A Game Approach for Multi-Channel Allocation in Multi-Hop Wireless Networks

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ABSTRACT

Channel allocation was extensively investigated in the framework of cellular networks, but it was rarely studied in the wireless ad-hoc networks, especially in the multi-hop ad-hoc networks. In this paper, we study the competitive multi-radio channel allocation problem in multi-hop wireless networks in detail. We model the channel allocation problem as a static cooperative game, in which some players collaborate to achieve high date rate. We propose the min-max coalition-proof Nash equilibrium (MMCPNE) channel allocation scheme in the game, which aims to max the achieved date rates of communication links. We analyze the existence of MMCPNE and prove the necessary conditions for MMCPNE. Furthermore, we propose several algorithms that enable the selfish players to converge to MMCPNE. Simulation results show that MMCPNE outperforms CPNE and NE schemes in terms of achieved data rates of the multi-hop links due to cooperation gain.

Categories and Subject Descriptors

H.4 [**Information Systems Applications**]: Communications Applications

General Terms

Theory

Keywords

Multi-Radio, Channel Allocation, Game Theory, Nash Equilibria

1. INTRODUCTION

Wireless communication system is often assigned a certain range of communication medium (e.g., frequency band). Usually this medium is shared by different users through multiple access techniques. *Frequency Division Multiple Access (FDMA)*, which enables more than one users to share a given frequency band, is one of the extensively used techniques in wireless networks [1], [2]. In FDMA, the total available bandwidth is divided permanently into a number of distinct sub-bands named *channels*. Commonly, we refer to the assignment of radio transceivers to these channels as the *channel allocation* problem. An efficient channel allocation is essential for the design of wireless networks.

In this paper, we present a game-theoretic analysis of fixed channel allocation strategies of devices that use multiple radios in the multi-hop wireless networks. Static non-cooperative game is a novel approach to solve the channel allocation problem in single-hop networks and Nash equilibrium (NE) provides an efficient criterion to evaluate a given channel allocation (e.g., in [3]). In multi-hop networks, however, non-cooperative game results in low achieved date rate of multi-hop links for the reasons mentioned in Section 4. Hence, we introduce static cooperative game with perfect information into our system. We mainly focus on the performance improvement of the multi-hop links, which is induced by cooperation gain, without sacrificing the performance of single-hop links. We first define the min-max coalition-proof Nash equilibrium (MMCPNE) in this game, which is aiming to achieve the maximal date rate of all links (single-hop links and multi-hop links). We also define three other equilibria schemes that approximate to MMCPNE, named as MCPNE, ACPNE and ICPNE respectively. Then, we study the existence of MMCPNE in the static cooperative game and our main result, Theorem 2, shows the necessary conditions for the existence of MMCPNE.

Furthermore, we propose the MMCP algorithm which enables the selfish players to converge to MMCPNE from an arbitrary initial configuration and the DCP-x algorithms which enable the players converge to approximate MMCPNE states (e.g., MCPNE, ACPNE and ICPNE). Finally, we present the simulation results of the pervious algorithms, which show that MMCPNE outperforms CPNE and NE channel allocation schemes in terms of achieved data rates of multi-hop links due to cooperative gain.

The paper is organized as follows. In Section 2, we present related work on channel allocation and channel access in wireless networks. In Section 3, we introduce the system model which contains multi-hop links. In Section 4, we introduce the game-theoretic description of competitive channel allocation problem in multi-hop wireless networks. In Section 5, we provide a comprehensive analysis of the Nash equilibrium and min-max coalition-proof Nash equilibrium in the channel allocation game. Additionally, we propose several algorithms to reach the exact and approximate MM-

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CPNE state in Section 6. In Section 7, we study their convergence properties and present the simulation results of previous algorithms. Finally, we conclude in Section 8.

2. RELATED WORK

There has been a considerable amount of research on channel allocation in wireless networks, especially in cellular networks. Three major categories of channel allocation schemes are always used in cellular networks: fixed channel allocation (FCA, e.g., as present in [4]), dynamic channel allocation (DCA, e.g., as present in [5]) and hybrid channel allocation (HCA, e.g., as present in [6]) which is a combination of both FCA and DCA techniques.

Recently, channel allocation problem is becoming a focus of research again due to the appearance of new communication technologies, e.g., wireless local area networks (WLANs), wireless mesh networks (WMNs, e.g., as present in [7] and [8]) and wireless sensor networks (WSNs, e.g., as present in [9] and [10]). Using weighted graph coloring method, Mishra *et al.* propose a channel allocation method for WLANs in [11]. In wireless mesh networks, many researchers have considered devices using multiple radios. Equipping multiple radios in the devices in WMNs, especially the devices acting as wireless routers, can improve the capacity by transmitting over multiple radios simultaneously using orthogonal channels. In the multi-radio communication context, channel allocation and access are also considered as the vital topics. By joint considering the channel assignment and routing problem, Alicherry *et al.* propose an algorithm to optimize the overall throughput of WMNs in [12].

In the above cited work, the authors make the assumption that the devices cooperate with the purpose of the achievement of high system performance. However, this assumption might not hold for the following two reasons. In one hand, players are usually selfish who would like to maximize their own performance without considering the other players' objective. In the other hand, the full cooperation of arbitrary devices is difficult to achieve due to the transmission distance limitation and transmission interference of neighboring devices.

Game theory provides a straightforward tool to study channel allocation problems in competitive wireless networks. As far as know, game theory has been applied to the CSMA /CA protocol [13], [14], to the Aloha protocol [15] and to the peer-to-peer system [16]. Furthermore, on the basis of graph coloring, Halldorsson *et al.* use game theory to solve a fixed channel allocation problem in [17]. Unfortunately, their model does not apply to multi-radio devices. In wireless ad-hoc networks (WANETs), Felegyhazi *et al.* present a game-theoretic analysis of fixed channel allocation strategies of devices that use multiple radios in [3]. However, their results can be only applied to single-hop wireless networks without considering multi-hop networks.

3. SYSTEM MODEL

We assume that the available frequency band is divided into N orthogonal channels of the same bandwidth using the FDMA method (e.g., 8 orthogonal channels in case of the IEEE 802.11a protocol). We denote the set of available orthogonal channels by $\mathbf{C} = \{c_1, c_2, \ldots, c_N\}$.

We assume that there exist L communication sessions[1] in our model and we denote the set of communication sessions by $\mathbf{L} = \{l_1, \ldots, l_L\}$. We further assume each user participates in only one session. Hence we can divide all users into L *disjoint groups*, denoted by g_i, according to different sessions. We denote the set of groups by $\mathbf{G} = \{g_1, \ldots, g_L\}$ and $g_i \rightleftharpoons l_i$, where "$\rightleftharpoons$" denotes a one-one mapping. Additionally, we denote the set of senders in all groups by $\mathbf{S}$ and the set of relaying users by $\mathbf{R}$. It is easy to see that $|\mathbf{S}| = |\mathbf{L}| = |\mathbf{G}| = L$. Figure 1 presents an example with three communication sessions, where $L = 3$, $\mathbf{L} = \{l_1, l_2, l_3\}$, $\mathbf{G} = \{\{s_1, d_1\}, \{s_2, r_{2_1}, d_2\}, \{s_3, d_3\}\}$, $\mathbf{S} = \{s_1, s_2, s_3\}$ and $\mathbf{R} = \{r_{2_1}\}$.

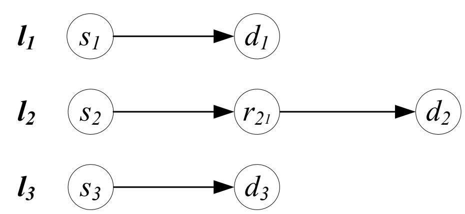

Figure 1: An example of 3 communication sessions.

