A Distributed-Centralized Scheme for Short-Term and Long-Term Spectrum Sharing with a Random Leader in Cognitive Radio Networks

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***Abstract*—In Cognitive Radio Networks (CRNs), Primary Users (PUs) can share their idle spectra with Secondary Users (SUs) under certain mechanisms. In this paper, we propose a distributed-centralized and incentive-aware spectrum sharing scheme for the multiple-PU scenario, which introduces a Random Leader who is elected randomly from SUs or PUs. The distributed aspect of our scheme lies in that it requires no central control entities, which can be independently implemented within a distributed spectrum market. The centralized aspect is that the leader draws up and assigns the socially optimal contracts for all PUs and SUs in a centralized manner, which maximizes the throughput of the whole network and attains the economic robustness (including Incentive Compatibility and Individual Rationality). Analysis shows that the proposed scheme takes in the advantages of both centralized and distributed schemes but overcomes their weaknesses. We use the proposed scheme to study two sharing scenarios: the short-term and the long-term spectrum sharing. The Short-Term Sharing (STS) focuses on distributing PUs' idle spectra within one time slot while the Long-Term Sharing (LTS) considers multiple slots, where the *spectrum mobility* must be investigated. As an integrated design of both STS and LTS, our scheme not only fulfils SUs' heterogeneous spectrum requirements but also obtains the socially optimal throughput while accounting for both SUs' and PUs' incentives.**

***Keywords*—Cognitive radio, spectrum sharing, spectrum mobility, Random Leader, economic robustness**

I. Introduction

Cognitive Radio (CR) has been proposed as a promising paradigm to solve the shortage of spectrum resources. In Cognitive Radio Networks (CRNs), Primary Users (PUs) can share their unused spectra with unlicensed Secondary Users (SUs) under certain incentives. Still, spectrum sharing in the CRN is an extremely challenging issue due to its complexity. On one hand, many factors such as power control and incentive issues must be considered. For example, as for power control, SUs must limit their power so as to cause as little interference to PUs as possible [19], [20]. Besides, the spectrum sharing scheme should also provide enough incentives for PUs and SUs to participate in the spectrum sharing [1], [3].

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On the other hand, SUs' heterogenous requirements for the spectrum usage result in another kind of complexity in designing spectrum sharing schemes. For example, some SUs may need to send a short message or browse a web page, which only requires the temporary usage of PUs' spectra and can be handled within one time slot in most cases. Meanwhile, some other SUs might need to watch an online movie or hold a distant conference, which demands a relatively long period for using PUs' spectra, thus imposing high requirements on the consistency of the spectrum usage within multiple time slots. The above two scenarios mainly differ in the duration of the spectrum sharing process, and we can accordingly divide the spectrum sharing in the CRN into two categories: Short-Term spectrum Sharing (STS) and Long-Term spectrum Sharing (LTS). In STS, PUs' idle spectra are distributed among SUs within one time slot, which is more likely to arise when SUs only have a small amount of data to transmit on PUs' channels. By comparison, LTS covers multiple slots, which is often the case when SUs require stable spectrum supply to sustain certain services over a long period. As a result, STS and LTS are supplementary to each other in terms of satisfying SUs' heterogenous requirements in the spectrum sharing, and different considerations should be given to them, respectively. A robust and practical spectrum sharing scheme should cover both scenarios in order to better cater to SUs' demands.

STS has been an extremely hot topic recently, but there hasn't been a commonly accepted STS mechanism because various flaws exist with them. So far, there have been two major considerations for STS: centralized methods and market-driven mechanisms. Centralized methods [10], [11] can be applied to STS to attain the socially optimal spectrum allocation in SUs. However, the centralized approaches are not suitable for many distributed CRNs such as ad-hoc CRNs.

