection between ESR and network performance is important for network design. For the network with ESR greater than critical ESR, the operation region will be full covered with probability one when n is large enough, which promises the sufficiency of critical ESR. While for those with ESR less than critical ESR, though n is large enough, the operation region still can't be full covered with probability one, which reflects the necessity of critical ESR. Let $\mathcal{C}$ denote the event that the operational region is fully covered. Then we can get the following definition. If

$$\lim_{n\to\infty} \mathbb{P}(\mathcal{C}) = 1, \text{if } r_\star \geq cR_\star(n) \text{ for any } c > 1;$$
$$\lim_{n\to\infty} \mathbb{P}(\mathcal{C}) < 1, \text{if } r_\star \leq \hat{c}R_\star(n) \text{ for any } 0 < \hat{c} < 1,$$

  $R_\star(n)$ is the critical ESR under the i.i.d. model. Similarly, we can define the critical ESR $R_\diamond(n)$ for the 1-dimensional random walk model.
- K-COVERAGE AT AN INSTANT — A point in the operational region is said to achieve k-coverage at an instant $t(t \geq 0)$ if it is sensed by at least k different sensors (they may come from various groups), where k is a positive integer. Let $\eta_{(t)}$ be the fraction of the whole operational region that achieves k-coverage at instant t.
- K-COVERAGE OVER A TIME INTERVAL — A point in the operational region is said to achieve k-coverage during a time interval, $\mathcal{T} = [0, t)$ if it has been sensed by at least k different sensors (they may come from various groups) at the end of the interval. Let $\eta_{(\mathcal{T})}$ denote the fraction of the whole area that achieves k-coverage within $\mathcal{T}$.
- DELAY OF DETECTION — Suppose certain point in the operational region is not covered at time $t = 0$. The delay of detection $\mathcal{D}$ is defined to be the time that initially uncovered point has been first ever covered. Similarly, $\mathcal{D}_k$ denotes the time for the initially uncovered point to be k-covered.

III. MAIN RESULTS

We summarize our main results in this paper as follows:

1. Under the uniform deployment scheme:

(1-a) With i.i.d. mobility model, the critical ESR is $R_\star(n) = \sqrt{\frac{\log n + \log\log n}{\pi n}}$.

(1-b) With 1-dimensional random walk mobility model, the critical ESR is $R_\diamond(n) = \frac{\log n + \log\log n}{\kappa n}$, where $\kappa = \kappa\left(f_D(d)\right) = 2\iint_{h\leq d} 2(1-h)\mathrm{d}h\mathrm{d}d$ is the function of $f_D(d)$.

2. Under the Poisson deployment scheme with the 2-dimensional random walk mobility model:

(2-a) $\mathbb{E}\{\eta_{(t)}\} = 1 - \frac{\Gamma\left(k, \pi n \sum_{y=1}^{u} c_y r_y^2\right)}{(k-1)!}$, where $\pi r_y^2 < |\mathcal{A}|$ holds for arbitrary y, and $\Gamma(a, b)$ is the upper incomplete gamma function, defined as $\Gamma(a,b) = \int_b^{+\infty} t^{a-1}e^{-t}\mathrm{d}t$.

(2-b) $\mathbb{E}\{\eta_{(\mathcal{T})}\} = 1 - \frac{\Gamma\left(k, n\sum_{y=1}^{u} c_y \mathbb{E}\{S_{(\mathcal{T},y)}\}\right)}{(k-1)!}$, where $\mathbb{E}\{S_{(\mathcal{T},y)}\}$ denotes the expected area covered by a sensor in group G_y during the time interval $\mathcal{T}$ and $\mathbb{E}\{S_{(\mathcal{T},y)}\} < |\mathcal{A}|$ holds for arbitrary y and $\mathcal{T}$, $\Gamma(a, b)$ is the upper incomplete gamma function.

3. Under the Poisson deployment scheme with 2-dimensional random walk mobility model and sensors move at constant velocity v_0 in sensing process:

(3-a) The delay of detection $\mathcal{D}_1$ is exponentially distributed $\mathcal{D}_1 \sim \text{Exponential}\left(2\lambda v_0 \sum_{y=1}^{u}(c_y r_y)\right)$.

(3-b) If $r_y = r$ and $f_{\Theta}^{(y)}(\theta) = \frac{1}{2\pi}$ for $y = 1, 2, \cdots, u$, the delay of full k-detection $\mathcal{D}_k$ is a random variable distributed according to the p.d.f. $f_{\mathcal{D}_k}(d) = \frac{\bar{\lambda}^k}{(k-1)!}d^{k-1}e^{-\bar{\lambda}d}$, where $\bar{\lambda} = 2\lambda r v_0$.

IV. ASYMPTOTIC COVERAGE UNDER UNIFORM DEPLOYMENT SCHEME

In this section, we analyze the asymptotic coverage under uniform deployment scheme with i.i.d. and 1-dimensional random walk mobility model, respectively.

A. Overview of Dense Grid

It is relatively difficult to analyze coverage by checking whether all points in the operational region are covered. To approach this problem, certain idea has been presented in [8] and [9], that is to transform the coverage of all points within the operational region to the coverage of certain set of points.

The set of points, denoted by $\mathbb{M}$, is all the grid points of a $\sqrt{m} \times \sqrt{m}$ grid on the operational region. From LEMMA 3.1

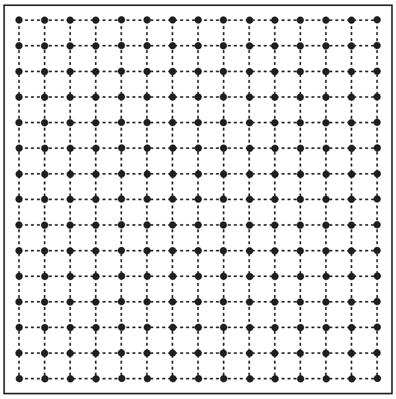

Fig. 2. Dense Grid Within the Operational Region.

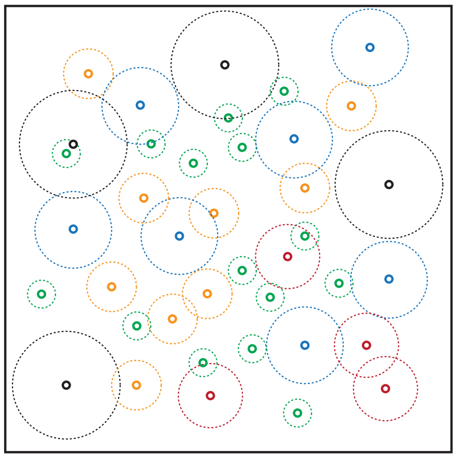

Fig. 3. Coverage Under I.I.D Mode.l

and THEOREM 4.1 in [9], we know that if m is large enough, such as $m = n\log n$ (i.e., the grid is dense enough), the sensing radius that is sufficient to guarantee the asymptotic coverage of points in $\mathbb{M}$ will be sufficient to ensure the asymptotic coverage of the entire operational region as well. And the necessary sensing radius for $\mathbb{M}$ is also necessary for the whole operational region to achieve full coverage. Then we can focus on the coverage of the dense grid. And we will derive the critical (both necessary and sufficient) ESR for sensor networks to achieve full coverage of the dense grid.

B. Critical ESR Under I.I.D. Mobility Model

Let $\mathcal{G}$ denote the event that the dense grid $\mathbb{M}$ is covered. And we derive the critical ESR to guarantee asymptotic full coverage of $\mathbb{M}$.

Definition 4.1: For mobile heterogeneous WSNs with i.i.d. mobility model, $r_\star(n)$ is the critical equivalent sensing radius for $\mathbb{M}$ if

$$\lim_{n\to\infty} \mathbb{P}(\mathcal{G}) = 1, \text{if } r_\star \geq c r_\star(n) \text{ for any } c > 1;$$
$$\lim_{n\to\infty} \mathbb{P}(\mathcal{G}) < 1, \text{if } r_\star \leq \hat{c} r_\star(n) \text{ for any } 0 < \hat{c} < 1.$$

We have the following useful lemmas.

