A Game Approach for Cell Selection and Resource Allocation in Heterogeneous Wireless Networks

Lin Gao, Xinbing Wang, Gaofei Sun and Youyun Xu

Abstract— Cell selection and resource allocation (CS-RA) are processes of determining cell and radio resource which provide service to mobile station (MS). Optimizing these processes is an important step towards maximizing the utilization of current and future networks. In this paper, we investigate the problem of CS-RA in heterogeneous wireless networks. Specifically, we propose a distributed cell selection and resource allocation mechanism, in which the CS-RA processes are performed by MSs independently. We formulate the problem as a two-tier game named as inter-cell game and intra-cell game, respectively. In the first tier, i.e. the inter-cell game, MSs select the best cell according to an optimal cell selection strategy derived from the expected payoff. In the second tier, i.e., the intra-cell game, MSs choose the proper radio resource in the serving cell to achieve maximum payoff. We analyze the existence of Nash equilibria of both games, the structure of which suggests the interesting property that we can achieve automatic load balance through the two-tier games. Furthermore, we propose distributed algorithms named as CS-Algorithm and RA-Algorithm to enable the independent MSs converge to Nash equilibria. Simulation results show that the proposed algorithms converge effectively to Nash equilibria and that the proposed CS-RA mechanism achieves better performance in terms of throughput and payoff compared to conventional mechanisms.

I. INTRODUCTION

One of the most important features of the evolving fourth generation (4G) wireless networks is heterogeneous wireless access [2], in which the mobile station (MS) has the ability of connecting to different wireless access networks (for example, WiMAX, LTE and WiFi). In the context of heterogeneous wireless access, to satisfy MSs' quality of services (QoS) requirement as well as service providers' revenue requirements, it is essential to design the joint mechanism of cell selection, radio resource management, congestion control, admission control, etc. Future, OFDM has been viewed as a promising paradigm to provide high performance physical layer for wireless access networks, due to the ability of combating inter symbol interference and achieving flexible and adaptive resource allocation [3], [4]. In this paper, we investigate the problem of cell selection and radio resource management in the heterogeneous OFDMA-based wireless networks.

We study the cell selection and resource allocation (CS-RA) problem from a different perspective, where the CS-RA processes are performed by MSs in a distributed manner.

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Specifically, We formulate the CS-RA problem as a two-tier game, namely *inter-cell game* and *intra-cell game*, respectively. In the first tier, i.e., the inter-cell game, each MS selects the best cell according to its cell selection strategy. The cell selection strategy is not a deterministic one but a stochastic one, i.e., a set of probabilities each representing the probability of selecting a particular cell. We derive the optimal strategy from MS's expected payoff, which depends on not only its channel qualities, but also the load distribution of all cells and the strategies of other MSs. In the second tier, i.e., the intra-cell game, MSs within the same cell choose the proper radio resource, typically sub-channels and power, to achieve their maximum payoff.

We prove the existence of Nash equilibria in both games and further propose distributed algorithms, named as CS-Algorithm and RA-Algorithm, to enable the independent MSs converge to Nash equilibria. It is interesting that although MSs update their strategies independently, they will finally converge to the Nash equilibria in both games.

Furthermore, we analyze the structure of the Nash equilibria, from which we find some attractive properties. Firstly, the whole system achieves load-balancing, both over the heterogeneous cells and over the sub-channels in a specific cell. Secondly, by adjusting the "price" parameters of cells, the load distribution in different cells can be regulated dynamically. Specifically, increasing "price" will drive load away, and decreasing "price" will attract load. Thirdly, by properly setting "price" of each cell and "DCR" of each MS, MSs with different interests can be directed towards the expected cells. Thus the proposed work can be applied to future's distributed and mobile user oriented wireless networks, e.g., cognitive radio networks [25].

The remainder of the paper is organized as follows. In Section II, we present related work on cell selection and resource allocation in wireless networks. In Section III, we present system model and problem formulation in cellular networks with heterogeneous OFDMA-based base stations. In Section IV, we introduce basic concepts in game theory and provide comprehensive analysis for the two-tier game. In Section V, we propose the distributed algorithms to achieve Nash equilibria. In Section VI, we study their convergence properties and present simulation results. Finally, we conclude in Section VII.

II. RELATED WORK

Cell selection is responsible for guaranteeing the required QoS for MS and keeping MS always camp on a cell with good

enough quality. Cell selection has received much attention in recent years, while the existing researches focus mainly on multiple access techniques, power control schemes and handoff protocols. In [5], Hanly *et al.* propose a cell selection algorithm to determine power allocation among different users so as to satisfy per-user SINR constraints. In [6], Wang *et al.* study an HSPA based handoff/cell-site selection technique to maximize the number of connected mobile stations, and propose a new scheduling algorithm to achieve this objective. The above researches take neither variable BSs' capacities nor MSs' QoS requirements into account. In [7], Mathar *et al.* provide an integrated design of optimal cell-site selection and frequency allocation, which maximizes the number of connected MSs and meanwhile maintains quasi-independence of radio based technology. However, the optimization problem in this model is NP-hard. In [8], Amzallag *et al.* formulate cell selection as an optimization problem called all-or-nothing demand maximization, and propose two algorithms to achieve approximate optimal solution. However, they do not consider the cell selection in heterogeneous wireless networks.

Resource allocation (RA), which determines how the radio resource (including time, frequency, power, etc) is assigned to each MS, is also a key element in providing guaranteed QoS. Adaptive subchannel-and-power allocation algorithms for multiuser OFDM systems have been investigated in [9]-[10], with purposes of maximizing overall data rate (rate-adaptive) or minimizing total transmit power (margin-adaptive). In [9], Wong *et al.* investigate margin-adaptive resource allocation problem and propose an iterative subcarrier and power allocation algorithm to minimize total transmit power given fixed data rates and bit error rate (BER). In [10], Jang *et al.* investigate rate-adaptive problem and propose a mechanism to maximize total data rate over all users subjected to power and BER constraints. However, neither of above works have taken sub-channel sharing into consideration. Further, the above algorithms work in a centralized manner, leading to high computational complexity and communication overhead.

Recently, game theory has been widely used in radio resource management (RRM) and power control problems, e.g., in [11] and [12]. However, the above work considers the problem in a single wireless access network. In a heterogeneous wireless access environment, most of recent researches focus on heterogeneous RRM, in particular vertical handoff mechanisms, e.g., in [13], [14] and [15]. In [16] and [17], the authors consider two-cell resource allocation problem, but they consider the CDMA based system and only focus on the power allocation problem in two cells. The multi-cell resource allocation problem are considered in [18] and [19], but both of them focus on multi-cell power allocation and do not consider sub-channel allocation problem. In this paper, we address the problems of cell selection, sub-channel allocation and power allocation together. It is notable that our results are not only applied to cellular networks, but also to ad-hoc networks and cognitive networks with minor modification. In [20][21], Wang *et al.* study the tradeoff of delay and throughput scaling in cognitive networks. However, we focus on the exact performance analysis from game theoretical perspective rather than asymptotic analysis in scaling law.
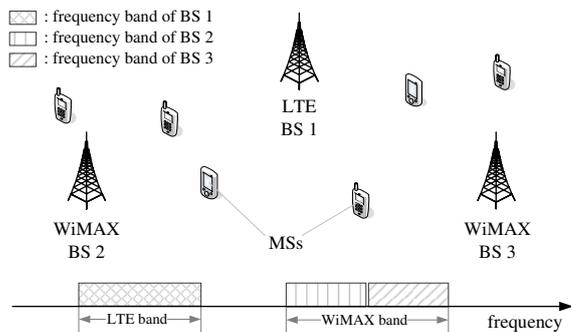

Fig. 1.   An example of cellular system with 3 BSs and 6 MSs.