By comparison, market-driven mechanisms are relatively flexible since they can be implemented in a distributed manner, and different market-driven STS mechanisms have been proposed: contract-based approaches [1], [13], auction-theoretic schemes [12], [23], game-theoretic methods [24], etc. However, many of these researches only addressed the single-PU scenario. In fact, there are often multiple PUs and multiple SUs coexisting in the network, and such a reality requires new design. To handle the multiple-PU scenario in STS, various extended mechanisms were proposed. For instance, the double-sided auction was proposed as an economic-robust method in [3], [8]. Unfortunately, similar to centralized schemes, double-sided auctions need a neutral and disinterested auctioneer, which is unlikely to arise in most cognitive radio systems. Evolutionary game was applied in [21] and [22] in order to investigate the network dynamics, where SUs can adapt their spectrum buying behaviors by observing the variations of spectrum quality and prices. However, most of these market-driven schemes only focus on the single-sided profit maximization in STS, either PUs' or SUs'.

When it comes to LTS, it differs from STS in two aspects. Firstly, the spectrum availability of PUs' spectra in the long term is dynamic due to PUs' uncertain behaviors of channel reclamation while it is definite knowledge in STS. Hence, LTS needs to be based on the *prediction* of PUs' spectrum availability. Secondly, in LTS, SUs must execute spectrum handovers when channels become unavailable in order to avoid conflicts with PUs, while an SU remains staying in the same PU's spectrum during the whole short-term spectrum sharing period. In many related literatures [16], [17], [18], the above two aspects were put together as *Spectrum Mobility Management*. Unfortunately, most of these researches are one-sided, which only focuses on SUs' optimal spectrum handover strategies in face of the spectrum mobility while imposing few considerations on PUs' incentives to accommodate those SUs when spectrum handover takes place. By comparison, LTS requires an integrate model of spectrum mobility management and spectrum sharing management, where we need to jointly design the spectrum allocation and handover strategies while accounting for both SUs' and PUs' incentives.

To address the above problems, we propose a distributed-centralized and incentive-aware spectrum sharing scheme with a **Random Leader** based on contracts. Both STS and LTS can be addressed under this common model. Our scheme has the following features:

• *Random Leader*: Our scheme introduces a Random Leader to draw up and assign socially optimal contracts for PUs and SUs, which shows the centralized aspect of our scheme. Meanwhile, since the Random Leader is elected within the spectrum market, which requires no central control entities to coordinate the spectrum sharing process, our scheme can be implemented in a distributed spectrum market. Under the Random Leader's leadership, all followers behave cooperatively and do not need to exchange information with each other, which mitigates the complexity and time overhead used for establishing contracts between PUs and SUs.

• *Socially Optimal Contracts*: In our scheme, the spectrum sharing results are socially optimal, i.e., the throughput of the whole network is maximized, overcoming the one-sidedness of many traditional spectrum sharing schemes which focused on either PUs' utilities or SUs' utilities.

• *Long-Term Spectrum Sharing*: To our best knowledge, this paper is the first to investigate the long-term spectrum sharing in the CRN. Our scheme provides an integrated framework for spectrum sharing and spectrum mobility management, where we consider the spectrum handover issue in the sharing process while accounting for PUs' incentives as well as spectrum allocation strategies in the spectrum mobility management.

• *Economic Robustness*: Although all PUs and SUs in our model are non-cooperative with each other, our scheme provides incentives for PUs and SUs to participate in the mechanism and behave in a cooperative manner by following the socially optimal contracts designed by the Random Leader. We also provide incentives for the leader to attain the "efficient leadership" (i.e., to lead its followers to obtain the optimal throughput of the whole network). In short, **Incentive Compatibility** and **Individual Rationality** are achieved in our scheme.

The rest of this paper is organized as followings. We will introduce the system model of STS in section II. Detailed procedures of our scheme in STS will be shown in section III, and the design of the socially optimal contracts in STS will be demonstrated in section IV. In section V, we will extend the system model to LTS and give its socially optimal contracts in section VI. Further in section VII, we will analyze the economic robustness of our scheme in both STS and LTS. Numerical results and the conclusions will be given in section VIII and IX, respectively. Due to the space constraints, we relegate some proofs to the online technical report [25].