We assume each user owns a device equipped with two independent sets of radio transceivers, denoted by $\mathbf{T}_s$ and $\mathbf{R}_s$, which used to originate and receive the data packets respectively. Each transceivers set contains $k < |\mathbf{C}|$ radio transceivers, all having the same communication capabilities. The communication between two devices is bidirectional and they always have some packets to exchange. Due to the bidirectional links, the originator radios in the sender and the receptor radios in the receiver are able to coordinate and thus to select the same channels to communicate. Hence, we omit the behaviors of $\mathbf{R}_s$ since they well correspond with the $\mathbf{T}_s$ in pre-hop users, and accordingly omit the behavior of receivers. Thus we define the *players* set, denoted by $\mathbf{U}$, as the summation of senders set and relay users set, i.e., $\mathbf{U} = \mathbf{S} \bigcup \mathbf{R}$.

We assume that there is a finite number of players. We further assume that each device can hear the transmissions of any other device if they are using the same channel. This means that the players reside in a single collision domain. Note however that one device cannot communicate directly with other devices except its neighboring devices, e.g., the device equipped by pre-hop or post-hop player. For a comprehensive understanding of this phenomenon, please refer to the definitions of *sensing range* and *transmission range* in [17].

We assume that there is a mechanism that enables the multiple radios in any $\mathbf{T}_s$ (or $\mathbf{R}_s$) to communicate simultaneously by using orthogonal channels (as it is implemented in [18] for example). We denote the number of radios of player u_i using channel c by $k_{u_i,c}$ for every $c \in \mathbf{C}$. For the sake of suppressing co-radios interference in device, we assume that different radios in any $\mathbf{T}_s$ (or $\mathbf{R}_s$) cannot use the same channel, i.e., $k_{u_i,c} \leq 1$ for arbitrary players and channels.

We formulate the channel allocation problem with a single stage game, which corresponds to a fixed channel allocation

[1]Note that in our paper, the meaning of communication session is equivalent to the active communication link.

among the players. Each player's *strategy* consists in defining the number of radios on each of the channels. Hence, we define the strategy of player u_i as its channel allocation vector:

$$\mathbf{x}_{u_i} = \left(k_{u_i,1},\ k_{u_i,2},\ \ldots,\ k_{u_i,|\mathbf{C}|}\right) \tag{1}$$

The strategy matrix, denoted by $\mathbb{X}$, is defined by all players' strategy vectors:

$$\mathbb{X} = \left(\mathbf{x}_{s_1}^T, .., \mathbf{x}_{s_{|\mathbf{S}|}}^T,\ \ \mathbf{x}_{r_1}^T, .., \mathbf{x}_{r_2}^T, .., \mathbf{x}_{r_{|\mathbf{R}|}}^T, ..\right)^T \tag{2}$$

Furthermore, we denote the strategy matrix except for the strategy of player u_i (or co_i) by $\mathbb{X}_{-u_i}$ (or $\mathbb{X}_{-co_i}$) and the strategy matrix of players set $\mathbf{U}_x \subset \mathbf{U}$ by $\mathbb{X}_{\mathbf{U}_x}$.

Figure 2 presents an example of channel allocation strategy in the system of Figure 1 with four available channels ($|\mathbf{C}| = 4$) and four players ($|\mathbf{U}| = |\mathbf{S}| + |\mathbf{R}| = 4$). Each player's device equipped by two radios sets which contain three radios transceivers ($k = 3$) respectively. The tubers at the left of node denote the radios of $\mathbf{R}_s$ and the remainder tubers denote the radios of $\mathbf{T}_s$. The number on each radio link denotes the channel used by this radio transceivers pair. We can easily write the strategy of players s_1 and s_2 as $\mathbf{x}_{s_1} = (1\ 1\ 0\ 1)$ and $\mathbf{x}_{s_2} = (1\ 1\ 0\ 0)$ respectively.

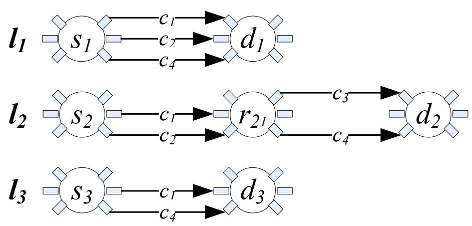

Figure 2: An example of channel allocation strategy, where $|\mathbf{C}| = 4$, $|\mathbf{S}| = 3$, $|\mathbf{R}| = 1$ and $k = 3$.

The total number of channels used by player u_i can be written as $k_{u_i} = \sum_c k_{u_i,c}$ and $k_{u_i} \leq k$ obviously. Similarly, the total number of radios using a particular channel c can be written as $k_c = \sum_{u_i} k_{u_i,c}$. In Figure 2, $k_{s_1} = 3$, $k_{s_2} = k_{s_3} = k_{r_{2_1}} = 2$, $k_{c_1} = k_{c_4} = 3$, $k_{c_2} = 2$ and $k_{c_3} = 1$.

We denote the total available bandwidth on channel c (i.e., the sum of the achieved date rate of all players on channel c) by $R_c(k_c)$. In fact $R_c(k_c)$ is independent of k_c for a TDMA protocol and for the CSMA/CA protocol using optimal backoff window values [19]. In practice, however, the backoff window values (e.g., in the 802.11 standard) are not optimal, and due to packet collisions $R_c(k_c)$ becomes a decreasing function of k_c for $k_c > 1$. In our model, we assume that $R_c(k_c)$ is independent of k_c and thus we can write $R_c(k_c)$ as R_c by omitting the parameter k_c. Note however, that our simulation shows similar results when $R_c(k_c)$ is a slowly decreasing function of k_c for $k_c > 1$.

We assume that the total available bandwidth on channel c (i.e., R_c) is shared *equally* among the radios deployed on this channel. We denote $R_{u_i,c}$ as the available bandwidth occupied by player u_i on channel c and we can write $R_{u_i,c}$ as following:

$$R_{u_i,c} = \frac{k_{u_i,c}}{k_c} \cdot R_c, \quad \forall u_i \in \mathbf{U},\ c \in \mathbf{C} \tag{3}$$

>From Equation (3), we can easily find that the higher the number of radios in a given channel is, the lower the bandwidth per radio is. In Figure 2, we have $R_{s_1,c_1} = R_{s_1,c_4} < R_{s_1,c_2}$ and $R_{s_2,c_1} < R_{s_2,c_2}$. We define the *utility* of player u_i, denote by $R_{u_i}^i$, as the total available bandwidth occupied by u_i and we can write $R_{u_i}^i$ as follows.

$$R_{u_i}^i = \sum_{c \in \mathbf{C}} R_{u_i,c}, \quad \forall u_i \in \mathbf{U} \tag{4}$$

In fact any player's utility is equivalent to its *one-hop rate*. In single-hop networks, the utility of any player exactly reflect its actual date rate. In multi-hop networks, however, the utility of any player[2] may not reflect its achieved data rate. We define the *end-to-end rate* of a communication link, denoted by $R_{l_i}^e$, as the minimal utility of players in the link and we can write $R_{l_i}^e$ as follows.

$$R_{l_i}^e = \min_{u_i \in g_i} R_{u_i}^i, \quad \forall l_i \in \mathbf{L} \tag{5}$$

where u_i is arbitrary player in group g_i [3], i.e., in communication link l_i. For single-hop link, $R_{l_i}^e = R_{u_i}^i$ since there is only one player u_i in the link.