III. SYSTEM MODEL AND PROBLEM FORMULATION

A. System Model

We consider a cellular system $\mathbf{G} = (M, N)$ which consists of M base stations (BSs) and N mobile stations (MSs). The set of BSs and MSs are denoted by $\mathbf{A} = \{1, ..., M\}$ and $\mathbf{U} = \{1, ..., N\}$, respectively. We allow the BSs to be heterogeneous, e.g., it can be base upon WiMAX, LTE or other OFDMA-based system. Without loss of generality, we assume that the number of cell in each BS is 1, and thus the meaning of cell is equivalent to BS in this work. We assume that all cells are interference-free by means of spectrum separating between different kinds of cells and frequency reusing between same kind of cells.

We denote the available bandwidth of cell i as B_i and the number of sub-channels of cell i as ω_i. Let c_k^i denote the k-th sub-channel of cell i. For simplicity, we omit the superscript i in c_k^i wherever no ambiguity is caused, and thus we can write the set of sub-channels of cell i as $\mathbf{C}_i = \{c_1, c_2, ..., c_{\omega_i}\}$. We assume that, for each cell $i \in \mathbf{A}$, all sub-channels have the same bandwidth, i.e., $\frac{B_i}{\omega_i}$.

We define the *required-subchannel-number* of player u in cell i, denoted by $k_{u,i}$, as the maximum number of sub-channels player u can occupy when connecting with cell i. We further denote the average channel gains of MS u in sub-channel c as $h_{u,c}$. In a long term perspective, for each MS u, the average channel gains in different sub-channels of a cell i are approximately same, i.e., $h_{u,c} \simeq h_{u,b}, \ \forall b, c \in \mathbf{C}_i$.[1]

Figure 1 presents an example of cellular system with 3 BSs and 6 MSs, where BS 1 is LTE base station, BS 2 and 3 are WiMAX base stations. Moreover, LTE frequency band is exclusively used by BS 1 while WiMAX frequency band is shared by BSs 2 and 3 by means of frequency reusing.

To facilitate reading, we list the major notations in Table I.

B. Problem Formulation

We formulate CS-RA problem as a two-tier game, wherein cell selection and resource allocation processes are performed by MSs in a distributed manner. Thus the *players* set can be defined as the set of MSs, i.e., $\mathbf{U}$.

In the inter-cell game, each player selects the cell with highest payoff. Thus the *strategy* of player u, denoted by

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[1]Note, however, our analysis and algorithm can be easily extended to the scenario with different sub-channel gains.

TABLE I
NOTATIONS

π_i	The unit price of power associated with cell i
q_u	The DCR parameter associated with player u
T_c	The number of players (load) in sub-channel c
N_i	The number of players (load) in cell i
$k_{u,i}$	The required-subchannel-number of player u in cell i
$\widetilde{k}_{u,i}$	The acquired-subchannel-number of player u in cell i
$Y_{u,c}$	The state of player u in sub-channel c
$P_{u,c}$	The transmitting power of player u in sub-channel c
$U_{u,c}$	The payoff of player u in sub-channel c
$\widetilde{U}_{u,c}$	The exclusive-payoff of player u in sub-channel c
$U_{u,i}$	The overall-payoff of player u in cell i
$\beta_{i,r}$	The probability of r players in cell i
$\mathbf{C}_i$	The set of sub-channels in cell i
$\mathbf{U}_i$	The set of players in cell i
X^*	The maximal (or optimal) X
$\mathbb{E}[X]$	The expectation of X

x_u, is defined as the cell he selected. It is obvious that $x_u \in \mathbf{A}, \ \forall u \in \mathbf{U}$. The strategy profile $\mathbb{X}$ is defined by all players' strategies:

$$\mathbb{X} \triangleq (x_1, \ x_2, \ \dots, \ x_N) \tag{1}$$

As long as each player selects a cell, all of the players are divided into M disjoint groups according to the cells they selected. We denote the set and number of players connecting with cell i as $\mathbf{U}_i$ and $N_i = |\mathbf{U}_i|$, respectively. Obviously $\bigcup_{i=1}^{M} \mathbf{U}_i = \mathbf{U}$ and $\sum_{i=1}^{M} N_i = N$. Each cell i is associated with an intra-cell game, with $\mathbf{U}_i$ as its players set. Without loss of generality, we consider the intra-cell game in cell i.

In the intra-cell game (of cell i), each player selects the proper sub-channels to achieve the highest payoff. We denote the state of player $u \in \mathbf{U}_i$ in sub-channel $c \in \mathbf{C}_i$ as $Y_{u,c}$ and we define $Y_{u,c} = 1$ if player u occupies sub-channel c and $Y_{u,c} = 0$ otherwise. Thus *strategy* of player u can be defined as its sub-channel state vector:

$$\mathbf{y}_{u,i} \triangleq \left(Y_{u,c_1}, \ Y_{u,c_2}, \ \dots, \ Y_{u,c_{\omega_i}}\right)^T, \tag{2}$$

where $(.)^T$ is the operator of matrix transposing. Obviously $\sum_{c \in \mathbf{C}_i} Y_{u,c} \leq k_{u,i}, \ \forall u \in \mathbf{U}_i$.

The strategy profile or strategy matrix, denoted by $\mathbb{Y}_i$, is defined by the strategy vectors of all players in cell i:

$$\mathbb{Y}_i \triangleq \left(\mathbf{y}_{u_1,i}, \ \mathbf{y}_{u_2,i}, \ \dots, \ \mathbf{y}_{u_{N_i},i}\right), \tag{3}$$

where u_k is the k-th player in cell i, i.e., $u_k \in \mathbf{U}_i$.

We present an example of strategy profiles for the inter-cell game and the intra-cell game[2] in Figure 2, where $M = 2$, $N = 4$ and players set $\mathbf{U} = \{1, 2, 3, 4\}$. The top floor of Figure 2 presents an example of the inter-cell game strategy profile: $x_1 = x_2 = x_4 = 2$ and $x_3 = 1$. The bottom floor of Figure 2 presents an example of the intra-cell game strategy profile in cell 2 with $\omega_2 = 8$ and players set $\mathbf{U}_2 = \{1, 2, 4\}$: $\mathbf{y}_{1,2} = (1, 1, 1, 1, 0, 0, 0, 0)^T$, $\mathbf{y}_{2,2} = (0, 0, 0, 0, 1, 1, 1, 1)^T$ and $\mathbf{y}_{4,2} = (0, 0, 1, 1, 0, 0, 1, 1)^T$.

We define the *load* of sub-channel c, denoted by T_c, as the number of players who are occupying sub-channel c, i.e.,

$$T_c = \sum_{u \in \mathbf{U}_i} Y_{u,c}, \quad \forall c \in \mathbf{C}_i \tag{4}$$

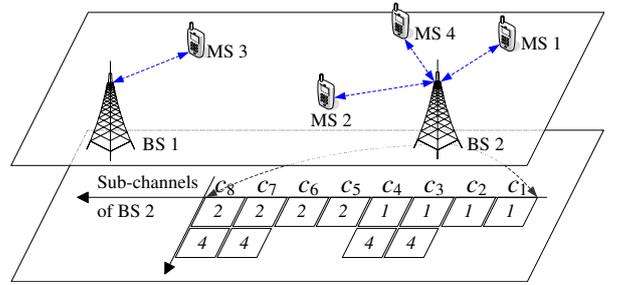[2]Without loss of generality, we present the intra-cell game in cell 2 only.