II. Short-Term Spectrum Sharing

A. Basic Model

We consider a cognitive radio network consisting of N PUs (denoted by PU_i, $i \in \mathcal{N} := \{1, 2, \cdots, N\}$) and M SUs (denoted by SU_k, $k \in \mathcal{M} := \{1, 2, \cdots, M\}$). We assume there're no central control entities in the network, and those PUs and SUs are supposed to be non-cooperative "participants" in a distributed spectrum market. Each PU can lease its idle spectra to SUs for their own data transmission but require payment from them at the same time. For simplicity, we assume every PU only owns one spectrum band exclusively and one channel with the same bandwidth on each band. In the market, PUs' spectra are heterogenous in two aspects. The first aspect is the difference in PUs' "spectrum quality", and we suppose that PU_i can provide SU_k with a spectrum band of the quality q_{ik}, reflecting how well SU_k can utilize a unit of PU_i's spectrum. In practice, q_{ik} can be measured by SU_k's transceiver and is determined by the fading properties of PU_i's channel, the noise level in the channel, the performance of SU_k's transceiver, etc. Besides, the other heterogeneity of PUs' spectra is indicated by the *resource vector* $\mathbf{L} := (L_1, L_2, \cdots, L_N)^T$, denoting the amount of PUs' idle spectra (i.e., their available spectrum resources offered to SUs) due to their various activity patterns. The system is time-slotted, so L_i ($\forall i \in \mathcal{N}$) is quantified by the idle time of PU_i's channel in a time slot. For simplicity, we normalize the length of a time slot to be 1. Note that the spectrum sharing period only covers one time slot in STS.

In our model, we focus on the time reuse of PUs' spectra, which means that one PU's channel is allowed to be accessed by multiple SUs during one time slot. At the SUs' side, we assume that SU_k ($\forall k \in \mathcal{M}$) could access no more than α_k channels in one time slot. Meanwhile, to better characterize SUs' demands for spectra, we introduce an M-dimensional *demand vector* $\mathbf{D} := (D_1, D_2, \cdots, D_M)^T$ for SUs, reflecting the upper bounds of SUs' demands for the time to occupy PUs' spectra in a time slot. This is reasonable in practical situations since excessive spectrum allocations to SUs will also leave the spectra unused, which lowers the spectrum utilization. In our model, all information is private except SUs' and PUs' indices.

B. Random Leader

We divide the spectrum market into two sides: sellers (PUs) and buyers (SUs). The Random Leader is elected from either side in the market. The one-sided election can make the election procedure more fair (see section III for details). Considering the computing ability, we assume that the Random Leader is elected from PUs in the followings of this paper. The Random Leader has double identities: the leader and the normal participant. As a leader, it acts like a central controller, which is responsible for collecting information from other PUs and SUs, designing and assigning socially optimal spectrum sharing contracts for them. As a normal participant, it participates in the spectrum sharing procedures and gains utility from the sharing process like its followers. Consequently, the leader's utility (see section II-C for details) should consist of two parts: the utility as a normal participant and the extra utility as a leader, which can be seen as the incentive for its leadership.

In our scheme, this leader is called the Random Leader because it is elected randomly. Since all participants' information is private at the beginning, it is inefficient to elect a leader in some interactive manners. As a result, the random election is the most efficient and feasible method. This is reasonable because our scheme guarantees that the leader always has the incentive to lead its followers to obtain the socially optimal throughput whichever PU is elected as the leader. Besides, the random election also shows the advantage in power conservation since it guarantees that the role of the leader will rotate among all PUs in the long run. Although a certain PU may be elected as the leader and experience relatively high workload in **a certain sharing period**, this PU won't always be the leader in the future, which means that it will also have great opportunities to save its power significantly as a normal participant. Hence, the workload is actually evenly distributed among all PUs in the long term.

C. Utility Function

Before showing the details of utility functions, we first introduce two important matrices in STS: the *spectrum allocation matrix* $\mathbf{\Theta}$ and the *payment matrix* $\mathbf{P}$. The element θ_{ik} in $\mathbf{\Theta}$ denotes the time that PU_i leases to SU_k for occupying PU_i's spectrum. However, due to PUs' heterogenous spectrum quality, SUs may gain various utilities from different PUs even if SUs are allocated the same amount of spectra. Hence, we will use $\theta_{ik}q_{ik}$ to indicate the "valid spectrum allocation" from PU_i to SU_k, reflecting the amount of PU_i's spectrum allocation from which SU_k can gain utility. The element p_{ik} in $\mathbf{P}$ denotes the payment that PU_i receives from SU_k.