Recall the example in Figure 2, we can easily obtain the normalized one-hop rates: $R_{s_1}^i = \frac{1}{3} + \frac{1}{2} + \frac{1}{3} = 1.17$, $R_{s_2}^i = \frac{1}{3} + \frac{1}{2} = 0.83$, $R_{s_3}^i = \frac{1}{3} + \frac{1}{3} = 0.67$ and $R_{r_{2_1}}^i = 1 + \frac{1}{3} = 1.33$. Accordingly, we can obtain the normalized end-to-end rate of communication links: $R_{l_1}^e = R_{s_1}^i = 1.17$, $R_{l_2}^e = \min\{R_{s_2}^i, R_{r_{2_1}}^i\} = 0.83$ and $R_{l_3}^e = R_{s_3}^i = 0.67$.

4. NASH EQUILIBRIA

We refer to each player as a *rational* and *self-interested* player, who will always choose action that maximize its payoff. Thus we can formulate the multi-radio channel allocation problem as a static game, which corresponds to a fixed channel allocation among the players.

In single-hop networks, the multi-radio channel allocation problem can be formulated as a static non-cooperative game (e.g., in [3]). We define the *payoff* of player u_i, denoted by $P_{u_i}(\mathbb{X})$, as the utility of u_i in the strategy matrix $\mathbb{X}$, i.e., $P_{u_i}(\mathbb{X}) = R_{u_i}^i$. In order to study the strategic interaction of the players in static non-cooperative game, we first introduce the concepts of Nash equilibrium [20].

Definition 1: (Nash Equilibrium - NE): The strategy matrix $\mathbb{X}^* = \left\{\mathbf{x}_{u_1}^*, \ldots, \mathbf{x}_{u_{|\mathbf{U}|}}^*\right\}$ defines a Nash Equilibrium (NE), if for every player u_i, we have:

$$P_{u_i}\left(\mathbf{x}_{u_i}^*, \mathbb{X}_{-u_i}^*\right) \geq P_{u_i}\left(\mathbf{x}_{u_i}', \mathbb{X}_{-u_i}^*\right) \tag{6}$$

for every strategy $\mathbf{x}_{u_i}'$.

The definition of NE expresses the resistance to the deviation of a single player in non-cooperative game. In other words, in a NE none of the players can unilaterally change its strategy to increase its utility.

However, non-cooperative game is not suitable for the multi-hop networks for the following two reasons. In one hand, the definition of payoff function is not suitable for multi-hop networks. Specifically, the achieved date rate of any player in multi-hop link is not only determined by the utility itself, but also by the utilities of other players in the same link. In the other hand, it is possible that the players in the same multi-hop link cooperatively choose their

[2]Specifically, the players belonging to the multi-hop links.
[3]Strictly speaking, u_i is arbitrary player in group $g_i - \{d_i\}$ since we do not consider the behaviors of d_i.

strategies for the purpose of high achieved date rate. Thus we formulate the problem as a static *cooperative* game in multi-hop networks.

In cooperative game, it might be possible that some players collude to increase their payoff at the expense of other players. Such a collusion is called a *coalition*, denoted by co_i. We denote the set of coalitions by $\mathbf{Q} = \{co_1,\ co_2,\ ...\}$. We can generalize the notion of classical coalition-proof Nash equilibrium as defined in [21].

Definition 2: (Coalition-Proof Nash Equilibrium - CPNE): The strategy matrix $\mathbb{X}^{cp}$ defines a coalition-proof Nash Equilibrium, if for every coalition $co_i \in \mathbf{Q}$, we have:

$$P_{u_i}\left(\mathbb{X}^{cp}_{co_i}, \mathbb{X}^{cp}_{-co_i}\right) \geq P_{u_i}\left(\mathbb{X}'_{co_i}, \mathbb{X}^{cp}_{-co_i}\right), \quad \forall u_i \in co_i \qquad (7)$$

for every strategy set $\mathbb{X}'_{co_i}$.

This means that no coalition can deviate from $\mathbb{X}^{cp}$ such that the payoff of at least one of its members increases and the payoffs of other members do not change. Note that our definition in (7) corresponds to the principle of *weak deviation*.

In our model, we define coalition as the players belonging to the same communication link, e.g., s_2 and r_{2_1} in Figure 1, since they can easily build up the cooperative process. Thus each communication link l_i corresponds to a coalition[4] co_i. In fact a coalition can be seen as a subset of $\mathbf{U}$ [5] and $\mathbf{Q}$ can be seen as a partition of $\mathbf{U}$, i.e., $\bigcup_{i=1}^{L} co_i = \mathbf{U}$ and $co_i \bigcap co_j = \emptyset,\ \forall i \neq j$. In Figure 1, $co_1 = \{s_1\}$, $co_2 = \{s_2,\ r_{2_1}\}$ and $co_3 = \{s_3\}$.

Unfortunately, we find that classical CPNE in definition 2 is *not* strictly suitable for multi-hop networks as we define the coalition as above. In detail, according to CPNE, it is not permission for any coalition to improve a member's (e.g., u_i) payoff with worsening any other member's (e.g., v_i) payoff, even the payoff of v_i is much higher than u_i. Thus the payoffs of players in the same coalition might be imbalance in classical CPNE, which will lead to poor performance in terms of achieved date rate.

Figure 3 presents an example of CPNE where $|\mathbf{C}| = 6,\ k = 3$, player $u_1,\ u_2$ formulate a coalition and $u_3,\ u_4,\ u_5$ are non-coalition players. In other word, the coalition (of u_1 and u_2) can not improve a member's payoff without worsening the other member's payoff by unilaterally changing the strategies of u_1 and u_2. It is obvious that $P_{u_1} = 1.0,\ P_{u_2} = 1.5$ and $P_{u_1} \ll P_{u_2}$. As we mentioned above, player u_1 and u_2 belong to the same communication link, and thus the actual data rates of u_2 is the minimal utility of u_1 and u_2 (i.e., 1.0), which is much lower than the payoff of itself.

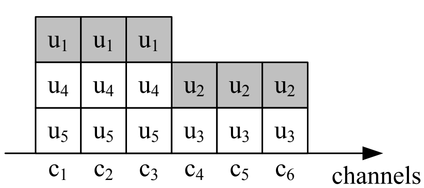

Figure 3: An example of CPNE channel allocation, where u_1 and u_2 formulate a coalition.

To overcome the shortcoming of payoff imbalance in CPNE, we define a novel coalition-proof Nash equilibrium in cooperative game, named as min-max coalition-proof Nash equilibrium (MMCPNE), in which players make their decisions so as to improve the minimal payoff of players in the coalition. We generalize the notion of MMCPNE as following:

Definition 3: (Min-Max Coalition-Proof Nash Equilibrium - MMCPNE): The strategy matrix $\mathbb{X}^{mm}$ defines a novel coalition-proof Nash Equilibrium, if for every coalition co_i, we have:

$$\min_{u_i \in co_i} R^i_{u_i}\left(\mathbb{X}^{mm}_{co_i}, \mathbb{X}^{mm}_{-co_i}\right) \geq \min_{u_i \in co_i} R^i_{u_i}\left(\mathbb{X}'_{co_i}, \mathbb{X}^{mm}_{-co_i}\right) \qquad (8)$$

for every strategy set $\mathbb{X}'_{co_i}$.

It is notable that MMCPNE points are not always CPNEs and vice versa. In fact, MMCPNE can be seen as a special coalition-proof Nash equilibrium with a judiciously designed payoff function, i.e., end-to-end rate $R^e_{l_i}$. Recall the example in Figure 3, if we do $(u_1, c_1) \leftrightarrows (u_2, c_4)$ [6], we obtain the MMCPNE channel allocation and we find the actual data rates of u_1 and u_2 increase to 1.17 !