Fig. 2. An example of the strategy matrix with 2 BSs and 4 MSs.

We assume that total available bandwidth on sub-channel c is *equally* shared among the players who are occupying this sub-channel. Thus we can write the available bandwidth of sub-channel c occupied by player u as:

$$B_{u,c} = \frac{B_i}{\omega_i} \cdot \frac{Y_{u,c}}{T_c}, \quad \forall u \in \mathbf{U}_i, \ c \in \mathbf{C}_i \tag{5}$$

where $\frac{B_i}{\omega_i}$ is the bandwidth of each sub-channel in cell i.

Let $P_{u,c}$ and $\Gamma_{u,c}$ denote the transmitting power and receiving signal-to-noise ratio (SNR) of player u in sub-channel c, respectively. Formally, we have:

$$\Gamma_{u,c} = \frac{P_{u,c} \cdot |h_{u,c}|^2}{\sigma^2}, \quad P_{u,c} \in [0, P_t] \tag{6}$$

where P_t is the maximum allowable transmitting power and σ^2 is the noise variance.

The achieved data rate of player u on sub-channel c, denoted by $R_{u,c}$, is given by Shannon-Hartley theorem [1]:

$$R_{u,c} = B_{u,c} \cdot \log_2\left(1 + \Gamma_{u,c}\right) \tag{7}$$

Similar to power auction game in [22], we introduce a *pricing* mechanism in the resource allocation process. Specifically, as long as a player selects a sub-channel (and the corresponding transmitting power) of cell i, the cell charges the player a fixed payment for each unit of power. For each sub-channel $c \in \mathbf{C}_i$, there are T_c players sharing the time slots of c, and thus we can define the payment of player u to cell i (for employing sub-channel c) as:

$$D_{u,c} \triangleq \frac{\pi_i \cdot P_{u,c}}{T_c} \tag{8}$$

where $\pi_i > 0$ is a cell-specific parameter denoting unit price of power in cell i.

Interpreting achieved data rate as income and payment as cost, *payoff* of player u in sub-channel c can be defined as:

$$\begin{aligned} U_{u,c} &\triangleq R_{u,c} - q_u D_{u,c} \\ &= \frac{B_i}{\omega_i T_c} \log_2\left(1 + \Gamma_{u,c}\right) - \frac{q_u \pi_i P_{u,c}}{T_c} \end{aligned} \tag{9}$$

where $q_u > 0$ is a player-specific parameter denoting its preference for currency and data rate. We call $q_u > 0$ as data-to-currency ratio (DCR) since it reflects the intension of player u willing to pay for a given data rate.

The *overall-payoff* of player u in cell i, denoted by $U_{u,i}$, is the aggregate payoff in all sub-channels he occupied:

$$U_{u,i} \triangleq \sum_{c \in \mathbf{C}_i} \left(Y_{u,c} \cdot U_{u,c}\right) \tag{10}$$

For simplicity, we further introduce the concepts of acquired-subchannel-number and exclusive-payoff. The *acquired-subchannel-number* of player u in cell i, denoted by $\widetilde{k}_{u,i}$,

is defined as the available fraction of total sub-channels he occupied:

$$\widetilde{k}_{u,i} \triangleq \sum_{c \in \mathbf{C}_i} \frac{Y_{u,c}}{T_c}, \quad \forall u \in \mathbf{U}_i \tag{11}$$

and we can easily find that $\frac{k_{u,i}}{|\mathbf{U}_i|} \leq \widetilde{k}_{u,i} \leq k_{u,i}$, $\forall u \in \mathbf{U}_i$.

The *exclusive-payoff* of player u in sub-channel c, denoted by $\widetilde{U}_{u,c}$, is defined as the achieved payoff of u in sub-channel c when no other players is using this sub-channel, i.e.,

$$\widetilde{U}_{u,c} \triangleq \frac{B_i}{\omega_i} \cdot \log_2 \left(1 + \Gamma_{u,c}\right) - q_u \pi_i P_{u,c} \tag{12}$$

and we can easily find that $\widetilde{U}_{u,c} = U_{u,c} \cdot T_c$. Substituting Eq. (12) into Eq. (10), we can rewrite the overall payoff as:

$$U_{u,i} = \sum_{c=1}^{\omega_i} \left(\frac{Y_{u,c}}{T_c} \cdot \widetilde{U}_{u,c} \right) \tag{13}$$

We refer to each player as *rational* and *self-interested* player, who will always choose the action maximizing its overall payoff. Thus the objective of each player $u \in \mathbf{U}$ is:

$$\max_{\{i, \, \mathbf{y}_{u,i}, \, \mathbf{P}_{u,i}\}} \quad U_{u,i} = \sum_{c \in \mathbf{C}_i} \left(Y_{u,c} \cdot U_{u,c} \right) \tag{14}$$

where i is the cell player u selected, $\mathbf{y}_{u,i}$ is the sub-channel state vector of player u in cell i, $\mathbf{P}_{u,i}$ is the power allocation vector of player u in cell i, i.e., $\mathbf{P}_{u,i} = (P_{u,c_1}, ..., P_{u,c_{\omega_i}})$.

IV. ANALYSIS OF THE TWO-TIER GAME

To solve the optimization problem of (14) in a distributed manner, each player u needs to compute the maximum overall-payoff in all cells, i.e., $U_{u,i}^* = \max_{\{\mathbf{y}_{u,i}, \mathbf{P}_{u,i}\}} U_{u,i}$, $\forall i \in \mathbf{A}$, and then select the cell i^* with maximal $U_{u,i}^*$, i.e., $i^* = \arg \max_{i \in \mathbf{A}} U_{u,i}^*$.

According to Eq. (13), the maximum overall-payoff of player u in cell i (i.e., $U_{u,i}^*$) can be achieved by maximizing the exclusive-payoff of player u in each sub-channel (of cell i), i.e., $\widetilde{U}_{u,c}$, and the acquired-subchannel-number of player u in cell i, i.e., $\widetilde{k}_{u,i}$. Additionally, the maximum exclusive-payoff $\widetilde{U}_{u,c}^*$ and the maximum acquired-subchannel-number $\widetilde{k}_{u,i}^*$ can be derived by optimizing power allocation vector $\mathbf{P}_{u,i}$ and sub-channel state vector $\mathbf{y}_{u,i}$, respectively.

It is worth noting that $\widetilde{k}_{u,i}$ is not only related to the sub-channel state vector of player u itself, but also related to the sub-channel state vectors of other players in cell i. In fact, $\widetilde{k}_{u,i}^*$ can be obtained only when the intra-cell game in cell i achieves Nash equilibria, which implies that player u has connected with cell i already. However, the best cell selection for player u is indirectly determined by $\widetilde{k}_{u,i}^*$, $\forall i \in \mathbf{A}$. Thus this leads to a *non-causal* problem in cell selection.

To overcome the non-causal problem mentioned above, we introduce the concept of mixed-strategy game in the inter-cell game instead of pure-strategy game. Specifically, in mixed-strategy game, each player u selects cell according to a set of probabilities derived from maximum expected overall-payoff in each cell, i.e., $\mathbb{E}[U_{u,i}^*]$, $\forall i \in \mathbf{A}$. Note that according to Eq. (13), $\mathbb{E}[U_{u,i}^*]$ is determined by the expectation of maximum acquired-subchannel-number, i.e., $\mathbb{E}[\widetilde{k}_{u,i}^*]$, which can be obtained without connecting with cell i.