1) SU's Utility Function:

$$U_{SU_k} = a_k \sum_{i\in\mathcal{N}} \theta_{ik}q_{ik} - \sum_{i\in\mathcal{N}} p_{ik} - Q_{SU_k},\ \forall k \in \mathcal{M}. \quad (1)$$

Here, a_k denotes SU_k's utility gain per unit of "valid spectrum allocations" (a_k can have various physical meanings such as the transmission rate), and Q_{SU_k} is the extra payment that SU_k gives to the leader. $\sum_{i\in\mathcal{N}} \theta_{ik}q_{ik}$ in (1) is the total amount of "valid spectra" that SU_k leases from all PUs, and $\sum_{i\in\mathcal{N}} p_{ik}$ is the sum of the payment that SU_k gives to all PUs.

2) PU's Utility Function:

$$U_{PU_i} = \sum_{k\in\mathcal{M}} p_{ik} - Q_{PU_i},\ \forall i \in \mathcal{N}. \quad (2)$$

Here, Q_{PU_i} is the extra payment that PU_i pays to the Random Leader. $\sum_{k\in\mathcal{M}} p_{ik}$ in (2) reflects the total payment that PU_i collects from all SUs.

3) Leader's Utility Function:

$$U_{leader} = U_o + \sum_{i\in\mathcal{N}} Q_{PU_i} + \sum_{k\in\mathcal{M}} Q_{SU_k}. \quad (3)$$

Let i^* be the index of the leader, then $U_o = U_{PU_{i^*}}$, which denotes the leader's original utility as a *normal participant*. The term $\sum_{i\in\mathcal{N}} Q_{PU_i} + \sum_{k\in\mathcal{M}} Q_{SU_k}$ in (3) is the total extra utility it gains as a leader. It should be mentioned that U_o also includes the term $Q_{PU_{i^*}}$ (as is shown by (2)), and when the leader collects its extra utility, it also gets a part of the extra utility from itself, which fully characterizes the leader's second identity as a normal participant. We will show in III that U_{leader} is actually proportional to the throughput of the whole CRN if we properly set the value of Q_{PU_i} and Q_{SU_k}, which motivates the leader to optimize the network throughput so as to maximize its own utility.

III. Detailed Procedures of Short-Term Spectrum Sharing

We divide the entire STS process with a Random Leader into three stages.

• ***Stage I***: A leader is elected randomly from all PUs in the CRN, and all followers submit their private information to the leader.

The detailed procedures of choosing a Random Leader could be various. One possible method is: SU_1 randomly generates a number within $[1, N]$ and broadcasts this random number to all PUs simultaneously. The leader would be the PU whose index coincides with this random number. Since SU_1 isn't a candidate for the leader and doesn't have any PUs' private information, it has no incentives to cheat in the election. Besides, SU_k's private information includes a_k, D_k and q_{ik} ($\forall i \in \mathcal{N}$), and PU_i's private information includes L_i. We will show that all participants have no incentives to submit untrue information to the leader in section VII-A.

• ***Stage II***: The leader designs the socially optimal *spectrum allocation sub-contracts* to optimize the network throughput in STS and draws up *payment sub-contracts* accordingly to attain the economic robustness of the scheme.

We first define *network throughput* $R_{network}$ in STS:

$$R_{network} = \sum_{i\in\mathcal{N}} \sum_{k\in\mathcal{M}} a_k\theta_{ik}q_{ik}. \quad (4)$$

Obviously, $R_{network}$ reflects the spectrum utilization of the whole network. Besides, we can also interpret $R_{network}$ as the sum of all participants' utilities, i.e.,

$$R_{network} = \sum_{i\in\mathcal{N}'} U_{PU_i} + \sum_{k\in\mathcal{M}} U_{SU_k} + U_{leader}, \quad (5)$$

where $\mathcal{N}'$ is the set of all PUs except the leader. This equation can be directly proved by taking (1), (2), (3) and (4) into (5).