Lemma 4.1: Given $x = x(n)$ and $y = y(n)$ both of which are positive functions of n, then $(1-x)^y \sim e^{-xy}$ if x and x^2y approach 0 as $n \to +\infty$.

Proof: Refer to Appendix. ■

Lemma 4.2: If $r_\star(n) = \sqrt{\frac{\log n + \log\log n + \xi}{\pi n}}$ and $m = m(n) = n\log n$, for any fixed $\theta < 1$,

$$m \prod_{y=1}^{u} \left(1 - \pi r_y^2(n)\right)^{c_y n} \geq \theta e^{-\xi}, \tag{1}$$

holds for all sufficiently large n.

Proof: See Appendix. ■

1) Necessary ESR for Full Coverage of Dense Grid: Let $\overline{\mathcal{G}}$ denote the event that the dense grid $\mathbb{M}$ is not fully covered and we have the following proposition.

Proposition 4.1: In the mobile heterogeneous WSNs with i.i.d. mobility model, if $r_\star(n) = \sqrt{\frac{\log n + \log\log n + \xi(n)}{\pi n}}$ and the density of the dense grid $\mathbb{M}$ is $m = n\log n$, then

$$\liminf_{n\to+\infty} \mathbb{P}(\overline{\mathcal{G}}) \geq e^{-\xi} - e^{-2\xi},$$

where $\xi = \lim_{n\to+\infty} \xi(n)$.

Proof: To begin with, we study the case where $r_\star(n) = \sqrt{\frac{\log n + \log\log n + \xi}{\pi n}}$ for a fixed ξ. Applying the Bonferroni inequality, we have that

$$\begin{aligned}
\mathbb{P}(\overline{\mathcal{G}}) &\geq \sum_{P_i\in\mathbb{M}} \mathbb{P}(\{\text{some point } P_i \text{ is not covered}\}) \\
&\geq \sum_{P_i\in\mathbb{M}} \mathbb{P}(\{P_i \text{ is the only uncovered point}\}) \\
&\geq \sum_{P_i\in\mathbb{M}} \mathbb{P}(\{P_i \text{ is not covered}\}) \\
&\quad - \sum_{\substack{P_i\neq P_j \\ P_i,P_j\in\mathbb{M}}} \mathbb{P}(\{P_i \text{ and } P_j \text{ are not covered}\})
\end{aligned} \tag{2}$$

Respectively, we can evaluate the two terms on the right hand side of (2). As for the first term, we have

$$\begin{aligned}
&\mathbb{P}(\{P_i \text{ is not covered}\}) \\
&= \prod_{y=1}^{u} \mathbb{P}(\{P_i \text{ is not covered by sensors in } G_y\}) \\
&= \prod_{y=1}^{u} \left(1 - \pi r_y^2(n)\right)^{c_y n}.
\end{aligned} \tag{3}$$

Using Lemma 4.2, we bound the first term that for any $\theta < 1$,

$$\sum_{P_i\in\mathbb{M}} \mathbb{P}(\{P_i \text{ is not covered}\}) \geq \theta e^{-\xi}, \tag{4}$$

for all $n > N_\xi$.

Let $s_y(n) = \pi r_y^2(n)$ and $s_\star(n) = \pi r_\star^2(n)$, then $s_\star(n) = \sum_{y=1}^{u} c_y s_y(n) = \frac{\log n + \log\log n + \xi}{n}$. Hence for all $y = 1, 2, \cdots, u$, $s_y(n) = \Theta\left(\frac{\log n + \log\log n + \xi}{n}\right)$ which indicates that $s_y(n)$ and $s_y^2(n)(c_y n)$ approach 0 as $n \to +\infty$. From Lemma 4.1, we obtain that for arbitrary positive constant α,

$$(1 - \alpha s_y(n))^{c_y n} \sim e^{-\alpha n (c_y s_y(n))}. \tag{5}$$

Thus, for two points P_i and P_j in $\mathbb{M}$, we obtain that

$$\begin{aligned} &\mathbb{P}(\{P_i \text{ and } P_j \text{ are not covered}\}) \\ &\geq \prod_{y=1}^{u} \left(1 - 4\pi r_y^2(n)\right)\left(1 - 2\pi r_y^2(n)\right)^{c_y n} \\ &\sim e^{-2n \sum_{y=1}^{u}(c_y s_y(n))}. \end{aligned} \tag{6}$$

and on the other hand, we have the upper bound

$$\begin{aligned} &\mathbb{P}(\{P_i \text{ and } P_j \text{ are not covered}\}) \\ &\leq \prod_{y=1}^{u} \pi r_y^2(n)\left(1 - \pi r_y^2(n)\right)^{c_y n} + \prod_{y=1}^{u}\left(1 - 2\pi r_y^2(n)\right)^{c_y n} \\ &\sim e^{-2n \sum_{y=1}^{u}(c_y s_y(n))}. \end{aligned} \tag{7}$$

From (6) and (7), we know that

$$\mathbb{P}(\{P_i \text{ and } P_j \text{ are not covered}\}) \sim e^{-2n \sum_{y=1}^{u}(c_y s_y(n))}. \tag{8}$$

Then the following result holds

$$\begin{aligned} &\sum_{P_i, P_j \in \mathbb{M}}^{P_i \neq P_j} \mathbb{P}(\{P_i \text{ and } P_j \text{ are not covered}\}) \\ &\sim m^2 e^{-2n \sum_{y=1}^{u}(c_y s_y(n))} \\ &= (n \log n)^2 e^{-2n s_\star(n)} \\ &= (n \log n)^2 e^{-2(\log n + \log\log n + \xi)} \\ &= e^{-2\xi}. \end{aligned} \tag{9}$$

Using (4) and (9) into (2), we have

$$\mathbb{P}(\overline{\mathcal{G}}) \geq \theta e^{-\xi} - e^{-2\xi}. \tag{10}$$

for any $\theta < 1$.

As for the case that ξ is a function of n with $\xi = \lim_{n\to+\infty} \xi(n)$, we know $\xi(n) \leq \xi + \delta$ for any $\delta > 0$ all $n > N_\delta$. Since $\mathbb{P}(\overline{\mathcal{G}})$ is monotonously decreasing in $r_\star$ and thus in ξ, we have

$$\mathbb{P}(\overline{\mathcal{G}}) \geq \theta e^{-(\xi+\delta)} - e^{-2(\xi+\delta)}, \tag{11}$$

for all $n > N_{\theta,\delta}$. Then the result follows. ■

From Proposition 4.1, $\mathbb{P}(\overline{\mathcal{G}})$ is bounded away from zero with positive probability if $\lim_{n\to+\infty} \xi(n) < +\infty$. Combined with Definition 4.1, we know that $r_\star \geq \sqrt{\frac{\log n + \log\log n}{\pi n}}$ is necessary to achieve the full coverage of the dense grid $\mathbb{M}$.

2) Sufficient ESR for Full Coverage of Dense Grid: Let $\mathcal{F}_i$ denote the event that point P_i in $\mathbb{M}$ is not covered. If $r_\star = c\sqrt{\frac{\log n + \log\log n}{\pi n}}$ where $c > 1$, then

$$\begin{aligned} \mathbb{P}\left(\bigcup_{i=1}^{m} \mathcal{F}_i\right) &\leq \sum_{i=1}^{m} \mathbb{P}(\mathcal{F}_i) \\ &\leq (n \log n) \prod_{y=1}^{u} \left(1 - \pi r_y^2(n)\right)^{c_y n} \\ &\sim (n \log n) e^{-n\pi (r_\star)^2} \\ &= \frac{1}{(n \log n)^{c^2 - 1}}. \end{aligned}$$

For any $c > 1$, we then have the following result

$$\lim_{n\to+\infty} \mathbb{P}\left(\bigcup_{i=1}^{m} \mathcal{F}_i\right) = 0.$$

Therefore, $r_\star \geq \sqrt{\frac{\log n + \log\log n}{\pi n}}$ is sufficient to guarantee the full coverage of the dense grid.