In summary, each player u first calculates $\widetilde{U}_{u,c}^*$, $\forall c \in \mathbf{C}_i$ and $\mathbb{E}[\widetilde{k}_{u,i}^*]$ for each cell $i \in \mathbf{A}$, from which player u can obtain the expectation of maximal overall-payoff for all cells, i.e., $\mathbb{E}[U_{u,i}^*]$, $\forall i \in \mathbf{A}$. Then player u selects the serving cell according to $\mathbb{E}[U_{u,i}^*]$, $\forall i \in \mathbf{A}$. After this, player u selects the best sub-channel state vector and best power allocation vector to achieve maximum payoff.

In what follows, we first introduce the concepts of best response function and Nash equilibrium in game theory in IV-A. Then, in IV-B, we derive the optimal power allocation vector and maximum exclusive-payoff. In IV-C, we derive the optimal sub-channel state vector in a given cell. Moreover, we analyze the characteristics of the intra-cell game Nash equilibrium and derive the expectation of maximum acquired-subchannel-number and maximum overall-payoff. In IV-D, based on the expectation of maximum overall-payoff in each cell, we derive the optimal cell selection strategy for each player. In IV-E, we present an example of the optimal cell selection strategy.

A. Best Response and Nash Equilibrium

In order to study the strategic interaction of players in a static non-cooperative game, we first introduce the concept of best response function [26], [27].

Definition 1: (Best Response Function): Each player u's best response (function) to the strategies of other players is the strategy x_u^* maximizing his payoff, i.e.,

$$x_u^* = \mathcal{B}_u \left(\mathbb{X}_{-u} \right) \triangleq \arg \max_{x_u} P_u \left(x_u, \mathbb{X}_{-u} \right) \tag{15}$$

where $P_u(\mathbb{X})$ denotes the payoff of player u in $\mathbb{X}$ and $\mathbb{X}_{-u}$ denotes the strategy profile except for strategy of player u.

In a game, each player can choose a particular strategy or randomly choose the strategies according to a set of probabilities. In the former case the player is said to choose a *pure strategy* while in the latter case the player chooses a *mixed strategy*. Now we introduce the concepts of pure-strategy Nash equilibrium [27].

Definition 2: (Pure-Strategy Nash Equilibrium): The strategy profile $\mathbb{X}^* = (x_1^*, x_2^*, ..., x_N^*)$ defines a pure-strategy Nash equilibrium, if for every player $u \in \mathbf{U}$, we have:

$$P_u \left(x_u^*, \mathbb{X}_{-u}^* \right) \geq P_u \left(x_u', \mathbb{X}_{-u}^* \right) \tag{16}$$

for every pure strategy $x_u' \neq x_u^*$.

In other words, in a pure-strategy Nash equilibrium, each player's strategy is the best response to the strategies of other players, i.e., $x_u^* = \mathcal{B}_u(\mathbb{X}_{-u}^*)$, $\forall u \in \mathbf{U}$.

It is notable that *not* every game possesses a pure-strategy Nash equilibrium as defined above. Moreover, as mentioned perviously, pure strategy leads to the non-causal problem in cell selection, and thus pure-strategy Nash equilibrium is *not* suitable for the inter-cell game, even if the game processes a pure-strategy Nash equilibrium. Thus we introduce the concept of mixed-strategy Nash equilibrium.

We denote θ_u as the number of pure strategies of player u and p_u^k as the probability of selecting the k-th pure strategy. Each player u's mixed strategy, denoted by $\mathbf{z}_u$, consists in defining its probability on each of the pure strategy, i.e.,

$$\mathbf{z}_u = \left(p_u^1, \, p_u^2, \, ... \, , \, p_u^{\theta_u} \right)^T \tag{17}$$

where $0 \leq p_u^k \leq 1$ and $\sum_{k=1}^{\theta_u} p_u^k = 1$.

The mixed-strategy profile or mixed-strategy matrix, denoted by $\mathbb{Z}$, is defined by all players' mixed-strategy vectors:

$$\mathbb{Z} = (\mathbf{z}_1, \mathbf{z}_2, \dots, \mathbf{z}_N) \tag{18}$$

For simplicity, we use the same notation $P_u(\mathbb{Z})$ denoting the expected payoff of player u in mixed-strategy matrix $\mathbb{Z}$. Formally, we have:

$$P_u(\mathbb{Z}) = \sum_{i_1=1}^{\theta_1} p_1^{i_1} \sum_{i_2=1}^{\theta_2} p_2^{i_2} \dots \sum_{i_N=1}^{\theta_N} p_N^{i_N} P_u\left(x_1^{i_1}, x_2^{i_2}, \dots, x_N^{i_N}\right) \tag{19}$$

where x_u^k denotes the k-th pure strategy of player u.

Similar to pure-strategy Nash equilibrium, the mixed-strategy Nash equilibrium is defined as follows [27]:

Definition 3: (Mixed-Strategy Nash Equilibrium): The strategy profile $\mathbb{Z}^* = (\mathbf{z}_1^*, \mathbf{z}_2^*, \dots, \mathbf{z}_N^*)$ defines a mixed-strategy Nash equilibrium, if for every player $u \in \mathbf{U}$, we have:

$$P_u\left(\mathbf{z}_u^*, \mathbb{Z}_{-u}^*\right) \geq P_u\left(\mathbf{z}_u', \mathbb{Z}_{-u}^*\right) \tag{20}$$

for every mixed strategy $\mathbf{z}_u' \neq \mathbf{z}_u^*$, where $\mathbb{Z}_{-u}$ denotes the mixed-strategy matrix except for the strategy of player u.

B. Optimal Power Allocation

Now we will derive the optimal power vector of player u in a given cell i, i.e., $\mathbf{P}_{u,i}^* = (P_{u,c_1}^*, P_{u,c_2}^*, \dots, P_{u,c_{\omega_i}}^*)$. We will further derive the maximum exclusive-payoff of player u in each sub-channel of cell i, i.e., $\widetilde{U}_{u,c}^*, \forall c \in \mathbf{C}_i$.

From Eq. (14), we can find that the optimal power allocation must satisfy the following optimization problems:

$$P_{u,c}^* = \widetilde{P}_{u,c}^* \triangleq \arg\max_{P_{u,c}} \widetilde{U}_{u,c}, \quad \forall c \in \mathbf{C}_i \tag{21}$$

We show this property as the following Lemma.

Lemma 1: The solution of (21), denoted by $\widetilde{P}_{u,c}^*$, is the optimal power allocation, i.e., $P_{u,c}^* = \widetilde{P}_{u,c}^*$.

The Lemma can be easily proved as we notice that $\widetilde{U}_{u,c} = U_{u,c} \cdot T_c$. Due to space limitation, we do not present the detailed proof.

By solving the optimization problem in (21), we can find the optimal power allocation, as shown in Lemma 2.

Lemma 2: The optimal power allocation for player u in sub-channel $c \in \mathbf{C}_i$ is given by:

$$P_{u,c}^* = \begin{cases} P_t & \pi_i \in (0, \underline{\pi}) \\ \frac{B_i}{q_u \pi_i \omega_i \ln 2} - \frac{\sigma^2}{|h_{u,c}|^2} & \pi_i \in [\underline{\pi}, \overline{\pi}] \\ 0 & \pi_i \in (\overline{\pi}, +\infty) \end{cases} \tag{22}$$

where $\underline{\pi} = \frac{B_i}{q_u \omega_i \ln 2 \cdot \left(\frac{\sigma^2}{|h_{u,c}|^2} + P_t\right)}$ and $\overline{\pi} = \frac{B_i}{q_u \omega_i \ln 2 \cdot \frac{\sigma^2}{|h_{u,c}|^2}}$.