However, it is very difficultly to find such a MMCPNE (or CPNE) strategy since we must jointly search the strategy in the strategies set of $|co_i|$ players. The computation of achieving MMCPNE (or CPNE) increases exponentially with the size of coalition, typically $O(\bar{\omega}^{|co_i|})$ where $\bar{\omega}$ is the expectation of ω and ω is the number of moves for a single player finding its best response strategy. In the worst case, a player must try all possible strategies to find its best response strategy, i.e., $\omega = \binom{|\mathbf{C}|}{k} = \frac{|\mathbf{C}|\cdot(|\mathbf{C}|-1)\cdot\ldots\cdot(|\mathbf{C}|-k+1)}{k\cdot(k-1)\cdot\ldots\cdot 1}$.

To reduce the large computation in finding MMCPNE, we introduce three approximate solutions, denoted by minimal coalition-proof Nash equilibrium (MCPNE), average coalition-proof Nash equilibrium (ACPNE) and **i** coalition-proof Nash equilibrium (ICPNE). The definitions of MCPNE, ACPNE and ICPNE are shown as follows.

Definition 4: (Minimal Coalition-Proof Nash Equilibrium - MCPNE): The strategy matrix $\mathbb{X}^m$ defines a special coalition-proof Nash Equilibrium, if for every player u_i, we have:

$$\min_{u_i \in co_i} R^i_{u_i}\left(\mathbf{x}^m_{u_i}, \mathbb{X}^m_{-u_i}\right) \geq \min_{u_i \in co_i} R^i_{u_i}\left(\mathbf{x}'_{u_i}, \mathbb{X}^m_{-u_i}\right) \qquad (9)$$

for every strategy $\mathbf{x}'_{u_i}$.

Definition 5: (Average Coalition-Proof Nash Equilibrium - ACPNE): The strategy matrix $\mathbb{X}^a$ defines a special coalition-proof Nash Equilibrium, if for every player u_i, we have:

$$\min_{u_i \in co_i} R^i_{u_i}\left(\mathbf{x}^a_{u_i}, \mathbb{X}^a_{-u_i}\right) > \min_{u_i \in co_i} R^i_{u_i}\left(\mathbf{x}'_{u_i}, \mathbb{X}^a_{-u_i}\right) \qquad (10)$$

or

$$\begin{cases} \min\limits_{u_i \in co_i} R^i_{u_i}\left(\mathbf{x}^a_{u_i}, \mathbb{X}^a_{-u_i}\right) = \min\limits_{u_i \in co_i} R^i_{u_i}\left(\mathbf{x}'_{u_i}, \mathbb{X}^a_{-u_i}\right) \\ \sum\limits_{u_i \in co_i} R^i_{u_i}\left(\mathbf{x}^a_{u_i}, \mathbb{X}^a_{-u_i}\right) \geq \sum\limits_{u_i \in co_i} R^i_{u_i}\left(\mathbf{x}'_{u_i}, \mathbb{X}^a_{-u_i}\right) \end{cases} \qquad (11)$$

for every strategy $\mathbf{x}'_{u_i}$.

Definition 6: (I Coalition-Proof Nash Equilibrium - ICPNE): The strategy matrix $\mathbb{X}^i$ defines a special coalition-proof Nash Equilibrium, if for every player u_i, we have:

$$\min_{u_i \in co_i} R^i_{u_i}\left(\mathbf{x}^i_{u_i}, \mathbb{X}^i_{-u_i}\right) > \min_{u_i \in co_i} R^i_{u_i}\left(\mathbf{x}'_{u_i}, \mathbb{X}^i_{-u_i}\right) \qquad (12)$$

[4]It is notable that the coalition may contain one player only.
[5]Strictly speaking, $co_i = g_i - \{d_i\}$ where d_i is the destination user in group g_i.

[6]Note that $(u_i, c_m) \leftrightarrows (u_j, c_n)$ means exchanging the radio of u_i in channel c_m and the radio of u_j in channel c_n.

or

$$\begin{cases} \min_{u_i \in co_i} R^i_{u_i}\left(\mathbf{x}^i_{u_i}, \mathbb{X}^i_{-u_i}\right) = \min_{u_i \in co_i} R^i_{u_i}\left(\mathbf{x}'_{u_i}, \mathbb{X}^i_{-u_i}\right) \\ R^i_{u_i}\left(\mathbf{x}^i_{u_i}, \mathbb{X}^i_{-u_i}\right) \geq R^i_{u_i}\left(\mathbf{x}'_{u_i}, \mathbb{X}^i_{-u_i}\right) \end{cases} \quad (13)$$

for every strategy $\mathbf{x}'_{u_i}$.

It is notable that MCPNE, ACPNE and ICPNE are totally different although they seem similar. In MCPNE, players in a coalition select their strategies to maximize the minimal utilities of players in the coalition. In ACPNE, players in a coalition select their strategies to maximize the average utility while do not decrease the minimal utility of players in the same coalition. In ICPNE, however, players in a coalition select their strategies to maximize their own utilities while do not decrease the minimal utility of players in the same coalition.

Obviously, players can select their strategies *independently* to achieve the above three approximate MMCPNE situations and thus the computations increase linearly to the size of coalition, i.e., $O(\bar{\omega} \cdot |co_i|)$. Strictly speaking, MCPNE (or ACPNE, ICPNE) is Nash equilibrium of non-cooperative game with a well-connected payoff function, rather than coalition-proof Nash equilibrium of cooperative game. In Section 6, we will show whether it is feasible to consider MCPNE (or ACPNE, ICPNE) as an approximation of MMCPNE through the simulation results.

5. EXISTENCE OF MMCPNE

In this section, we study the existence of Nash equilibria and min-max coalition-proof Nash equilibria in the static cooperative game.

In our model, we assume that $|\mathbf{U}| \cdot k > |\mathbf{C}|$, hence the devices have a conflict during the channel allocation process. We first retrospect the work done by Mark Felegyhazi in [3]. The authors study in detail the problem of competitive multi-radio multi-channel allocation in *single-hop* wireless networks, i.e., $\mathbf{R} = \emptyset$ or $\mathbf{U} = \mathbf{S}$, and propose the conditions for Nash equilibria as the following theorem[7].

Theorem 1: Assume that $|\mathbf{S}| \cdot k > |\mathbf{C}|$. Then a channel allocation $\mathbb{X}^*$ is a NE iff the following conditions hold:

- $k_{u_i,c} \leq 1$ and $k_{u_i} = k$ for any $u_i \in \mathbf{S},\ c \in \mathbf{C}$ and
- $\delta_{b,c} \leq 1$ for any $b, c \in \mathbf{C}$

where $\delta_{b,c} = k_b - k_c$ denotes the difference of radios number between channel b and c.

As mentioned in Section 4, the NE of non-cooperative game is not suitable for the multi-hop networks. In the following, we study the existence of MMCPNE for coalition set $\mathbf{Q}$ in multi-hop networks. For simplicity, we assume that any communication session contains at most 2 hops, i.e., any coalition co_i contains at most 2 players. Note however, that it can be easily extended to the system in which any session contains more than 2 players.

Similar as Nash equilibria, there exist multiple MMCPNE states in the system. We divide the MMCPNE states into two sets according to theorem 1. We denote the MMCPNE states which satisfy the theorem 1 by MMCPNE-1 and denote the remainder MMCPNE states by MMCPNE-2. We find that the multi-hop links in MMCPNE-1 states always occupy more bandwidth compared with those in MMCPNE-2 states. We show this property as the following proposition.

[7]Note that we omit the second type of Nash equilibria proposed by Mark Felegyhazi, in which some players use multiple radios in the same channel.

Proposition 1: Assume that there exists a MMCPNE channel allocation $\mathbb{X}$ with *high* coalition utility[8] (for the multi-user coalitions), then $\mathbb{X}$ is a Nash equilibrium, i.e., the conditions of theorem 1 hold.