3) Critical ESR for Full Coverage of Operational Region: The density of the dense grid $m = n \log n$ is sufficiently large to evaluate the coverage problem of the whole operational region. Referring to LEMMA 3.1 in [9] and using similar approach as THEOREM 4.1 in [9], we can demonstrate that $r_\star \geq \sqrt{\frac{\log n + \log\log n}{\pi n}}$ is sufficient to achieve the full coverage of the whole operational region. On the other hand, points in $\mathbb{M}$ constitute a subregion of the operational region, which indicates that the necessary condition for the dense grid is surely the necessary condition for the whole operational region.

Hence, we have the following theorem

Theorem 4.1: Under the uniform deployment scheme with i.i.d. mobility model, the critical ESR for mobile heterogeneous WSNs to achieve asymptotic full coverage is $R_\star(n) = \sqrt{\frac{\log n + \log\log n}{\pi n}}$.

C. Critical ESR Under 1*-Dimensional Random Walk Mobility Model*

Under the 1-dimensional random walk mobility model, the sensing process is slotted and sensors make use of each time slot to move and sense. We use $\mathcal{G}^\tau$ to denote the event that $\mathbb{M}$ is covered in a given time slot τ, and $\mathbb{P}_\tau(\mathcal{G}^\tau)$ to denote the corresponding probability. Similarly, we define the critical ESR for 1-dimensional random walk model.

Definition 4.2: For mobile heterogeneous WSNs with 1-dimensional random walk mobility model, $r_\diamond(n)$ is the critical equivalent sensing radius for the dense grid if

$$\lim_{n\to\infty} \mathbb{P}_\tau(\mathcal{G}^\tau) = 1, \text{if } r_\diamond \geq c r_\diamond(n) \text{ for any } c > 1;$$
$$\lim_{n\to\infty} \mathbb{P}_\tau(\mathcal{G}^\tau) < 1, \text{if } r_\diamond \leq \hat{c} r_\diamond(n) \text{ for any } 0 < \hat{c} < 1.$$

1) Failure Probability of a Point in $\mathbb{M}$: We use $\mathcal{F}_i$ to denote the event that point P_i in $\mathbb{M}$ is not covered by any sensor and use $\mathbb{P}(\mathcal{F}_i)$ to denote the probability that point P_i in $\mathbb{M}$ is not covered.

We first consider the probability that an arbitrary point P_i in $\mathbb{M}$ can be successfully covered by a sensor X_y in group G_y, and denote the probability as $\mathbb{P}_{(i,y)}$. When sensor moves under 1-dimensional random walk mobility, it can cover more space (a rectangular of size $r_y \times D$) during a time slot τ than stationary condition, thus increasing $\mathbb{P}_{(i,y)}$. For $\mathbb{P}_{(i,y)}$ we have $\mathbb{P}_{(i,y)} = \pi r_y^2 + \mathbb{P}_{(i,y,\tau)}$, where πr_y^2 denotes the probability that X_y can cover P_i when it is stationary, and $\mathbb{P}_{(i,y,\tau)}$ denotes the probability that X_y can cover P_i considering its mobility. In the following part, we first derive $\mathbb{P}_{(i,y,\tau)}$ and then obtain $\mathbb{P}_{(i,y)}$ as well as $\mathbb{P}(\mathcal{F}_i)$.

Because of the symmetry of the topology, we only need to take into account the case that X_y moves horizontally.

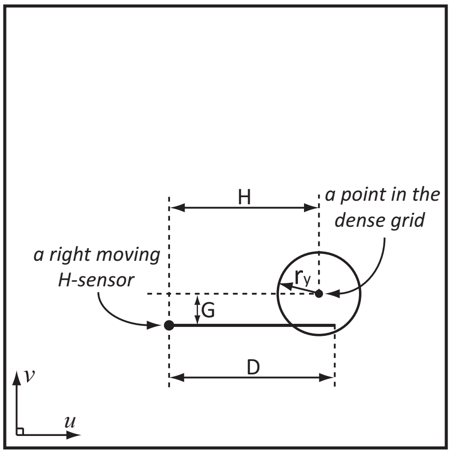

Fig. 4. Coverage of a Single Point.

Since X_y chooses to move left or right with equal probability, we suppose X_y moves right. Initially, sensors are uniformly deployed and according to the 1-dimensional random walk mobility model, sensors are always uniformly distributed at each time slot τ in the operational region seen by the points in $\mathbb{M}$. On the other hand, the dense grid is randomly and uniformly located in the operational region.

We build a Cartesian coordinate system in the operational region and denote the position of X_y and P_i with (u_1, v_1) and (u_2, v_2), respectively. Then we know that u_1, v_1, u_2, v_1 are uniformly distributed from 0 to 1.

Since X_y does not move vertically, when considering the vertical distance between X_y and P_i which is denoted as G, we should recognize the fact that the upper boundary of the operational region is adjacent to the lower boundary (considering the operational region is a torus). Hence, we have $G = \min\{|v_1 - v_2|, 1 - |v_1 - v_2|\}$. However, on the horizontal dimension, X_y moves in certain direction and might sense P_i on its way, which leads the horizontal distance to be $H = |u_1 - u_2|$.

Then, we can have the p.d.f. of G and H

$$f_G(g) = \begin{cases} 2, & 0 \le g \le \frac{1}{2}; \\ 0, & \text{otherwise}. \end{cases}$$

$$f_H(h) = \begin{cases} 2(1-h), & 0 \le h \le 1; \\ 0, & \text{otherwise}. \end{cases}$$

Point P_i can be successfully covered by X_y if and only if X_y can enter the circle with radius r_y centered at P_i. For $\mathbb{P}_{(i,y)}$,

$$\begin{aligned} \mathbb{P}_{(i,y,\tau)} &= \mathbb{P}(G \le r_y) \cdot \mathbb{P}(H \le D) \\ &= (2r_y) \iint_{h \le d} 2(1-h) \mathrm{d}h \mathrm{d}d \;=\; \kappa r_y. \end{aligned} \quad (12)$$

Here $\kappa = \iint_{h \le d} 2(1-h)\mathrm{d}h\mathrm{d}d$ is a constant if $f_D(d)$ is known.

$$\mathbb{P}_{(i,y)} = \pi r_y^2 + \mathbb{P}_{(i,y,\tau)} = \pi r_y^2 + \kappa r_y \quad (13)$$

Before we derive the critical ESR, firstly we would like to derive the range of d_0, to promise $\mathbb{P}_{(i,y)}$ together with $\mathbb{P}_{(i,y,\tau)}$ nonzero. For κ, we have

$$\begin{aligned} \kappa &= \iint_{h \le d} 2(1-h)\mathrm{d}h\mathrm{d}d \\ &= \int_{d_0}^{1} f_D(d) \int_0^d 2(1-h)\mathrm{d}h\mathrm{d}d \\ &= \int_{d_0}^{1} (2d - d^2) f_D(d)\mathrm{d}d \end{aligned}$$

According to this integration, if $f_D(d)$ is independent of d (uniform deployment as an example), the necessary condition for $\mathbb{P}_{(i,y,\tau)}$ to be nonzero is

$$\begin{aligned} &\int_{d_0}^{1} (2d - d^2)\mathrm{d}d \\ &= \frac{d_0^3 - 3d_0^2 + 2}{3} \\ &= \frac{(d_0 - 1)(d_0 - 1 + \sqrt{3})(d_0 - 1 - \sqrt{3})}{3} > 0 \end{aligned}$$

Thus we get $0 < d_0 < 1$.