The above Lemma can be proved by $\frac{\partial \widetilde{U}_{i,c_k}}{\partial P_{i,c_k}} = 0$. Note that $\frac{\partial \widetilde{U}_{i,c_k}}{\partial P_{i,c_k}}$ is always larger (or smaller) than zero if $\pi_j < \overline{\pi}$ (or $\pi_j > \underline{\pi}$), which implies the upper-bound (or lower-bound) of available power as the optimal one.

As mentioned in Section III-A, for each player u, we assume that $h_{u,b} \simeq h_{u,c}$, $\forall b, c \in \mathbf{C}_i$, and thus we can replace sub-channel index as cell index in the subscript of $h_{u,c}$, $\forall c \in \mathbf{C}_i$, and write $h_{u,c}$ as $h_{u,i}$ in the rest of the paper. Based on the above assumption, the optimal power allocations in different

sub-channels are also the same according to Lemma 2. Thus we can further write $P_{u,c}^*$ as $P_{u,i}^*$ in the rest of the paper. [3]

Substituting $P_{u,i}^*$ into Eq. (12), we can obtain the maximum exclusive-payoff of player u in each sub-channel of cell i, i.e., $\widetilde{U}_{u,c}^*, \forall c \in \mathbf{C}_i$. Similarly, the maximum exclusive-payoff of player u in different sub-channels of cell i are also the same. Thus we can write $\widetilde{U}_{u,c}^*$ as $\widetilde{U}_{u,i}^*$, i.e.,

$$\widetilde{U}_{u,i}^* = \frac{B_i}{\omega_i} \log_2\left(1 + \frac{P_{u,i}^* |h_{u,i}|^2}{\sigma^2}\right) - q_u \pi_i P_{u,i}^* \tag{23}$$

C. Analysis of Intra-Cell Game

In this subsection, we will analyze the intra-cell game in detail and derive closed form solutions to the game outcomes. Specifically, we will derive the best response, i.e., the optimal sub-channel state vector $\mathbf{y}_{u,i}^*$, for each player u in a given cell i. Then we will study the existence of pure-strategy Nash equilibria and propose the necessary and sufficient conditions for Nash equilibria. Furthermore, we will analyze the characteristics of Nash equilibria and derive the expectation of maximum acquired-subchannel-number of player u in cell i, i.e., $\mathbb{E}[\widetilde{k}_{u,i}^*]$, and the expectation of maximum overall-payoff of player u in cell i, i.e., $\mathbb{E}[U_{u,i}^*]$

As shown in Lemma 2, we can derive the optimal power allocation vector of player u in any cell i based on Eq. (22). Substituting $\mathbf{P}_{u,i}^*$ into Eq. (13) and noticing that $\widetilde{U}_{u,b}^* = \widetilde{U}_{u,c}^* = \widetilde{U}_{u,i}^*, \forall b, c \in \mathbf{C}_i$, we have:

$$U_{u,i} = \sum_{c=1}^{\omega_i} \left(\frac{Y_{u,c}}{T_c} \cdot \widetilde{U}_{u,c}^*\right) = \widetilde{U}_{u,i}^* \cdot \widetilde{k}_{u,i} \tag{24}$$

It is worth noting that the term of $\widetilde{U}_{u,i}^*$ is independent of the intra-cell game strategies of other players in cell i. Thus we can derive $\mathbf{y}_{u,i}^*$ by optimizing $\widetilde{k}_{u,i}$ solely, i.e.,

$$\mathbf{y}_{u,i}^* = \arg\max_{\mathbf{y}_{u,i}} \widetilde{k}_{u,i}$$
$$\text{s.t.} \sum_{c \in \mathbf{C}_i} Y_{u,c} \leq k_{u,i} \text{ and } Y_{u,c} \in \{0, 1\} \tag{25}$$

Let $T_c^{-u} = T_c - Y_{u,c} = \sum_{v \in \mathbf{U}_i, v \neq u} Y_{v,c}$ denote the number of players other than player u selecting sub-channel c. We define $\mathcal{C}_i \triangleq \{\varsigma_1, \varsigma_2, \dots, \varsigma_{\omega_i}\}$ as a permutation of $\mathbf{C}_i$ according to ascending order of T_c^{-u}, i.e., $T_{\varsigma_1}^{-u} \leq T_{\varsigma_2}^{-u} \leq \dots \leq T_{\varsigma_{\omega_i}}^{-u}$. The best response of player u is shown in Lemma 3.

Lemma 3: The best response of player u is $\mathbf{y}_{u,i}^* = (Y_{u,c_1}, Y_{u,c_2}, \dots, Y_{u,c_{\omega_i}})$ with:

$$Y_{u,c} = \begin{cases} 1, & \forall c \in \{\varsigma_1, \varsigma_2, \dots, \varsigma_k\} \\ 0, & \forall c \in \{\varsigma_{k+1}, \dots, \varsigma_{\omega_i}\} \end{cases}$$

where $k = k_{u,i}$ is required-subchannel-number of u in cell i.

Lemma 3 shows that each player u will select the $k_{u,i}$ sub-channels with lowest load.

A pure-strategy matrix $\mathbb{Y}_i^* = (\mathbf{y}_{1,i}^*, \dots, \mathbf{y}_{N,i}^*)$ is a Nash equilibrium if for every player $u \in \mathbf{U}_i$, its strategy $\mathbf{y}_{u,i}^*$ is the best response to other players' strategies. Similar to [24], we

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[3]Note that the above assumption is used to facilitate the description of Nash equilibrium state. Without this assumption, we cannot split the overall-payoff $U_{u,i}$ into two independent terms $\widetilde{U}_{u,i}^*$ and $\widetilde{k}_{u,i}$ as shown in Eq. (24). However, we can still split the expectation of maximum overall-payoff $\mathbb{E}[U_{u,i}^*]$ into two terms.

provide the necessary and sufficient conditions for the pure-strategy Nash equilibria in the following theorem.

Theorem 1: A strategy matrix $\mathbb{Y}_i^*$ is a Nash equilibrium of the intra-cell game (in cell i) iff the following conditions hold:

$$(i) \qquad \sum_{c \in \mathbf{C}_i} Y_{u,c} = k_{u,i}, \quad \forall u \in \mathbf{U}_i$$

$$(ii) \qquad \Omega_{b,c} \leq 1, \quad \forall b, c \in \mathbf{C}_i$$

where $\Omega_{b,c} \triangleq T_b - T_c$ denotes the difference of load on sub-channel b and c.

The first condition in Theorem 1 states that each player will occupy as many sub-channels as possible. The second condition establishes an interesting property about the pure-strategy Nash equilibria. In fact, the players are (approximately) *equally* distributed in all sub-channels in Nash equilibria, which implies the difference of load on arbitrary two sub-channels does not exceed one. Hence, all Nash equilibria sub-channel allocations achieve *load-balancing* over the sub-channels in a cell.