Proof. It is straightforward to see that the first condition in theorem 1 always holds in MMCPNE due to the co-radios interference in device and the selfish nature of players. We validate the second condition in theorem 1 by contradiction. Assume there exists two channels b and c such that $\delta_{b,c} \geq 2$ in a (high coalition utility) MMCPNE strategy $\mathbb{X}$. We denote the set of individual players in channel b by $U_b = \{u_1,\ u_2,\ ...\}$, i.e., $k_{u_i,b} = 1,\ \forall u_i \in U_b$. It is obvious that $k_{u_i,c} = 1$ otherwise u_i can improve its payoff by move its radio from channel b to c. We denote the set of remainder players in channel b by $U_{co} = \{v_1, v_2, ...\}$. Similarly, we denote U_x as the remainder players in channel c excluding the players in U_b. It is easy to see that $|U_{co}| - |U_x| = k_b - k_c \geq 2$, i.e., there exist at least two players v_{j_1} and v_{j_2} such that $v_{j_1} \in U_{co},\ v_{j_2} \in U_{co}$ and $v_{j_1} \notin U_x,\ v_{j_2} \notin U_x$. We show this situation as Figure 4. Now suppose that player v_{j_1} (or v_{j_2}) moves its radio from channel b to c, the utility U_b and U_x occupied decreases. Thus the utility of U_{co} increases since the total available bandwidth is constant. □

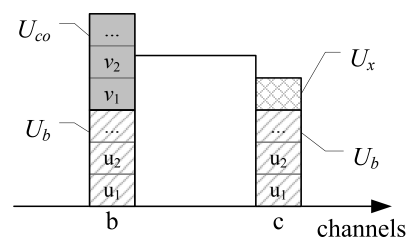

Figure 4: An example of MMCPNE channel allocation corresponding to Proposition 1.

It is obvious that the coalition with high utility is likely to achieve high date rate. The value of Proposition 1 is that it provides a method to choose the MMCPNE with the high coalition utility[9], i.e., MMCPNE-1. Thus we will focus on the MMCPNE-1 strategies for the remainder of the paper.

We divide the channels in NE channel allocation $\mathbb{X}^*$ into two sets. We define the set of channels $\mathbf{C}^+$ with the maximum number of radios, i.e., where any $b \in \mathbf{C}^+$ has $k_b = \max_{c \in \mathbf{C}} k_c$. We denote the set of the remainder channels by $\mathbf{C}^-$. We denote the number of radios of any channel in $\mathbf{C}^+$ and $\mathbf{C}^-$ by δ^+ and δ^- respectively. It is obvious that $\mathbf{C} = \mathbf{C}^+ \bigcup \mathbf{C}^-$ and $\delta^+ = \delta^- + 1$ according to theorem 1 [10].

Although none of the players can unilaterally change its strategy to increase its payoff in NE, it is possible that a player change its strategy to improve the payoff of another player he is in a coalition with, e.g., u_1 and u_2 in Figure 3.

[8]Coalition utility is defined as the summation of all members' utilities in the coalition. High coalition utility is defined as the fact that the coalition can not improve its utility by unilaterally changing its members' strategies.

[9]It is notable that the utilities of individual players do not decrease in MMCPNE-1 compared with those in NE state.

[10]Note that the second equation holds when $|\mathbf{C}^-| > 0$, otherwise δ^- is meaningless.

Players in a coalition can help each other in two ways. The first possibility is that a player *relocates* its radios to improve the payoff of other when two players share any channels. This property is expressed as the following lemma.

Lemma 2: Assume that there exists a coalition $co_i = \{u_1, u_2\}$ and $R^i_{u_1} \neq R^i_{u_2}$ in a NE channel allocation $\mathbb{X}$. If there exist two channels $c_1 \in \mathbf{C}^+$ and $c_2 \in \mathbf{C}^-$ such that $k_{u_i,c_1} = 1, \forall i$ and $k_{u_i,c_2} = 0, \forall i$, then $\mathbb{X}$ is not MMCPNE.

Proof. Without loss of generality, we assume $R^i_{u_1} > R^i_{u_2}$ and thus the actual date rate $R^e_{l_i}(\mathbb{X}) = \min\{R^i_{u_1}, R^i_{u_2}\} = R^i_{u_2}$. Suppose that u_1 moves its radio from channel c_1 to c_2, the rate of u_1 does not change whereas the rate of u_2 change to $R^i_{u_2} + 1/\delta^- - 1/\delta^+$. We can write the new rate as $R^e_{l_i}(\mathbb{X}') = \min\{R^i_{u_1}, R^i_{u_2} + 1/\delta^- - 1/\delta^+)\} > R^i_{u_2}$ since $\delta^+ = \delta^- + 1$. So we declare that $\mathbb{X}$ is not MMCPNE. □

An example of any NE channel allocation corresponding to Lemma 2 is shown in Figure 5, where $|\mathbf{C}| = 6$, $k = 4$ and u_1, u_2 formulate a coalition. According to Lemma 2, it cannot be a MMCPNE, since we can increase the actual date rates of u_1 and u_2 by moving u_1 from channel c_4 to c_6.

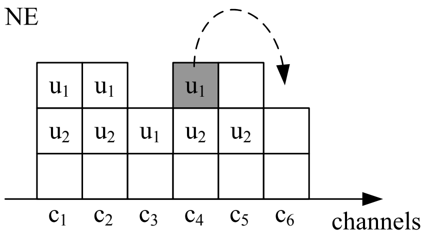

Figure 5: An example of a NE channel allocation corresponding to Lemma 2.

In some cases the assumption of unequal payoffs of two players, i.e., $R^i_{u_1} \neq R^i_{u_2}$, might not hold. In such cases, the Lemma 2 may no longer hold. Thus we show another necessary condition as follows.

Lemma 3: If there exists a coalition $co_i = \{u_1, u_2\}$ and multiple channels $\{x_1, x_2, ...\} \in \mathbf{C}^+$ and $\{y_1, y_2, ...\} \in \mathbf{C}^-$ such that $k_{u_i,x_j} = 1$, $\forall i, j$ whereas $k_{u_i,y_j} = 0$, $\forall i, j$ in a NE channel allocation $\mathbb{X}$, then $\mathbb{X}$ is not MMCPNE.

Proof. Suppose that u_1 moves its radio in channel x_1 to y_1 and u_2 moves its radio in channel x_2 to y_2, the payoffs of player u_1 and u_2 both increase, and thus the minimal payoff (i.e., actual date rate) increases. □

The second possibility for coalition members helping each other is that they mutually *exchange* some radios with each other. We show this necessary condition as the following lemma.

Lemma 4: Assume that there exists a coalition $co_i = \{u_1, u_2\}$ and $R^i_{u_2} - R^i_{u_1} > (1/\delta^- - 1/\delta^+)$ in a NE channel allocation $\mathbb{X}$. If there exists two channels $c_1 \in \mathbf{C}^+$ and $c_2 \in \mathbf{C}^-$ such that $k_{u_1,c_1} = 1$ and $k_{u_2,c_1} = 0$ whereas $k_{u_1,c_2} = 0$ and $k_{u_2,c_2} = 1$, then $\mathbb{X}$ is not MMCPNE.

We can proof the lemma by exchanging their radios in channel c_1 and c_2. Due to space limitation, we do not present the detail proof. We show an example of any NE channel allocation corresponding to Lemma 4 in Figure 6, where $|\mathbf{C}| = 6$, $k = 3$ and u_1, u_2 formulate a coalition. According to Lemma 4, it cannot be a MMCPNE, since we can increase the actual date rates of u_1 and u_2 by exchanging their radios in channel c_3 and c_5.

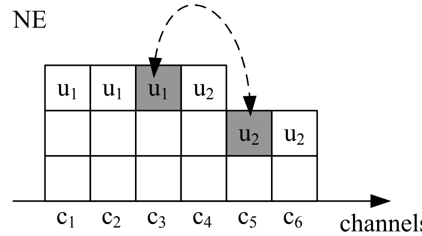

Figure 6: An example of a NE channel allocation corresponding to Lemma 4.