For cases when $f_D(d)$ is a function of d, intuitively, if $0 < d_0 < 1$, the sensor always has the chance to move long enough to cover the point i in one step, since the maximum distance in the operational region is 1, which promises the necessity that $\mathbb{P}_{(i,y,\tau)}$ is nonzero.

Therefore $0 < d_0 < 1$ is necessary for $\mathbb{P}_{(i,y)}$ to be nonzero in one move or a fixed number of moves, since $\mathbb{P}_{(i,y)} > \mathbb{P}_{(i,y,\tau)}$ and $\mathbb{P}(\mathcal{F}_i)$ can be calculated according to the distribution of D.

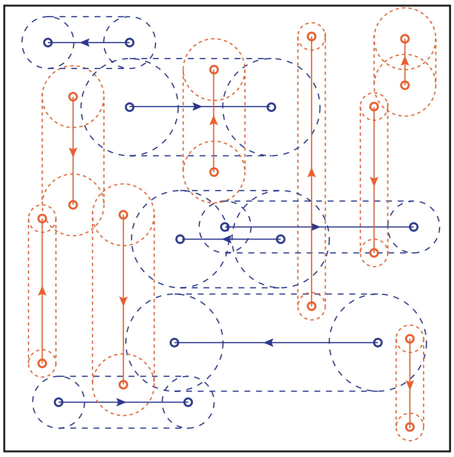

Fig. 5. Coverage Under 1-Dimensional Random Walk Mobility Model.

2) Necessary ESR for Full Coverage of Dense Grid: Let $\overline{\mathcal{G}^\tau}$ denote the event that the dense grid $\mathbb{M}$ is not fully covered in the given time slot τ. We present the following proposition regarding the necessary condition on ESR.

Proposition 4.2: In the mobile heterogeneous WSNs with 1-dimensional random walk mobility model, if $r_\diamond = \frac{\log n + \log\log n + \xi(n)}{\kappa n}$ and the density of the dense grid $\mathbb{M}$ is $m = n \log n$, then

$$\liminf_{n\to+\infty} \mathbb{P}_\tau(\overline{\mathcal{G}^\tau}) \geq e^{-\xi} - e^{-2\xi}.$$

where $\xi = \lim_{n\to+\infty} \xi(n)$.

Proof: The technique used in this proof is similar to that used in the proof of Proposition 4.1, and we present the main steps here. We begin with the case that $r_\diamond = \frac{\log n + \log\log n + \xi}{\kappa n}$ for a fixed ξ.

$$\mathbb{P}_\tau(\overline{\mathcal{G}^\tau}) \geq \sum_{P_i\in\mathbb{M}} \mathbb{P}_\tau(\{P_i \text{ is not covered}\}) - \sum_{P_i,P_j\in\mathbb{M}}^{P_i\neq P_j} \mathbb{P}_\tau(\{P_i, P_j \text{ are not covered}\}). \quad (14)$$

Based on (13), we can evaluate the first term of on the right hand of side of (14) and have

$$\begin{aligned} &\mathbb{P}_\tau(\{P_i \text{ is not covered}\}) \\ &= \prod_{y=1}^{u} \left(1 - (\pi r_y^2(n) + \kappa r_y(n))\right)^{c_y n} \\ &= \prod_{y=1}^{u} \left(1 - (1+\zeta_y)\kappa r_y(n)\right)^{c_y n}. \end{aligned} \quad (15)$$

where $\zeta_y = \frac{\pi r_y(n)}{\kappa}$. Let $\zeta = \sum_{y=1}^{u} c_y \zeta_y$, we can easily obtain $\zeta = o(r_\diamond)$. Taking the logarithm of the right side of (15) and using Lemma 4.1, we have

$$\begin{aligned} &\log(\text{R.H.S. of (15)}) \\ &= \sum_{y=1}^{u} \left(c_y n \left(\log \left(1 - (1+\zeta_y)\kappa r_y(n) \right) \right) \right) \\ &\sim -\sum_{y=1}^{u} \left(c_y n \kappa r_y(n) \right) = -\log n - \log\log n - \xi \end{aligned}$$

for all sufficiently large n, considering $\zeta = o(r_\diamond)$. Therefore,

$$\sum_{P_i\in\mathbb{M}} \mathbb{P}_\tau(\{P_i \text{ is not covered}\}) = \Theta(me^{-\log n - \log\log n - \xi}) \geq \theta e^{-\xi} \quad (16)$$

for any fixed $\theta < 1$ and all $n > N(\xi, \theta)$.

Let $t_y(n) = \kappa r_y(n)$ and $t_\diamond(n) = \kappa r_\diamond(n)$. And we can similarly obtain the lower bound

$$\begin{aligned} &\mathbb{P}_\tau(\{P_i, P_j \text{ are not covered}\}) \\ &\geq \prod_{y=1}^{u} \left(1 - 4\pi r_y^2(n)\right) \left(1 - 2(1+\zeta_y)\kappa r_y(n)\right)^{c_y n} \\ &\sim e^{-2n\sum_{y=1}^{u}(c_y t_y(n))}, \end{aligned} \quad (17)$$

and the upper bound

$$\begin{aligned} &\mathbb{P}_\tau(\{P_i, P_j \text{ are not covered}\}) \\ &\leq \prod_{y=1}^{u} \pi r_y^2(n) \Big(1 - (1+\zeta_y)\kappa r_y(n)\Big)^{c_y n} + \prod_{y=1}^{u} \Big(1 - (1+\zeta_y)\kappa r_y(n)\Big)^{c_y n} \\ &\leq \prod_{y=1}^{u} \pi r_y^2(n) (1 - \kappa r_y(n))^{c_y n} + \prod_{y=1}^{u} (1 - \kappa r_y(n))^{c_y n} \\ &\sim e^{-2n\sum_{y=1}^{u}(c_y t_y(n))}. \end{aligned}$$

Combining (17) and (18), we have

$$\begin{aligned} &\sum_{P_i,P_j\in\mathbb{M}}^{P_i\neq P_j} \mathbb{P}_\tau(\{P_i, P_j \text{ are not covered}\}) \\ &\sim m^2 e^{-2n\sum_{y=1}^{u}(c_y t_y(n))} = e^{-2\xi}. \end{aligned} \quad (18)$$

Thus, using (16) and (18) into (14), we obtain

$$\mathbb{P}_\tau(\overline{\mathcal{G}^\tau}) \geq \theta e^{-\xi} - e^{-2\xi}. \quad (19)$$

Taking into account the case that ξ is a function of n, the conclusion still holds. ■

From Proposition 4.2, we know that $r_\diamond \geq \frac{\log n + \log\log n}{\kappa n}$ is necessary to achieve the full coverage of $\mathbb{M}$.

3) Sufficient ESR for Full Coverage of Dense Grid: If $r_\diamond = c \cdot \frac{\log n + \log\log n}{\kappa n}$ where $c > 1$, then

$$\begin{aligned} \mathbb{P}_\tau\left(\bigcup_{i=1}^m \mathcal{F}_i\right) &\leq \sum_{i=1}^m \mathbb{P}_\tau(\mathcal{F}_i) \\ &\leq (n\log n)\prod_{y=1}^u \Big(1-(1+\zeta_y)\kappa r_y(n)\Big)^{c_y n} \\ &\leq (n\log n)\prod_{y=1}^u \big(1-\kappa r_y(n)\big)^{c_y n} \\ &\sim (n\log n)e^{-n\pi(r_\diamond)^2} \\ &= \frac{1}{(n\log n)^{c^2-1}} \to 0 \text{ as } n\to+\infty. \end{aligned}$$

Therefore, $r_\diamond \geq \frac{\log n + \log\log n}{\kappa n}$ is sufficient to guarantee the full coverage of the dense grid $\mathbb{M}$.

4) Further Discussion: If we only take the rectangular area the sensor covers when it moves into account(i.e. use $\mathbb{P}_{(i,y,\tau)}$ to replace $\mathbb{P}_{(i,y)}$), we can also prove the necessity and sufficiency of $r_\diamond \geq \frac{\log n + \log\log n}{\kappa n}$ to guarantee the full coverage of dense grid $\mathbb{M}$ using similar approach.