We denote $\mathbb{E}[T_c]$ as the average load in sub-channel c under Nash equilibria. Due to load-balancing property, we have:

$$\mathbb{E}[T_c] = \max \left\{ \frac{\sum_{u \in \mathbf{U}_i} k_{u,i}}{\omega_i}, 1 \right\}, \quad \forall c \in \mathbf{C}_i \qquad (26)$$

Substituting Eq. (26) into Eq. (11), we can obtain the expectation of maximum acquired-subchannel-number of player u in cell i as follows:[4]

$$\mathbb{E}\left[\widetilde{k}_{u,i}^*\right] \simeq \frac{k_{u,i}}{\mathbb{E}[T_c]} = \min \left\{ \frac{\omega_i}{\sum_{l \in \mathbf{U}_i} k_{l,i}}, 1 \right\} \cdot k_{u,i} \qquad (27)$$

For simplicity, we assume that $k_{u,i} = k_{l,i}, \forall u \neq l$. Thus $\mathbb{E}[\widetilde{k}_{u,i}^*]$ is determined by the number of players in cell i rather than the detail strategies of players, and thus he can obtain $\mathbb{E}[\widetilde{k}_{u,i}^*]$ without connecting with the cell i.[5]

Substituting Eqs. (23) and (27) into Eq. (24), we have:

$$\mathbb{E}\left[U_{u,i}^*\right] = \widetilde{U}_{u,i}^* \cdot \mathbb{E}\left[\widetilde{k}_{u,i}^*\right] \qquad (28)$$

Intuitively, the first term in Eq. (28) $\widetilde{U}_{u,i}^*$ denotes the maximum payoff of player u in one sub-channel, and $\mathbb{E}[\widetilde{k}_{u,i}^*]$ denotes the expectation of maximum acquired sub-channels number. Hence, $\mathbb{E}[U_{u,i}^*]$ denotes the expectation of maximum overall payoff of player u in cell i. The essentiality of (28) is that it provides a straightforward expected outcome of the intra-cell game without converging to any Nash equilibrium.

D. Analysis of Inter-Cell Game

In this subsection, we will derive the optimal cell selection strategy for each player and study the existence of Nash equilibria in the inter-cell game. Further, we will propose an iterative updating process to converge to Nash equilibria.

It is worth noting that the pure strategy is *not* suitable for the inter-cell game. In pure strategy Nash equilibria, each player u will select a particular cell i with highest $U_{u,i}^*$ according to the optimization problem in (14). However, such a process will lead to the non-causal problem.

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[4]Note that the approximating equation follows because $\frac{1}{\mathbb{E}[T]} \simeq \mathbb{E}\left[\frac{1}{T}\right]$.

[5]Note without this assumption, the $\mathbb{E}[\widetilde{k}_{u,i}^*]$ is determined by the number of players in cell i and the required-subchannel-number of each player.

Hence, we model the inter-cell game as a mixed-strategy game, wherein each player u selects cell in a random manner and thus we can use $\mathbb{E}[U_{u,i}^*]$ as the expected payoff of player u. Specifically, each player u selects a cell according to its mixed-strategy, which is derived by its expectation of maximum payoff in each cell, i.e., $\mathbb{E}[U_{u,i}^*], \forall i \in \mathbf{A}$.

We define the mixed strategy of player u as follows:

$$\mathbf{z}_u = \left(p_u^1, \ p_u^2, \ ... \ , \ p_u^M\right)^T \qquad (29)$$

where p_u^k is the probability of player u selecting cell k.

According to Eq. (19), the expected payoff of player u is:

$$P_u(\mathbb{Z}) = \sum_{i_1=1}^{M} p_1^{i_1} ... \sum_{i_N=1}^{M} p_N^{i_N} \mathbb{E}\left[U_{u,i_u}^*\left(x_1^{i_1}, ..., x_N^{i_N}\right)\right] \qquad (30)$$

where i_k denotes the cell player k selected, and $\mathbb{E}[U_{u,i_u}^*(\mathbb{X})]$ denotes the expectation of maximum overall-payoff of player u in cell i_u under $\mathbb{X}$, a realization of all players' strategies.

We provide the existence of mixed-strategy Nash equilibrium in the inter-cell game by the following proposition.

Proposition 1: The cellular system $\mathbf{G} = (M, N)$ with finite MSs and BSs has at least one mixed-strategy Nash equilibrium in the inter-cell game.

The Proposition can be proved by Kakutani's fixed point theorem. Due to space limitations, we do not present the detailed proof here. Besides showing the existence of Nash equilibrium, the above Proposition does not provide any methodological suggestion in finding mixed-strategy Nash equilibrium. As far as calculating mixed-strategy Nash equilibria is concerned, the following Lemma is useful.

Lemma 4: The mixed-strategy matrix $\mathbb{Z}^* = (\mathbf{z}_1^*, \mathbf{z}_2^*, ..., \mathbf{z}_N^*)$ is a mixed-strategy Nash equilibrium if for each player $u \in \mathbf{U}$, the following condition holds:

$$\mathbf{z}_u^* = \mathcal{B}_u\left(\mathbb{Z}_{-u}^*\right) \triangleq \arg\max_{\mathbf{z}_u} P_u\left(\mathbf{z}_u, \mathbb{Z}_{-u}^*\right) \qquad (31)$$

where $\mathcal{B}_u(.)$ is the best response function of player u.

The proof can be addressed by the definition of Nash equilibrium. Lemma 4 states that Nash equilibrium can be derived by jointly solving N best response functions in (31).

In a practical system, however, each player does *not* have enough information to calculate the best responses of other players, which prevents it from directly calculating the Nash equilibria. Nevertheless, the Nash equilibria can be achieved in a distributed fashion if we allow the players to *iteratively* update their strategies based on best response functions. In greater detail, each player updates its strategy at time t according to the strategies of other players at time $t - 1$. We call this iterative updating process as best response dynamic. Formally, we can write the best response dynamic as follows:

$$\mathbf{z}_u(t) = \mathcal{B}_u\left(\mathbb{Z}_{-u}(t - 1)\right), \quad \forall u \in \mathbf{U} \qquad (32)$$

where $\mathbf{z}_u(t)$ and $\mathbb{Z}_{-u}(t)$ denote the strategy of player u and the strategies of other players at time t, respectively.

E. An Example of Mixed-strategy Nash Equilibrium

To facilitate the reader's comprehension, we present in this subsection an example of mixed-strategy Nash equilibrium for the inter-cell game. For simplicity, we consider the cellular system with 2 BSs, i.e., $\mathbf{A} = \{1, 2\}$. Thus we can write the

mixed strategy of each player $u \in \mathbf{U}$ as $\mathbf{z}_u = (p_u, 1 - p_u)^T$, where $p_u \in [0, 1]$ denotes the probability of selecting cell 1.

Without loss of generality, we assume that $k_{u,i} = \omega_i, \forall u \in \mathbf{U}, i \in \mathbf{A}$, i.e., the player always occupies all sub-channels of the cell it selected. It is easy to see that $\widetilde{k}_{u,i}^* = \frac{\omega_i}{N_i}$ since each sub-channel in cell i is occupied by N_i players.

We first consider the scenario with 2 MSs, i.e., $\mathbf{U} = \{1, 2\}$. Clearly, there are 4 total outcomes depending on the choices made by two players. We can succinctly summarize the payoffs gained in these four outcomes via the two-by-two matrix in Figure 3, where $F_{u,i} = \widetilde{U}_{u,i}^* \omega_i$ denotes the maximum overall-payoff of player u in cell i if player u exclusively occupies cell i. The strategies of player 1 correspond to the rows and the strategies of player 2 correspond to the columns of the matrix. The entries of the matrix are payoffs gained by the players in each situation.