Then the problem of optimizing *network throughput* in STS is given by:

NETWORK OPT (STS):

$$\mathbf{\Theta}^* = \arg\max_{\mathbf{\Theta}} R_{network} \quad (6)$$

$$\begin{aligned} s.t \qquad & \textstyle\sum_{k\in\mathcal{M}} \theta_{ik} \le L_i \quad \forall i \in \mathcal{N}, \\ & \textstyle\sum_{i\in\mathcal{N}} \theta_{ik} \le D_k \quad \forall k \in \mathcal{M}, \\ & e(\mathbf{\Theta}_{SU_k}) \le \alpha_k \qquad \forall k \in \mathcal{M}. \end{aligned}$$

Here, the first and second constraint reflect the resource and demand limitation respectively, as is discussed in section II-A. Besides, $\mathbf{\Theta}_{SU_k} := (\theta_{1k}, \theta_{2k}, \cdots, \theta_{Nk})^T$ and $e(\mathbf{\Theta}_{SU_k})$ denotes the number of non-zero elements in $\mathbf{\Theta}_{SU_k}$. As a result, the third constraint in (6) corresponds with our discussion that SU_k can at most access α_k channels at the same time. We will give an algorithm to solve this problem in section IV-A.

• ***Stage III***: The leader collects its extra payment from all participants and then assigns the optimal contracts to them. Finally all participants carry out their contracts.

Here, we give Q_{PU_i} and Q_{SU_k} mentioned in II-C:

$$\begin{aligned} Q_{PU_i} &= \frac{\sum_{k\in\mathcal{M}} a_k \theta_{ik} q_{ik}}{2R_{network}} (bR_{network} - U_o); \\ Q_{SU_k} &= \frac{\sum_{i\in\mathcal{N}} a_k \theta_{ik} q_{ik}}{2R_{network}} (bR_{network} - U_o). \end{aligned} \quad (7)$$

The implication of (7) will be discussed at the end of this section (in the second last paragraph) since some further deductions are needed to assist the understanding of (7). Taking (7) into (3), we can rewrite the leader's utility by:

$$U_{leader} = bR_{network}. \quad (8)$$

b in (7) and (8) is the leader's utility gain per unit of network throughput, which is a dynamic parameter varying with the spectrum allocation results. The strategy of setting b will be given section VII-A. (8) is the actual utility that the leader can obtain in the CRN, and it's easy to see that the leader's utility is proportional the throughput of the whole network, which motivates it to optimize the network throughput.

Now we give the explanation of (7). In the first factor, $\sum_{k\in\mathcal{M}} a_k \theta_{ik} q_{ik}$ (or $\sum_{i\in\mathcal{N}} a_k \theta_{ik} q_{ik}$) in the numerator is the total throughput PU_i (or SU_k) "creates" in the CRN while term $2R_{network}$ in the denominator is the normalization constant. The second factor $(bR_{network} - U_o)$ denotes the total *extra* utility the leader gains. Hence, (7) means that the leader's extra utility is paid by all the participants proportional to the network throughput they "create" in the network. It should be mentioned that the expressions of Q_{PU_i} and Q_{SU_k} are artificially designed to obtain the economic robustness of our scheme (see section VII for details).

Finally, we briefly discuss some implementation issues in STS. To implement the Random Leader in the practical systems, there would be a separate common control channel (CCC) different from channels possessed by PUs. At the beginning of each time slot, a Random Leader is elected in the way shown in **Stage I**. Then the followers submit their private information to the leader through the CCC. Afterwards, the leader locally processes received information to obtain the socially optimal spectrum allocation sub-contracts as well as payment sub-contracts, and assigns those contracts to followers through the CCC. In the rest of the slot, all participants execute their assigned contracts.

IV. Optimal Contracts for Short-Term Spectrum Sharing

A. Socially Optimal Spectrum Allocation Sub-contracts

Allocating multiple PUs' spectra among multiple SUs to maximize the network throughput in STS is an NP-hard problem since it can be reduced to the classical multi-knapsack problem. We will use **Dynamic Programming (DP)** [7] to solve **NETWORK OPT** in STS, which is pseudo-polynomial in both time and space complexity.