Thus we can conclude that when we consider the critical ESR for sensors under 1-dimensional random walk mobility, the rectangular area the sensor covers when it moves contributes most for coverage rather than the circle area it covers when it is static.

5) Critical ESR for Full Coverage of Operational Region: Similar to the analysis in the i.i.d. mobility model, we can reach the following theorem.

Theorem 4.2: Under the uniform deployment scheme with 1-dimensional random walk mobility model, the critical ESR for mobile heterogeneous WSNs to achieve asymptotic full coverage is $R_\diamond(n) = \frac{\log n + \log\log n}{\kappa n}$.

D. The Impact of Mobility and Heterogeneity on Sensing Energy Consumption

We discuss the impact of mobility and heterogeneity on sensing energy consumption based on the results obtained in previous parts of this section.

We used the sensing energy model as $E_y \propto r_y^2$, where E_y is the energy consumption of sensors with sensing radius r_y. Let $\overline{E}$ denote the average energy consumption of the mobile heterogenous WSNs , thus $\overline{E} \propto \sum_{y=1}^u c_y r_y^2$.

1) Impact of Mobility: First, we consider the impact of mobility and sensors are assumed to have identical sensing radius, i.e., $r_y = r_\star$ or $r_y = r_\diamond (y = 1, 2, \cdots, u)$ under i.i.d and 1-dimensional random walk mobility model, respectively. We have the following results

(a) Under I.I.D. Mobility Model:

$$\overline{E}_{i.i.d.} = \Theta\left(\frac{\log n + \log\log n}{n}\right). \quad (20)$$

(b) Under 1-Dimensional Random Walk Mobility Model:

$$\overline{E}_{r.w.} = \Theta\left(\left(\frac{\log n + \log\log n}{n}\right)^2\right). \quad (21)$$

From the derivation of ESR under i.i.d. mobility model, we can realize that i.i.d. mobility is actually quasi-static since the reshuffle of sensor positions does not increase the area of sensed region in a time slot compared with the stationary case. The energy consumption $\overline{E}_{stat.}$ in static WSNs equals that in WSNs with i.i.d model. In COROLLARY 5.1 of [9], the author presented that in a static and homogeneous network under uniform deployment,

$$c(n) \geq 1 + \frac{\phi(np) + k\log\log(np)}{\log(np)}$$

is sufficient for an unit square to be asymptotically k-covered, where $c(n) = \frac{np\pi r^2}{\log(np)}$, $\phi(np) = o(\log\log(np))$ and p is the probability that a sensor is currently operating. By assuming $p = 1$, $k = 1$ and ignoring $\phi(np)$ as $n \to \infty$, we translate this landmark result to our model. We obtain

$$\begin{aligned} \frac{n\pi r^2}{\log n} &\geq 1 + \frac{\log\log n}{\log n} \\ r &\geq \sqrt{\frac{\log n + \log\log n}{\pi n}} \end{aligned}$$

which matches our results under i.i.d. mobility model, verifying that $\overline{E}_{i.i.d.} = \overline{E}_{stat.}$. Therefore, we obtain that

$$\overline{E}_{r.w.} = \Theta\left(\frac{\log n + \log\log n}{n}\right) \cdot \overline{E}_{stat.},$$

which indicates that 1-dimensional random walk mobility model can decrease energy consumption in WSNs.

However, this improvement in energy efficiency sacrifices the timeliness of detection since under 1-dimensional random walk mobility model we evaluate the coverage performance of WSNs in a time slot τ while in stationary WSNs full coverage is maintained in any time instant. The delay to achieve full coverage is upper bounded by $\Theta(1)$. This is the trade-off between energy consumption and delay of coverage in mobile WSNs. Practically, designers should also consider another possible source of energy consumption: agents motion. Sometimes sensors move by motors and wheels equipped on them, which must consume the electricity in the battery they carry. As sensors keep moving in all time slots under both 1-dimensional and 2-dimensional random walk mobility model, energy consumption of this part should be of large quantity. However, the specific value of energy consumed depends mainly on the physical entity of the agents, and therefore is totally another topic which is beyond our scope. Otherwise, sensors are fixed on moving vehicles or flyers and move passively with their ”hosts”. Energy consumption by motion needs not be discussed in these scenarios. Then designers can balance energy consumption and delay of coverage by choosing sensors to be mobile or not.

2) Impact of Heterogeneity: Second, we consider the impact of heterogeneity. We have the following technical lemma.

Lemma 4.3: For arbitrary q that $0 < q < 1$,

$$\left(\sum_{y=1}^{u} c_y r_y^2\right) < \left(\sum_{y=1}^{u} c_y r_y\right)^{\frac{3}{2}} \left(\frac{1}{n^q}\right)^{\frac{1}{2}}, \tag{22}$$

where c_y, r_y, u are as defined in the system model, and $r_y = \Theta\left(\frac{\log n}{n}\right)$.

Proof: Please refer to Appendix. ∎

Then we can have the following results.

(a) Under I.I.D. Mobility Model:

$$\overline{E}_{i.i.d.} = \Theta\left(\frac{\log n + \log\log n}{n}\right). \tag{23}$$

(b) Under 1-Dimensional Random Walk Mobility Model:

$$\overline{E}_{r.w.} > \Theta\left(\left(\frac{\log n + \log\log n}{n}\right)^2\right); \tag{24}$$

$$\overline{E}_{r.w.} < \Theta\left(\frac{(\log n + \log\log n)^{\frac{3}{2}}}{n^{\frac{3}{2}+\frac{1}{2}q}}\right). \tag{25}$$

The lower bound in (b) comes from the Cauchy Inequality that

$$\left(\sum_{y=1}^{u} c_y\right)\left(\sum_{y=1}^{u} c_y r_y^2\right) > \left(\sum_{y=1}^{u} c_y r_y\right)^2.$$

And the upper bound results from Lemma 4.3.

Hence, with the sensing energy model $E_y \propto r_y^2$, heterogeneity does not make any difference to sensing energy in WSNs under i.i.d. mobility model or stationary WSNs, which can be seen from (20) and (23). However, from (21) and (24) we know that under 1-dimensional random walk mobility model, sensing energy consumption will increase due to heterogeneity. And there is a trade-off in mobile WSNs that designers must face: on one hand, heterogeneous WSNs composed of high-end sensors with large sensing range and low-end sensors with small sensing range can reduce the cost of WSNs and guarantee a satisfactory sensing performance; on the other hand, heterogeneity will increase sensing energy consumption. From the upper bound in (25), the order of energy consumption in terms of n approaches the order (i.e., n^2) in the case without heterogeneity. Hence, the energy efficiency will not be dramatically deteriorated by the heterogeneity.

Remark 4.1: The sensing process in WSNs depends on the area covered by each sensor. And under the 1-dimensional random walk mobility model, the area covered by sensors is on the same order of sensing radius r. In stationary WSNs or WSNs with i.i.d. mobility model, however, the covered area is on the order of r^2. This discrepancy of dependence on sensing radius leads the impact of heterogeneity to be different under the two models.

E. Full Coverage Control and Sensing Energy Consumption Control

From previous discussions, we can know that critical ESR is the critical condition for full coverage of the operational region. Therefore, we can control the ESR to promise the network achieve full coverage under different random mobility patterns, as shown in Figure 6. As the total number of sensors increases, the critical ESR decreases for I.I.D mobility model and 1-dimensional random walk mobility model. When we want to control the full coverage, we can make the network achieve the critical ESR under I.I.D mobility and 1-dimensional random walk mobility model as illustrated in the figure.