Strategy of Player 2

	$x_2 = 1$	$x_2 = 2$
$x_1 = 1$	$\left(\frac{F_{1,1}}{2}, \frac{F_{2,1}}{2}\right)$	$(F_{1,1}, F_{2,2})$
$x_1 = 2$	$(F_{1,2}, F_{2,1})$	$\left(\frac{F_{1,2}}{2}, \frac{F_{2,2}}{2}\right)$

Strategy of Player 1 (left label)

Fig. 3. The payoff matrix for the system with 2 BSs and 2 MSs.

According to Eq. (30), we can easily write the expected payoff of player 1 as follows:

$$P_1(\mathbf{z}_1, \mathbf{z}_2) = \frac{p_1(2 - p_2)}{2} F_{1,1} + \frac{(1 - p_1)(1 + p_2)}{2} F_{1,2} \tag{33}$$

from which we can derive the best response of player 1 as a function of p_2, i.e., $p_1^* = \mathcal{B}_1(p_2)$. Similarly, the best response of player 2 is a function of p_1, i.e., $p_2^* = \mathcal{B}_2(p_1)$.

According to Lemma 4, we have $p_1^* = \mathcal{B}_1(p_2^*)$ and $p_2^* = \mathcal{B}_2(p_1^*)$, if (p_1^*, p_2^*) defines a Nash equilibrium. Thus we can derive that $(p_1^*, p_2^*) = (1, 0)$, $(0, 1)$ or $(\tilde{p}_1, \tilde{p}_2)$.

We can draw the best response function with a line for each player in a unit square strategy space, as shown in Figure 4. The solid line in Figure 4 shows the optimal p_1^* (shown in the y-axis) as a function of p_2 (shown in the x-axis), i.e., $p_1^* = \mathcal{B}_1(p_2)$. The dotted line shows the optimal p_2^* as a function of p_1, i.e., $p_2^* = \mathcal{B}_2(p_1)$. The Nash equilibria occur at the points where the two player's best responses agree, i.e., the red circles in Figure 4.

Furthermore, for the system with more than 2 BSs, we can use the same method to find the mixed-strategy Nash equilibria, and we do not present the detailed solutions due to space limitations.

V. CONVERGENCE TO NASH EQUILIBRIA

In this section, we will propose the distributed algorithms to enable the players to converge to Nash equilibrium state. It is worth noting that although the closed-form solutions proposed in Section IV need the complete information of all players, the algorithm proposed in this section works in a totally *distributed* manner.

We divide the algorithm into two stages, namely CS-Algorithm and RA-Algorithm, respectively. To apply the al-
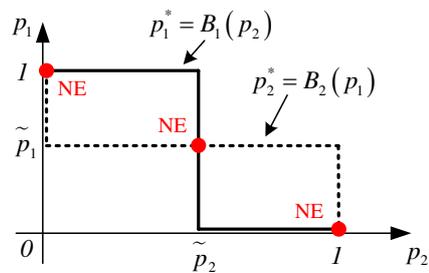

Fig. 4. The best response function for the system with 2 BSs and 2 MSs.

gorithm in a practical system, the following three essential assumptions are necessary.

First, we assume that each MS has ability to initiate inter-frequency measurement, from which each MS can obtain the average channel gain in every cell. Second, we assume that each cell periodically broadcasts the number of MSs connecting with this cell. Third, we assume that each cell counts the load on sub-channels and multicasts this information to all MSs connecting with this cell.

A. CS-Algorithm

In this subsection, we propose the distributed algorithm, denoted by CS-Algorithm, which enables the players converge to mixed-strategy Nash equilibria of the inter-cell game.

To exploit the CS-Algorithm, we must settle the following two major difficulties. Firstly, the iterative updating process in Eq. (32) implies that each player has ability to observe the strategies of other players. In our model, however, the mixed strategy of one player can never be observed by other players, even though the particular action of player can be observed by others, which makes the calculation of expected payoff impractical. Secondly, the mixed strategy will degenerate to pure strategy due to the *non-smooth* characteristic of the best response functions. That is, the players will put all probability, i.e., probability 1, in the strategy with highest expected payoff, which leads to a "jumping" effect in best response as shown in Figure 4.

We solve the first problem by the second essential assumption mentioned above. Specifically, we find a way of computing the expected payoff without knowing the mixed strategies of other players. Formally, we show this in the following Lemma.

Lemma 5: The expected payoff of player u, i.e., $P_u(\mathbb{Z})$ in (30), is equivalent to the $Q_u(\mathbf{z}_u)$ defined as follows:

$$Q_u(\mathbf{z}_u) \triangleq \sum_{i=1}^M \left(p_u^i \sum_{r=0}^{N-1} \beta_{i,r} \mathbb{E}\left[U_{u,i}^*(r+1)\right] \right) \tag{34}$$

where $\{\beta_{i,r}\}$ is the probability distribution function of cell load (CLPDF) in cell i, and $\mathbb{E}[U_{u,i}^*(r+1)]$ is the expectation of maximum overall-payoff of player u in cell i if there exist r other players in cell i.

The above Lemma can be proved by transforming the random cell selection probabilities of all players into the probability distribution function of cell load CLPDF. Note that each player can gradually learn the CLPDF $\{\beta_{i,r}\}$ by

TABLE II

CS-Algorithm in Inter-Cell Game Stage

1:	**for** $t = 0$ to T_e step by τ **do**
2:	each cell i counts load N_i and broadcasts to all players
3:	**for** $u = 1$ to $|\mathbf{U}|$ **do**
	/* cell selection stage */
4:	measure the channel gains of all cells, i.e., $h_{u,i}, \forall i \in \mathbf{A}$
5:	decode the load information of all cells, i.e., $N_i, \forall i \in \mathbf{A}$
6:	**if** t_u is equal to 0 **then**
7:	update the probabilities $\{\beta_{i,r}\}$
8:	calculate $Q_{u,i}, \forall i \in \mathbf{A}$
9:	calculate $\mathbf{z}_u$ according to (35)
10:	select a cell with the probability in $\mathbf{z}_u$ and handoff to the cell
11:	reset the cell selection timer $t_u = T_u$
12:	**else**
13:	decrease the t_u value by τ, i.e., $t_u = t_u - \tau$
14:	**end if**
	/* sub-channel allocation stage */
15:	perform the RA-Algorithm in Table B
16:	**end for**
17:	**end for**

TABLE III

RA-Algorithm in Intra-Cell Game Stage

1:	**for** $t = 0$ to T_e step by τ **do**
2:	each cell i counts load N_i and broadcasts to all players
3:	**for** $u = 1$ to $|\mathbf{U}|$ **do**
	/* cell selection stage */
4:	perform the CS-Algorithm in Table A
	/* sub-channel allocation stage */
5:	set i as the index of the cell player u connecting with
6:	get the load of all sub-channels in cell i, i.e., $T_c, \forall c \in \mathbf{C}_i$
7:	**if** W_u is equal to 0 **then**
8:	find $k_{u,i}$ sub-channels according to Lemma 3
9:	send the sub-channels state vector $\mathbf{y}_{u,i}$ to cell i
10:	reset W_u to a new value from the set $\{1, ..., W\}$
11:	**else**
12:	decrease the W_u value by one, i.e., $W_u = W_u - 1$
13:	**end if**
14:	**end for**
15:	**end for**

statistically recording the load information of each cell, which is broadcasted by every cell periodically.