We divide the radios of any player u_i in a NE channel allocation $\mathbb{X}^*$ into two sets. We denote the number of radios deployed in $\mathbf{C}^+$ by $k^+_{u_i}$. Similarly, we denote the number of radios deployed in $\mathbf{C}^-$ by $k^-_{u_i}$. In Figure 6, $k^+_{u_1} = 3$, $k^-_{u_1} = 0$, $k^+_{u_2} = 1$ and $k^-_{u_2} = 2$. Now we can extend Lemma 4 to more general situation.

Lemma 5: Assume that there exists a coalition $co_i = \{u_1, u_2\}$ and $|k^+_{u_1} - k^+_{u_2}| > 1$ in a NE channel allocation $\mathbb{X}$, then $\mathbb{X}$ is not MMCPNE.

Proof. Without loss of generality, we assume that $k^+_{u_1} > k^+_{u_2}$. As mentioned above, any player cannot use multiple radios in the same channel, thus there exists at least one channel $c_1 \in \mathbf{C}^+$ such that $k_{u_1,c_1} = 1$ and $k_{u_2,c_1} = 0$. Similarly, there exists at least one channel $c_2 \in \mathbf{C}^-$ such that $k_{u_1,c_2} = 0$ and $k_{u_2,c_2} = 1$. Furthermore, we can write the utilities of two players as $R^i_{u_1} = k^+_{u_1}/\delta^+ + k^-_{u_1}/\delta^-$ and $R^i_{u_2} = k^+_{u_2}/\delta^+ + k^-_{u_2}/\delta^-$ respectively. Note that $k^+_{u_i} + k^-_{u_i} = k$, thus we can write the utility difference of two players as:

$$\begin{aligned} R^i_{u_2} - R^i_{u_1} &= \frac{k^+_{u_2} - k^+_{u_1}}{\delta^+} + \frac{k^-_{u_2} - k^-_{u_1}}{\delta^-} \\ &= \left(k^+_{u_1} - k^+_{u_2}\right)\left(\frac{1}{\delta^-} - \frac{1}{\delta^+}\right) \end{aligned} \tag{14}$$

Using the conditions of the lemma, we can find that $R^i_{u_2} - R^i_{u_1} > (1/\delta^- - 1/\delta^+)$. Hence, the two conditions of Lemma 4 hold, and we achieve the proof directly from Lemma 4. □

>From equation (14), we can easily find that $|k^+_{u_1} - k^+_{u_2}| > 1$ if $|R^i_{u_2} - R^i_{u_1}| > (1/\delta^- - 1/\delta^+)$. Thus we can immediately relieve some restrictions in Lemma 4. We express this property as the following corollary.

Corollary 6: If there exists a coalition $co_i = \{u_1, u_2\}$ and $R^i_{u_2} - R^i_{u_1} > (1/\delta^- - 1/\delta^+)$ in a NE channel allocation $\mathbb{X}$, then $\mathbb{X}$ is not MMCPNE.

It is notable that lemma 2 and lemma 3 are also available for CPNE state whereas the other lemmas are exclusively used in MMCPNE. Based on the previous lemmas, we prove the necessary conditions that enables a given NE allocation to be MMCPNE and we present it as the following theorem.

Theorem 2: Assume that there exists a coalition $co_i = \{u_1, u_2\}$ and $R^i_{u_1} \geq R^i_{u_2}$ in a NE channel allocation $\mathbb{X}$, if $\mathbb{X}$ is MMCPNE, the following conditions hold:

- $R^i_{u_1} - R^i_{u_2} \leq (1/\delta^- - 1/\delta^+)$ and
- *case 1*: if $R^i_{u_1} \neq R^i_{u_2}$ then there *does not exist* two channels $b \in \mathbf{C}^+$ and $c \in \mathbf{C}^-$ such that $k_{u_1,b} = k_{u_2,b} = 1$ whereas $k_{u_1,c} = k_{u_2,c} = 0$,
- *case 2*: if $R^i_{u_1} = R^i_{u_2}$ then there *does not exist* four channels $\{b_1, b_2\} \in \mathbf{C}^+$ and $\{c_1, c_2\} \in \mathbf{C}^-$ such that

$k_{u_1,b_i} = k_{u_2,b_i} = 1, \ \forall i$ whereas $k_{u_1,c_j} = k_{u_2,c_j} = 0, \ \forall j$.

We could not prove that the conditions in Theorem 2 are sufficient to enable a NE channel allocation to be MMCPNE, neither could we find a counterexample, where the conditions hold and the NE channel allocation is not MMCPNE. Hence, we formulate the following conjecture.

Conjecture 1: Assume that there exists a coalition $co_i = \{u_1, u_2\}$ and $R^i_{u_1} \geq R^i_{u_2}$ in a NE channel allocation $\mathbb{X}$. If the condition in theorem 2 hold, then $\mathbb{X}$ is MMCPNE. Hence the above conditions are necessary and sufficient conditions.

6. CONVERGENCE TO MMCPNE

We have demonstrated the necessary conditions to enable
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C. DCP-M Algorithm

random channel allocation
while not in a MCPNE **do**
  get the current channel allocation
  for $\mathsf{i} = 1$ to $|\mathbf{U}|$ **do**
    if backoff counter is 0 **then**
      assume that user u_i belongs to coalition co_i
      //reorganize the radios of u_i *according to Def.4:*
      for $\mathsf{j} = 1$ to k **do**
        assume that radio j uses channel b
        $\Omega := \{\mathsf{c} | \mathsf{c} \in \mathbf{C}, \mathsf{k}_{u_i,c} = 0\}$
        for $\mathsf{m} = 1$ to $|\Omega|$ **do**
          assume the m_{th} element in Ω is c
          suppose that move radio j from b to c and record $\mu_c = \min_{u \in co_i}(\mathsf{R}^i_u)$
        end for
        move the radio j from b to channel a where $\mathsf{a} = \arg\max_{c \in \Omega} \mu_c$
      end for
      reset the backoff counter to a new value from the set $\{1, ..., \mathsf{W}\}$
    else
      decrease the backoff counter value by one
    end if
  end for
end while

(i.e., the coalition co_x) in our simulation since there is no cooperative gain in the single-hop link. We first introduce three criterions, i.e., coalition utility, coalition efficiency and coalition usage factor, to evaluate the performance of links in the system and we present the concepts of them as follows.

- *Coalition Utility*: the coalition utility of any coalition co_i is defined as the ratio of the total bandwidth co_i occupied to the average bandwidth per user, denoted by $\varphi_{co_i} = \frac{\sum_{u_i \in co_i}(R^i_{u_i})}{(|\mathbf{C}|/|\mathbf{U}|)}$.
- *Coalition Usage Factor*: the coalition usage factor of any coalition co_i is defined as the ratio of the achieved date rate to the total bandwidth co_i occupied, denoted by $\tau_{co_i} = \frac{\min_{u_i \in co_i}(R^i_{u_i})}{\sum_{u_i \in co_i}(R^i_{u_i})}$.
- *Coalition Efficiency*: the coalition efficiency of any coalition co_i is defined as the product of coalition utility and usage factor, denoted by $\phi_{co_i} = \varphi_{co_i} \times \tau_{co_i}$.