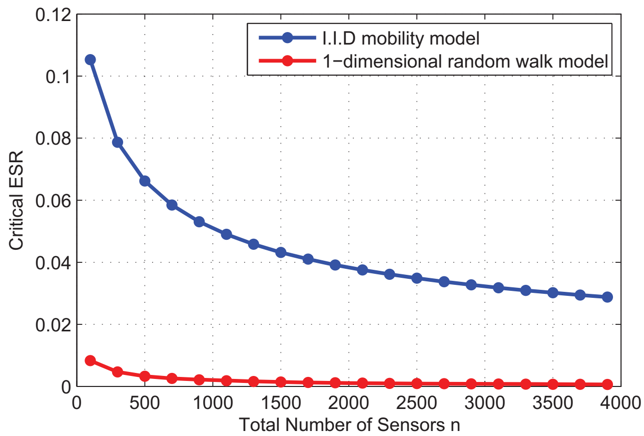

Fig. 6. Relationship between ESR and total number of sensors n.

Since the sensing energy consumption is positively correlated to the ESR as discussed previously, if we want to further decrease the energy consumption, we have to decrease the ESR. This further reduction of ESR will make the network not be full covered, thus sacrificing the coverage performance, which is actually a tradeoff control realized by the value of ESR.

V. K-Coverage In Mobile Heterogeneous WSNs Under Poisson Deployment Model

In this section, we study k-coverage at an instant and over a time interval in mobile heterogeneous WSNs. The sensor locations within the operational region are modeled as 2-dimensional Poisson process with density n. Thus, the coverage problem can be described by the frequently used Poisson-Boolean model. Besides, the 2-dimensional random walk mobility model is employed in this part.

A. K-coverage at an Instant

We start with the results regarding k-coverage at an instant.

Theorem 5.1: Given an instant $t > 0$, the expectation of the fraction of operational region that is k-covered at instant t is

$$\mathbb{E}\{\eta_{(t)}\} = 1 - \frac{\Gamma\left(k, \pi n \sum_{y=1}^{u} c_y r_y^2\right)}{(k-1)!},$$

where $\Gamma(a, b)$ is the upper incomplete gamma function, defined as $\Gamma(a, b) = \int_b^{+\infty} t^{a-1} e^{-t} \mathrm{d}t$.

Proof: For any point B in the operational region $\mathcal{A}$, define an indicator function I_B on whether B is k-covered at instant t as follows,

$$I_B = \begin{cases} 1, \text{ point } B \text{ is } k\text{-covered at instant } t, \\ 0, \text{ otherwise.} \end{cases}$$

Then we acquire that

$$\mathbb{E}\{I_B\} = \mathbb{P}(\{\text{point } B \text{ is } k\text{-covered at instant } t\}). \tag{26}$$

Denote the total area in $\mathcal{A}$ that is k-covered as $|\mathcal{V}|$. We have $|\mathcal{V}| = \int_{\mathcal{A}} I_B \mathrm{d}B$. By Fubini's theorem, exchanging the order of integral and expectation, we obtain

$$\mathbb{E}\{|\mathcal{V}|\} = \int_{\mathcal{A}} \mathbb{E}\{I_B\} \mathrm{d}B. \tag{27}$$

Let $\eta_{(t)}$ be the fraction of area that is k-covered at the instant t. Clearly, since we do not consider the boundary effects, for any point B in $\mathcal{A}$, the probability that B is k-covered is identical. Therefore, substituting (26) into (27),

$$\begin{aligned} \mathbb{E}\{\eta_{(t)}\} &= \frac{\int_{\mathcal{A}} \mathbb{P}(\{B \text{ is } k\text{-covered at instant } t\}) \mathrm{d}B}{|\mathcal{A}|} \\ &= \mathbb{P}(\{B \text{ is } k\text{-covered at instant } t\}), \end{aligned} \tag{28}$$

where B is any given point in $\mathcal{A}$. The equality (28) means that the expected value of $\eta_{(t)}$ is equal to the probability that any given point in $\mathcal{A}$ is k-covered.

Let $C(B, r)$ be the circle centered at point B with radius r. If a sensor X with sensing range r is in $C(B, r)$, then sensor X covers point B. Use $m_{(t,y,B,r_y)}$ to denote the number of sensors in group G_y which have ever been in $C(B, r_y)$ at the given instant t. Define $M(B, t) := \sum_{y=1}^{u} m_{(t,y,B,r_y)}$. Then $M(B, t)$ is the number of sensors that cover point B at the instant t. Thus, point B is k-covered at instant t if and only if the following inequality holds: $M(B, t) \geq k$.

Since sensors are Poisson distributed at time $t = 0$ and follow the random mobility pattern, the locations of sensors at arbitrary instant t can still be modeled as Poisson point process of density n. Further, sensors in the group $G_y (y = 1, 2, \cdots, u)$ can be seen to follow a Poisson point process with density $n_y = c_y n (y = 1, 2, \cdots, u)$. Therefore, at the given instant t, $m_{(t,y,B,r_y)}$ follows the Poisson distribution with density $n_y \pi r_y^2$. Namely, $m_{(t,y,B,r_z)} \sim \text{Poisson}(n_y \pi r_y^2)$.

As $M(B, t)$ is the sum of independent Poisson-distributed random variables $m_{(t,y,B,r_y)}$ $(y = 1, 2, \cdots, u)$, thereby $M(B, t)$ obeys a Poisson distribution with parameter $\sum_{y=1}^{u} n_y \pi r_y^2$. Then, we have

$$M(B, t) \sim \text{Poisson} \left(\sum_{y=1}^{u} n_y \pi r_y^2 \right),$$

and thus we obtain the probability mass function

$$\begin{aligned} &\mathbb{P}(M(B, t) = m) \\ &= \frac{1}{m!} \left(\sum_{y=1}^{u} n_y \pi r_y^2 \right)^m \exp \left(- \sum_{y=1}^{u} n_y \pi r_y^2 \right). \end{aligned} \tag{29}$$

From (29), we can derive

$$\begin{aligned} &\mathbb{P}(\{B \text{ is not } k\text{-covered at instant } t\}) \\ &= P(M(B, t) \leq k - 1) \\ &= \sum_{m=0}^{k-1} \frac{1}{m!} \left(\sum_{y=1}^{u} n_y \pi r_y^2 \right)^m \exp \left(- \sum_{y=1}^{u} n_y \pi r_y^2 \right) \\ &= \frac{\Gamma \left(k, \sum_{y=1}^{u} n_y \pi r_y^2 \right)}{(k-1)!} \\ &= \frac{\Gamma \left(k, n \sum_{y=1}^{u} c_y \pi r_y^2 \right)}{(k-1)!}. \end{aligned} \tag{30}$$

Then from (28) and (30), the result follows. ■

B. K-coverage over a Time Interval

Similar results can be obtained for k-coverage over a time interval $\mathcal{T}$.

Theorem 5.2: Given a time interval $\mathcal{T} = [0, t)$, the expectation of the fraction of the operational region that is k-covered during this interval is

$$\mathbb{E}\{\eta_{(\mathcal{T})}\} = 1 - \frac{\Gamma\Big(k, n \sum_{y=1}^{u} c_y \mathbb{E}\{S_{(\mathcal{T},y)}\}\Big)}{(k-1)!},$$

where $\mathbb{E}\{S_{(\mathcal{T},y)}\}$ denotes the expected area covered by a sensor in group G_y during the time interval $\mathcal{T}$ and $\mathbb{E}\{S_{(\mathcal{T},y)}\} = \pi r_y^2 + \mathbb{E}\{v_y\} < |\mathcal{A}|$ holds for arbitrary y and $\mathcal{T}$, $\Gamma(a, b)$ is the upper incomplete gamma function.