To solve the second problem, we introduce the concept of *smoothed* best response functions, which move "smoothly" from one pure strategy to another. There are many functions that represent smoothed best response functions. In this work, we define the smoothed best response of player u as follows:

$$p_u^i = \frac{e^{\frac{1}{\gamma} Q_{u,i}}}{e^{\frac{1}{\gamma} Q_{u,1}} + e^{\frac{1}{\gamma} Q_{u,2}} + ... + e^{\frac{1}{\gamma} Q_{u,M}}}, \ \forall i \in \mathbf{A} \quad (35)$$

where γ is the smoothing factor, $Q_{u,i}$ is the expected payoff of player u in cell i, i.e., $Q_{u,i} = \sum_{r=0}^{N-1} \beta_{i,r} \mathbb{E}\left[U_{u,i}^*(r+1)\right]$.

Based on the above discussion, we present the CS-Algorithm in Table II, where T_e is the time of simulation's end, τ is the minimum time scale of simulation (also named as time slot), t_u is the cell selection timer of player u and T_u is the length of t_u. In practice, the time scale τ can be set as the minimum scheduling interval of all cells. Further, To avoid the frequently handoff in different cells, the timer length T_u must be set large enough.

B. RA-Algorithm

Now we propose the distributed algorithm, denoted by RA-Algorithm, which enables the players within a cell converge to Nash equilibrium of the intra-cell game.

Benefitting from the third essential assumption mentioned above, each player can easily obtain the load of each sub-channel in the serving cell, i.e., $T_c, \forall c$. Thus the best response of player u, i.e., the best sub-channels set, can be obtained according to Lemma 3.

Table III presents the detail pseudo-code of RA-Algorithm. Note that, to avoid the unstable sub-channel allocations caused by simultaneously moving of different players, we use the technique of backoff mechanism well known in the IEEE 802.11 medium access technology similar to [24]. We denote the backoff window as W and each player u chooses a random initial value for his backoff counter w_u with uniform probability from the set $\{1, ..., W\}$.

It is notable that the CS-Algorithm is performed every T_u seconds while RA-Algorithm is performed every τ seconds for player u. In practice, τ is in millisecond-level and T_u is in second-level. We will show in the simulations that $T_u = 1$s is enough for the players within the cell converging to Nash equilibrium of the intra-cell game.

VI. Simulation Results And Analysis

To evaluate the performance of our proposed scheme and decide what strategy each player should adopt, we perform simulations for the cellular system with multiple MSs and multiple BSs.

In our simulation, all MSs and BSs are uniformly distributed in the square of $1000m \times 1000m$. The propagation loss factor is set to 2, and the sub-channel gains are distance based (i.e., time-varying fading is not considered here). Moreover, the length of cell selection timer is $T_u = 1$s and the minimal scheduling interval is $\tau = 1$ms. We run each simulation for 2000 cell selection rounds, which corresponds to 2000s according to the setting of T_u.

We first show the convergence of RA-Algorithm in a given cell. To provide the quantitative analysis of different sub-channels allocations, we introduce the concept of *variance ratio* $\phi(\mathbb{Y}_i)$ as defined in [24]. A sub-channels allocation $\mathbb{Y}_i$ is Nash equilibrium iff $\phi(\mathbb{Y}_i) = 1$. We show that the average variance ratio of 100 runs in Figure 5, where $W = 16$, $|\mathbf{C}_i| = 64$, $|\mathbf{U}_i| = 50$ and $k_{u,i} = 5$, $\forall u \in \mathbf{U}_i$, from which we find that the variance ratio ϕ converges to 1 within 50ms $(\ll 1s)$.

Then we investigate the expectation of maximum acquired-subchannel-number of each player, which is essential for the calculating of expectation of maximum overall-payoff. We show the simulate results of the first 15 players in Figure 6, where $W = 16$, $|\mathbf{C}_i| = 64$, $|\mathbf{U}_i| = 50$ and $k_{u,i} = 5$, $\forall u \in \mathbf{U}_i$. Note that the simulation results is the average of 100 runs. The estimation of "expectation of maximum acquired-subchannel-number" adopted in our work is given by Eq. (27), i.e., $\min\{\frac{64}{50 \cdot 5}, 1\} \cdot 5 = 1.28$. From Figure 6, we can see that the adopted expectation of maximum acquired-subchannel-number well coincides with the simulation result.

We show the mixed-strategy Nash equilibrium of the inter-cell game in Figure 7, where $|\mathbf{A}| = 3$, $|\mathbf{U}| = 100$, $\pi_1 = \pi_2 = \pi_3 = 1$ and $B_1 = B_2 = B_3 = 100$KHz. The triangles and dots denote the BSs and MSs, respectively. The arrow
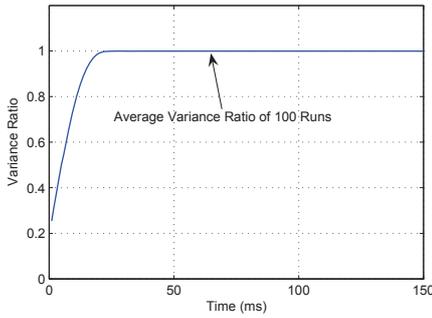
Fig. 5. Dynamic of variance ratio using $W = 16$, $|\mathbf{C}_i| = 64$, $|\mathbf{U}_i| = 50$, $k_{u,i} = 5$.
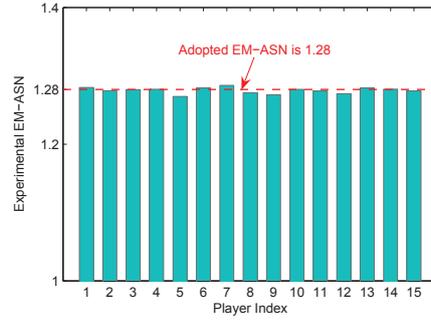

Fig. 6. Simulate expectation of maximal acquired-subchannel-number of players using $W = 16$, $|\mathbf{C}_i| = 64$, $|\mathbf{U}_i| = 50$, $k_{u,i} = 5$.
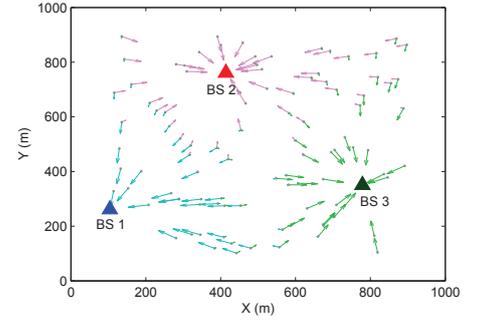

Fig. 7. Mixed-strategy Nash equilibrium of inter-cell game using $|\mathbf{A}| = 3$, $|\mathbf{U}| = 100$, $\pi_1 = \pi_2 = \pi_3 = 1$ and $B_1 = B_2 = B_3 = 100$KHz.

denotes the probability of the associated player selecting the cell which the arrow aiming at. For example, the red arrows denote the probabilities of players selecting cell 2. We can find that the mixed strategies in Nash equilibrium are dramatically influenced by the distances between MSs and BSs.

VII. CONCLUSION

We propose a distributed cell selection and resource allocation mechanism, in which the CS-RA processes are performed by MSs independently. We formulate the problem as a two-tier game named as inter-cell game and intra-cell game, respectively. In the inter-cell game, MSs select the best cell according to optimal cell selection strategy derived from the expected payoff. In the intra-cell game, MSs choose the proper sub-channels to achieve maximum payoff. We illustrate the structure of Nash equilibria in both games. Furthermore, we propose distributed algorithms to enable the independent MSs converge to Nash equilibria. Our results provide the insights on understanding the future heterogeneous wireless network.

ACKNOWLEDGMENT

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