The criterion of coalition utility reflects the ability of any coalition[11] to scrabble for the channel bandwidth. The criterion of coalition usage factor is used to measure the usage ratio of total bandwidth co_i occupied. We can easily find that (1) $\tau_{co_i} \leq 1/|co_i|$ and (2) If $\tau_{co_i} \neq 1/|co_i|$, the total bandwidth co_i occupied is not fully used, i.e., any bandwidth wasted. From Lemma 5 in Section 5, players in the same coalition tend to achieve the same utility in a MMCPNE channel allocation, and thus τ_{co_i} is close to its up-bound, i.e., $1/|co_i|$. Furthermore, the criterion of coalition efficiency allows us to define the ability of any coalition to achieve a given date rate. It is easy to see that $\phi_{co_i} = \frac{\min_{u_i \in co_i}(R^i_{u_i})}{(|\mathbf{C}|/|\mathbf{U}|)}$, i.e., the ratio of the end-to-end rate of any link to the average bandwidth per user.

We define *average coalition utility* as the average of coalition utility per round over a long period of time. Similarly, we define *average coalition usage factor* and *average coalition efficiency* as the average of coalition usage factor and efficiency respectively. We also introduce the notion of *Efficiency Ratio* defined in [3] to valid whether a channel allocation is Nash equilibria. Due to space limitation, we do not present the definition in detail.

[11] Specifically, the multi-user coalition.

We assume that the duration of one round in the updating algorithm is $10ms$. This duration of one round corresponds roughly to the time needed for all these devices to transmit one MAC layer packet, i.e., the time that the devices can learn about other devices in the channel. We run each simulation for 600 rounds, which corresponds to $6s$ according to the assumption above. Each average value is the result of 1000 simulation runs.

We firstly investigate the *efficiency ratio* of all algorithms. Figure 7 presents the simulation results in terms of efficiency ratio, where $W = 15$, $|\mathbf{C}| = 8$, $k = 4$, $|\mathbf{U}| = 5$ and players u_1 and u_2 formulate a coalition[12]. From Figure 7, we find that MMCP, DCP-M, DCP-A and DCP-I algorithms all converge to the Nash equilibrium, i.e., their efficiency ratios converge to one.

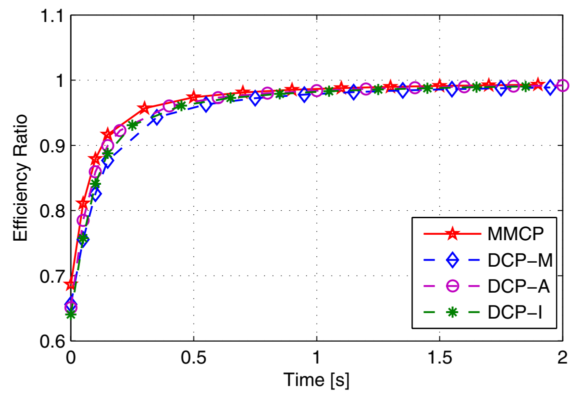

Figure 7: Efficiency ratio vs. time using $W = 15$, $|\mathbf{C}| = 8$, $k = 4$, $|\mathbf{U}| = 5$ and $co_x = \{u_1, u_2\}$.

Next, we present the simulation results of *coalition utility* of co_x in Figure 8. Note that the NE algorithm in the figure is the algorithm in [3], which enables the players converge to any Nash equilibrium from an arbitrary initial configuration. The CPNE algorithm is a algorithm which enables the players converge to conventional CPNE state. We do not present the pseudo-code of CPNE due to space limitation. From Figure 8, we find that MMCP, CPNE and DCP-A algorithms show higher coalition utility compared with other algorithms, specifically, the curves of MMCP and CPNE are almost overlapped. In other word, the coalition co_x tends to occupy more bandwidth in the state of MMCPNE, CPNE and ACPNE. This phenomenon in MMCPNE (or CPNE) can be seen as the results of Lemma 2 and 3. In ACPNE, this phenomenon is caused by the fact that all players in the coalition are willing to improve the total bandwidth. The coalition utility of CPNE is a little higher than DCP-A due to the cooperation gain.

As mentioned above, the criterion of coalition utility cannot reflect the ability of a coalition to achieve a given data rate. We show this phenomenon by CPNE and DCP-A algorithms in Figure 9. We present the simulation results

[12] Note that we present the results of the first 2 second since the curves tend to be steady in the latter time.

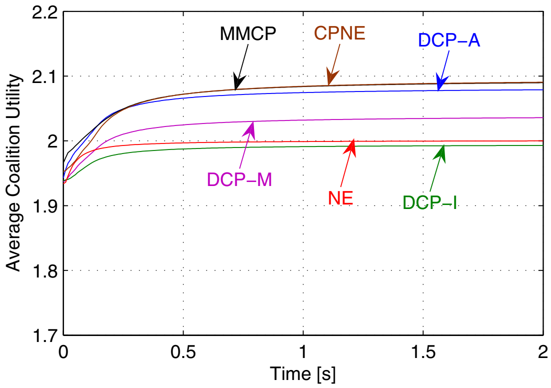

Figure 8: Average coalition utility vs. time using $W = 15,\ |\mathbf{C}| = 8,\ k = 4,\ |\mathbf{U}| = 5$ and $co_x = \{u_1, u_2\}$.

of *coalition efficiency* of co_x in Figure 9, which exactly reflects the achieved data rate. We can observe that CPNE and DCP-A algorithms converge with low coalition efficiency (i.e., low data rate). However, MMCP algorithm still shows highest coalition efficiency. The NE and DCP-I algorithms show lowest performance in terms of both coalition utility and efficiency in the multi-hop networks.

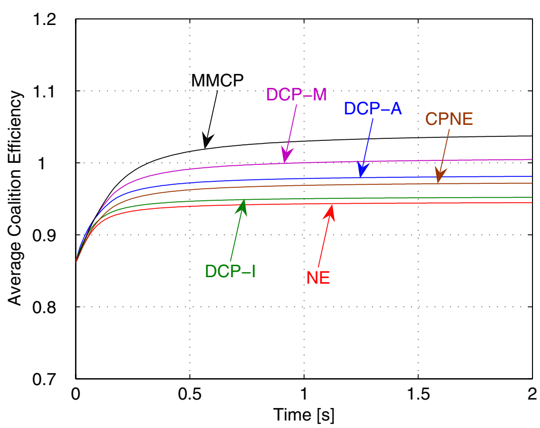

Figure 9: Average coalition efficiency vs. time using $W = 15,\ |\mathbf{C}| = 8,\ k = 4,\ |\mathbf{U}| = 5$ and $co_x = \{u_1, u_2\}$.

Then we present the simulation results of *coalition usage factor* of co_x in Figure 10. We find that MMCP algorithm converges to its up-bound, i.e., $\tau_{co_x} \approx 1/|co_x| = 0.5$. CPNE and DCP-A algorithms show low coalition usage factor because they occupy high total bandwidth while the available bandwidth is low. It is well coincident with former explanation. We find that NE algorithm also shows low coalition usage factor, specifically, the performance of NE is closely to DCP-A.

>From Figure 8 to 10, we find that DCP-M algorithm and MMCP algorithm show tiny performance difference, e.g., less than 1% in terms of coalition usage factor and 5% in terms of coalition efficiency. The DCP-A algorithm and MMCP algorithm show large performance difference in terms of the coalition usage factor due to the large total bandwidth DCP-A occupied and low bandwidth it availably

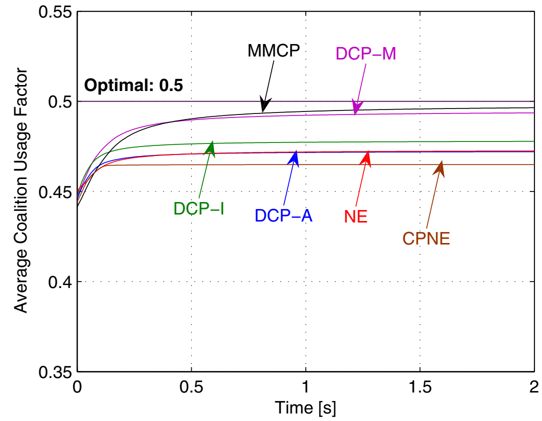

Figure 10: Average coalition usage factor vs. time using $W = 15,\ |\mathbf{C}| = 8,\ k = 4,\ |\mathbf{U}| = 5$ and $co_x = \{u_1, u_2\}$.

used. The DCP-I algorithm and MMCP algorithm show large performance difference in terms of the coalition efficiency due to the low total bandwidth DCP-I occupied.