Proof: As demonstrated in [1], area coverage depends on the distribution of the random shapes only through its expected area. Hence, similar to the proof of *Theorem* 5.1, we can easily obtain that

$$\mathbb{E}\{\eta_{(\mathcal{T})}\} = 1 - \frac{\Gamma \left(k, n \sum_{y=1}^{u} c_y \mathbb{E}\{S_{(\mathcal{T},y)}\} \right)}{(k-1)!}. \tag{31}$$

Note the condition that $\mathbb{E}\{S_{(\mathcal{T},y)}\} < |\mathcal{A}|$ which means the area covered by a sensor during $\mathcal{T}$ cannot exceed the total area of the operational region otherwise the conclusion derived in [1] will not apply. ■

Based on *Theorem* 5.2, we can consider $\eta_{(\mathcal{T})}$ in several special cases as below.

1. If $k = 1$, the studied problem becomes 1-coverage. Substituting $k = 1$ into (31) and applying $\Gamma(1, b) = e^{-b}$, we have

$$\begin{aligned} \mathbb{E}\{\eta_{(\mathcal{T})}\} &= 1 - \Gamma \left(1, n \sum_{y=1}^{u} c_y \mathbb{E}\{S_{(\mathcal{T},y)}\} \right) \\ &= 1 - \exp \left(-n \sum_{y=1}^{u} c_y \mathbb{E}\{S_{(\mathcal{T},y)}\} \right). \end{aligned}$$

2. We regard the situation where $\mathbb{E}\{S_{(\mathcal{T},y)}\}$ are all identical for $y = 1, 2, \cdots, u$. Denote the same value as $\mathbb{E}\{S_{(\mathcal{T})}\}$. This scenario can happen if both r_y and $\mathbb{E}\{v_y\}$ are identical for $y = 1, 2, \cdots, u$. Replacing $\mathbb{E}\{S_{(\mathcal{T},y)}\}$ with $\mathbb{E}\{S_{(\mathcal{T})}\}$ in (31) and then using $\sum_{y=1}^{u} c_y = 1$, we obtain

$$\mathbb{E}\{\eta_{(\mathcal{T})}\} = 1 - \frac{\Gamma\left(k, n\mathbb{E}\{S_{(\mathcal{T})}\}\right)}{(k-1)!}. \quad (32)$$

3. If both conditions above holds, i.e., we have $k = 1$ and $\mathbb{E}\{S_{(\mathcal{T},y)}\}$ are all identical for $y = 1, 2, \cdots, u$. Substituting $k = 1$ into (32),

$$\begin{aligned}\mathbb{E}\{\eta_{(\mathcal{T})}\} &= 1 - \Gamma\left(1, n\mathbb{E}\{S_{(\mathcal{T})}\}\right) \\ &= 1 - \exp\left(-n\mathbb{E}\{S_{(\mathcal{T})}\}\right).\end{aligned}$$

Comparing $\mathbb{E}\{\eta_t\}$ and $\mathbb{E}\{\eta_{(\mathcal{T})}\}$ in *Theorem* 5.1 and *Theorem* 5.2, we notice that the difference caused by mobility lies in parameter b of $\Gamma(a, b)$. Since here $\Gamma(a, b)$ is an decreasing function of b, and $\mathbb{E}\{S_{(\mathcal{T},y)}\} > \pi r_y^2$, the expectation of fraction of k-covered area will increase due to mobility. This again verifies the performance improvement brought by mobility. The same as stated in Section IV, such benefit is also at the cost of $\Theta(1)$ delay, so network designers still face the trade-off between energy consumption and delay when considering k-coverage under Poisson deployment.

VI. Delay of Detection Under Poisson Deployment Model

In this section, we assess the performance of the mobile WSN from another perspective, which can reveal an advantage of mobile networks over static networks.

In static WSN, the network topology is determined after sensors are deployed. That means the area that is initially covered will be always under detection while the uncovered area can never be detected in the rest of time. Hence, there are blind spots in static WSN. In mobile WSN, however, the initially uncovered area might be detected later on. And the whole operational region may be fully detected within a certain period of time.

In this part, sensors follow the 2-dimensional random walk mobility model but with the assumption that they move with constant velocity v_0 throughout time.

A. Delay of Detection in Heterogeneous Networks

We first explore the delay of detection in heterogeneous WSNs.

Theorem 6.1: Under Poisson deployment model with 2-dimensional random walk mobility model at constant velocity v_0, the delay of detection $\mathcal{D}_1$ is exponentially distributed $\mathcal{D}_1 \sim$ Exponential $\left(2\lambda v_0 \sum_{y=1}^{u} (c_y r_y)\right)$.

Proof: The method used here comes from [4] and we would like to show the main argument. Consider an arbitrary point B in the operational region. We divide the space around B evenly into l directions ($l \to \infty$). Then all the sensors can be partitioned into l classes in the sense of the direction and sensors of class i move in the direction $\theta_{(i)} = \frac{2\pi i}{l}$. Recall that each sensor in group G_y independently chooses its moving direction according to the same p.d.f., then sensor in group G_y of each class i is a thinning of the original Poisson point process with density $\lambda_{(y,i)} = \lambda_y f_\Theta^{(y)}(\theta_{(i)})\Delta\theta$, where $\Delta\theta = 2\pi/l$ and $\lambda_y = c_y\lambda$.

Let $\mathcal{D}_{(y,i)}$ denote the time that point B is first ever covered by a sensor of group G_y in class i. Hence, the time that B is first covered by one sensor should be $\mathcal{D}_1 = \min\{\mathcal{D}_{(y,i)}\}$.

Same with proceeding section, $C(B, r)$ denotes the circle centered at point B with radius r. The initial distance from the first sensor of group G_y in class i that cover point B to the perimeter of circle C is denoted as $L_{(y,i)}$, and $L_{(y,i)} \sim$ Exponential $\left(2\lambda_{(y,i)} r_y\right)$ according to [12].

Since $\mathcal{D}_{(y,i)} = L_{(y,i)}/v_0$, then $\mathcal{D}_{(y,i)}$ follows the exponential distribution with parameter $2\lambda_{(y,i)} r_y v_0$.

Since the minimum of exponentially distributed random variables follow an exponential distribution with parameter equal to the sum of the parameter of those exponential distributions.

$$\begin{aligned}&\sum_{y=1}^{u}\left(\lim_{l\to\infty}\sum_{i=1}^{l} 2\lambda_{(y,i)} r_y v_0\right) \\ &= \sum_{y=1}^{u}\left(\lim_{l\to\infty}\sum_{i=1}^{l} 2\lambda_y f_\Theta^{(y)}(\theta_{(i)})\Delta\theta r_y v_0\right) \\ &= \sum_{y=1}^{u}\left(2\lambda_y r_y v_0 \int_0^{2\pi} f_\Theta^{(y)}(\theta)\mathrm{d}\theta\right) \\ &= 2\lambda v_0 \sum_{y=1}^{u}(c_y r_y). \quad (33)\end{aligned}$$

Therefore, the result follows. ■

B. Delay of k-Detection

We consider k-detection in this part in which case the uncovered area is guaranteed to be k-covered within a period of time. We assume that all the sensors are identical in the sense of sensing radius and mobility pattern. Besides, we assume that sensors uniformly choose the moving direction ($f_\Theta(\theta) = \frac{1}{2\pi}$) and move at constant velocity v_0.

Theorem 6.2: In the mobile heterogeneous WSN, all sensors are identical, having the same sensing radius r and following the same 2-dimensional random walk mobility model at constant velocity v_0 with $f_\Theta(\theta) = \frac{1}{2\pi}$. The delay of full k-detection $\mathcal{D}_k$ is a random variable distributed according to the p.d.f.

$$f_{\mathcal{D}_k}(d) = \frac{\bar{\lambda}^k}{(k-1)!} d^{k-1} e^{-\bar{\lambda} d},$$

where $\bar{\lambda} = 2\lambda r v_0$. We say $\mathcal{D}_k$ follows the order distribution with parameters k, λ, that is $\mathcal{D}_k \sim$ Order (k, λ).