Finally, we present the effect of number of players on coalition usage factor in Figure 11. We can see that MMCP algorithm always keeps the system in a state of high coalition usage factor whereas CPNE, NE and DCP-A algorithms show low usage factor in most case. It is interesting that all algorithms converge to the same value when total players number $|\mathbf{U}| = 6$ or 8. It is due to the fact that all channels are shared by the same number of radios, and thus the NE state is equivalent with MMCPNE (or ACPNE, MCPNE, ICPNE) state.

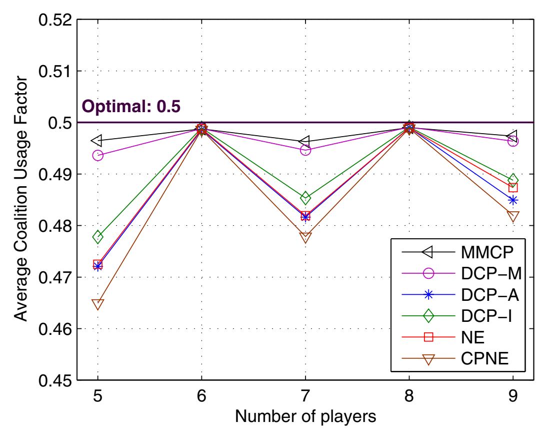

Figure 11: Average coalition usage factor vs. user using $W = 15,\ |\mathbf{C}| = 8,\ k = 4$ and $co_x = \{u_1, u_2\}$.

In summary, we can observe that, the proposed MMCP algorithm based on MMCPNE ensures high performance in terms of coalition efficiency and usage factor due to cooperation gain. NE algorithm proposed in [3] shows low performance in all terms in multi-hop system. Furthermore, we find that MCPNE can be seen as a feasible approximation of MMCPNE while ACPNE and ICPNE show poor performance in terms of coalition efficiency and usage factor respectively compared with MMCPNE.

8. CONCLUSION

In this paper, we have studied the problem of competitive channel allocation among devices which use multiple radios in the multi-hop system. We first analyze that NE and CPNE channel allocation schemes cannot work in multi-hop networks due to the poor performance of achieved date rate of the multi-hop links. Then we propose a novel coalition-proof Nash equilibrium, denoted by MMCPNE, to ensure the multi-hop links to achieve high date rate without worsening the date rates of single-hop links. We investigate the existence of MMCPNE and propose the necessary conditions for the existence of MMCPNE. Finally, we provide several algorithms to achieve the exact and approximate MMCPNE states. We study their convergence properties theoretically. Simulation results show that MMCPNE outperforms CPNE and NE schemes in terms of achieved data rates of links due to cooperation gain.

9. ACKNOWLEDGEMENTS

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10. REFERENCES

[1] T. S. Rappaport. *Wireless Communications: Principles and Practice (2nd).* Prentice Hall, 2002.

[2] M. Schwartz. *Mobile Wireless Communications.* Cambridge Univ. Press, 2005.

[3] M. Felegyhazi, M. Cagalj, S. S. Bidokhti, *etc.* Non-cooperative Multi-radio Channel Allocation in Wireless Networks. In *Proceedings of the IEEE Conference on Computer Communications (INFOCOM '07)*, March 13-17 2007.

[4] J. van den Heuvel, R. A. Leese, and M. A. Shepherd. Graph labeling and radio channel assignment. *Journal of Graph Theory*, 29:263-283, 1998.

[5] I. Katzela and M. Naghshineh. Channel assignment schemes for cellular mobile telecommunication systems: a comprehensive survey. *IEEE Personal Communications*, 3(3):10-31, Jun 1996.

[6] Hac A and Z. Chen. Hybrid channel allocation in wireless networks. In *Proceedings of the IEEE Conference on Vehicular Technology Conference (VTC '99)*, 50(4):2329-2333, Sept. 1999.

[7] I. F. Akyildiz, X. Wang, and W. Wang. Wireless mesh networks: A survey. *Computer Networks, Elsevier North-Holland, Inc.*, 47:445-487, March 2005.

[8] A. Raniwala and T. C. Chiueh. Architecture and algorithms for an IEEE 802.11-based multi-channel wireless mesh network. In *Proceedings of the IEEE Conference on Computer Communications (INFOCOM '05)*, March 13-17 2005.

[9] M. Li and Y. H. Liu, Rendered Path: Range-Free Localization in Anisotropic Sensor Networks with Holes, In *Proceedings of the ACM Conference on Mobile Computing and Networking (MOBICOM '07)*, Montreal, Quebec, Canada, Sept 2007.

[10] M. Li and Yunhao Liu, Underground Structure Monitoring with Wireless Sensor Networks, In *Proceedings of ACM/IEEE Conference on Information Processing In Sensor Networks (IPSN '07)*, Cambridge, Massachusetts, USA, April, 2007.

[11] A. Mishra, S. Banerjee, and W. Arbaugh. Weighted coloring based channel assignment for WLANs. *Mobile Computing and Communications Review (MC2R)*, 9(3), 2005.

[12] M. Alicherry, R. Bhatia, and L. (Erran) Li. Joint channel assignment and routing for throughput optimization in multi-radio wireless mesh networks. In *Proceedings of the ACM Conference on Mobile Computing and Networking (MOBICOM '05)*, pages 58-72, Aug. 28-Sep. 2 2002.

[13] M. Cagalj, S. Ganeriwal, I. Aad, *etc.* On selfish behavior in CSMA/CA networks. In *Proceedings of the IEEE Conference on Computer Communications (INFOCOM '05)*, March 13-17 2005.

[14] J. Konorski. Multiple access in ad-hoc wireless LANs with noncooperative stations. In *Proceedings of Networking 2002*, pages 1141-1146, 2002.

[15] A. B. MacKenzie and S. B. Wicker. Stability of multipacket slotted aloha with selfish users and perfect information. In *Proceedings of the IEEE Conference on Computer Communications (INFOCOM '03)*, March 30-April 3 2003.

[16] M. Y. Kao, X. Y. Li and W. Z. Wang. Towards Truthful Mechanisms for Binary Demand Games: A General Framework. In *Proceedings of the ACM Conference on Electronic Commerce (EC '05)*, pages 213-222, 2005.

[17] M. M. Halldorsson, J. Y. Halpern, L. (Erran) Li, *etc.* On spectrum sharing games. In *Proceedings of the 23rd annual ACM symposium on Principles of Distributed Computing (PODC '04)*, pages 107-114, July 25-28 2004.

[18] A. Adya, P. Bahl, J. Padhye, *etc.* A multi-radio unification protocol for IEEE 802.11 wireless networks. In *Proceedings of the IEEE conference on Broadband Communications, Networks, and Systems (BROADNETS '04)*, pages 344-354, 2004.

[19] G. Bianchi. Performance analysis of the IEEE 802.11 distributed coordination function. *IEEE Journal on Selected Areas in Communication*, 18(3), March 2000.

[20] D. Fudenberg and J. Tirole. *Game Theory.* MIT Press, 1991.

[21] B. D. Bernheim, B. Peleg, and M. D. Whinston. Coalition-proof Nash equilibria: I concepts. *Journal of Economic Theory*, 42(1), June 1987.