Proof: Since sensors in different groups have identical sensing radius r and select their moving direction according to p.d.f. , sensors are just partitioned into different classes based on the direction with respect to point B. Similarly, sensors class i $(i = 1, 2, \cdots, l)$ is Poisson point process with density $\lambda_{(i)} = \lambda f_\Theta(\theta)\Delta\theta$, where $\Delta\theta = 2\pi/l$. Note that $f_\Theta(\theta) = \frac{1}{2\pi}$, we have $\lambda_{(i)} = \lambda/l$. Thus different sensors classes are identical distributed with parameter $\hat{\lambda} = \lambda/l$.

Let $\mathcal{D}_{(i)}$ be the time that point B is first ever covered by a sensor in class i. Now $\mathcal{D}_k$ denotes the time that point B is k-th covered by sensors. Namely, $\mathcal{D}_k$ is the k-th smallest of $\mathcal{D}_{(i)}$, which is a problem of order statistics. We know that $\mathcal{D}_{(i)} \sim$ Exponential $\left(2\hat{\lambda}rv_0\right), i = 1, 2, \cdots, l$.

As shown in [24], the p.d.f. of $\mathcal{D}_k$ is

$$f_{\mathcal{D}_k}(d) = l\binom{l-1}{k-1} F_{\mathcal{D}_{(i)}}(d)\left(1 - F_{\mathcal{D}_{(i)}}(d)\right)^{l-k} f_{\mathcal{D}_{(i)}}(d), \tag{34}$$

where $F_{\mathcal{D}_{(i)}}(d)$ and $f_{\mathcal{D}_{(i)}}(d)$ are the cumulative distribution function (c.d.f.) and p.d.f. of random variable $\mathcal{D}_{(i)}$, respectively.

Taking limits $l \to \infty$ and using Stirling's approximation $n! \sim \sqrt{2\pi n}\left(\frac{n}{e}\right)^n$, we obtain the p.d.f. of $\mathcal{D}_k$

$$f_{\mathcal{D}_k}(d) = \frac{(2\lambda rv_0)^k}{(k-1)!} d^{k-1} e^{-(2\lambda rv_0)d}.$$

■

VII. Concluding Remarks

In this paper, we have studied coverage in mobile and heterogeneous wireless sensor networks. Specifically, we have investigated asymptotic coverage under uniform deployment model with i.i.d. and 1-dimensional random walk mobility model, respectively. Mobility is found to decrease sensing energy consumption. On the other hand, we demonstrate that heterogeneity increases energy consumption under 1-dimensional random walk mobility model but imposes no impact under the i.i.d. mobility model. The k-coverage under Poisson deployment scheme with 2-dimensional random walk mobility model has been discussed, which also identifies the coverage improvement brought by mobility.

There are several directions for future work. First, we plan to extend the results for asymptotic coverage to include the case that sensors follow Poisson deployment scheme. Second, it is interesting to consider asymptotic k-coverage under certain mobility models. Finally, we would like to consider coverage under uniform deployment model for a more general n rather than the asymptotic case.

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APPENDIX

Proof of Lemma 4.1: Taking logarithm, we have

$$\begin{aligned}\log(1-x)^y &= y\log(1-x)\\ &= -y\sum_{i=1}^{+\infty}\frac{x^i}{i}\\ &= -y\left(x+\frac{x^2}{2}+\delta(x)\right),\end{aligned}\tag{35}$$

where

$$0<\delta(x)=\sum_{i=3}^{+\infty}\frac{x^i}{i}<\sum_{i=3}^{+\infty}\frac{x^i}{3}=\frac{1}{3}\frac{x^3}{1-x}<\frac{x^2}{3}.\tag{36}$$

The last step in (36) comes from the fact that $1-x>x$ when n is sufficiently large.

Therefore, from (35) and (36) we have

$$e^{-xy-\frac{5}{6}x^2y}<(1-x)^y<e^{-xy-\frac{1}{2}x^2y}.\tag{37}$$

Then the result follows. ■

Proof of Lemma 4.2: Let $s_y=\pi r_y^2(n)$ and take logarithm of the left hand side of (1), then we obtain

$$\begin{aligned}&\log\left(m\prod_{y=1}^{u}\left(1-\pi r_y^2(n)\right)^{c_y n}\right)\\ &= \log(n\log n)+\sum_{y=1}^{u}\Big((c_y n)\log(1-s_y)\Big)\\ &= \log(n\log n)-\sum_{y=1}^{u}\left((c_y n)\sum_{i=1}^{+\infty}\frac{(s_y)^i}{i}\right)\\ &= \log(n\log n)-\sum_{y=1}^{u}\left(c_y n\left(\sum_{i=1}^{2}\frac{(s_y)^i}{i}+\delta_y\right)\right),\end{aligned}\tag{38}$$

where, similar to (36),

$$\delta_y=\delta_y(n)=\sum_{i=3}^{+\infty}\frac{(s_y)^i}{i}\le\frac{1}{3}(s_y)^2,\tag{39}$$

Substituting (39) into (38), we have

$$\begin{aligned}&\log\left(m\prod_{y=1}^{u}\left(1-\pi r_y^2(n)\right)^{c_y n}\right)\\ &\ge \log(n\log n)-\sum_{y=1}^{u}\left((c_y n)\left((s_y)+\frac{5}{6}(s_y)^2\right)\right)\\ &\ge \log(n\log n)-n\left(\pi r_\star^2(n)\right)-\frac{5}{6}n\sum_{y=1}^{u}\left(c_y(s_y)^2\right)\\ &= -\xi-\frac{5}{6}n\sum_{y=1}^{u}\left(c_y(s_y)^2\right).\end{aligned}\tag{40}$$

Note that $s_i<c_j$ for $i,j=1,2,\cdots,u$ when n is sufficiently large. We then have

$$\sum_{y=1}^{u}\left(c_y(s_y)^2\right)\le\left(\sum_{y=1}^{u}(c_y s_y)\right)^{3/2}=\left(\pi r_\star^2(n)\right)^{\frac{3}{2}}.$$

Hence, we know that

$$\frac{5}{6}n\sum_{y=1}^{u}\left(c_y(s_y)^2\right)\le\frac{5}{6}n\left(\pi r_\star^2(n)\right)^{\frac{3}{2}}\to 0,\ \text{as } n\to+\infty.$$

Therefore, for any $\epsilon>0$, the following holds

$$\log\left(m\prod_{y=1}^{u}\left(1-\pi r_y^2(n)\right)^{c_y n}\right)\ge-\xi-\epsilon,\tag{41}$$

for all $n>N_\epsilon$. Let $\theta=e^{-\epsilon}$ and take the exponent of both sides, then the results follows. ■

Proof of Lemma 4.3: Since $c_y=\Theta(1)$, $r_y=\Theta\left(\frac{\log n}{n}\right)$ and $q<1$, we have

$$\frac{c_y}{n^q}>r_z(z=1,2,\cdots,u).\tag{42}$$

Then, we can obtain that

$$\begin{aligned}
&\left(\sum_{y=1}^{u} c_y r_y\right)^{\frac{3}{2}} \left(\frac{1}{n^q}\right)^{\frac{1}{2}} = \left(\sum_{y=1}^{u} c_y r_y\right) \left(\sum_{y=1}^{u} \frac{c_y}{n^q} r_y\right)^{\frac{1}{2}} \\
&> \left(\sum_{y=1}^{u} c_y r_y\right) \left(\sum_{y=1}^{u} r_y^2\right)^{\frac{1}{2}} = \left[\left(\sum_{y=1}^{u} c_y r_y\right)^2 \left(\sum_{y=1}^{u} r_y^2\right)\right]^{\frac{1}{2}} \\
&> \left[\left(\sum_{y=1}^{u} c_y^2 r_y^2\right) \left(\sum_{y=1}^{u} r_y^2\right)\right]^{\frac{1}{2}} \\
&\geq \left[\left(\sum_{y=1}^{u} c_y r_y \cdot r_y\right)^2\right]^{\frac{1}{2}} = \left(\sum_{y=1}^{u} c_y r_y^2\right). \qquad (43)
\end{aligned}$$

■

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