We divide the whole spectrum programming process into $N \times M$ stages, and $stage_{ik}$ denotes that the leader is programming PU_i and SU_k. Let θ_{ik} denote the *strategy variable* at $stage_{ik}$, s_{ik} be the *state variable* denoting the amount of PU_i's left spectrum at $stage_{ik}$, $f_{ik}(s_{ik})$ be the *optimal value function* denoting optimal network throughput gained from $stage_{ik}$ to $stage_{NM}$ under s_{ik}, and $v_{ik}(\theta_{ik})$ be the *stage value function* denoting the throughput gained at $stage_{ik}$ if we choose θ_{ik} as the strategy. For simplicity of denotation, we reduce $stage_{ik}$ to $stage_j$ ($j = 1, 2, \cdots, N \times M$) and will use $stage_{ik}$ and $stage_j$ interchangeably in the following, i.e., j has a corresponding i and k, which is given by:

$$j = (i-1) \times M + k, \quad (9)$$

This means that $stage_{11}$, $stage_{12}$, $\cdots$, $stage_{NM}$ $\Leftrightarrow$ $stage_1$, $stage_2$, $\cdots$, $stage_{N\times M}$.

We first give a lemma here:

Lemma: The best strategy θ_j^* is chosen from $\{D_k, s_j, 0\}$.

Due to the space limitation, we present the proof to the above lemma in [25].

The expression of $f_j(s_j)$ is as the following.

$$f_j(s_j) = \max_{0\le\theta_j\le\min\{s_j,D_k\}\mathbf{1}(d_k^j<\alpha_k)} \{f_{j+1}(s_{j+1}) + v_j(\theta_j)\}. \quad (10)$$

In (10), d_k^j denotes the number of channels that SU_k has accessed before $stage_j$, and the notation $\mathbf{1}(d_k^j < \alpha_k) = 1$ when $d_k^j < \alpha_k$ otherwise $\mathbf{1}(d_k^j < \alpha_k) = 0$. For convenience, we define $I_k^j := \min\{s_j, D_k\}\mathbf{1}(d_k^j < \alpha_k)$.

The expression of $v_j(\theta_j)$ is:

$$v_j(\theta_j) = a_k \theta_j q_j. \quad (11)$$

The state transfer equation is:

$$s_{j+1} = \begin{cases} s_j - \theta_j & if\ j \ne i \times M \\ L_{i+1} & if\ j = i \times M \end{cases} \quad (12)$$

where i is derived from (9). Note that if $j = i \times M$, the next state will involve a new PU (i.e., PU_{i+1}), and $s_{j+1} = L_{i+1}$.

Using the lemma, (11) and (12), we can rewrite (10) by:

$$f_j(s_j) = \begin{cases} \max\{f_{j+1}(s_j - I_k^j) + a_k I_k^j q_j, f_{j+1}(s_j)\} & j \neq i \times M \\ f_{j+1}(L_{i+1}) + a_k I_k^j q_j & j = i \times M \end{cases} \quad (13)$$

Based on (13), we design **Algorithm 1**, where $Path_{ik}(s_{ik})$ records the best choice of strategies at $stage_{ik}$ under s_{ik} and is chosen from $\{D_k, s_{ik}, 0\}$. Besides, Δ is the step length for the loop of s_{ik} from 0 to L_i.

Algorithm 1 Socially Optimal Spectrum Allocation in STS

```
Initializing Stage
for s_NM = 0 : Δ : L_N do
    f_NM(s_NM) = min{a_M q_NM D_M, a_M q_NM s_NM};
    Path_NM(s_NM) = min{D_M, s_NM};
end for
Dynamic Programming Stage
for each i = N : 1,  k = M : 1, s_ik = 0 : Δ : L_i do
    Record f_ik(s_ik) according to (13) except f_NM(s_NM);
    Record Path_ik(s_ik) except Path_NM(s_NM);
end for
Output Stage
for i = 1 : N do
    s_i = L_i;
    for k = 1 : M do
        θ*_ik = Path_ik(s_i);  s_i = s_i − θ*_ik;
    end for
end for
END.
```

The computational space complexity in each stage is $O(NM)$ because we need to record the optimal path for each SU from the current stage to $stage_{NM}$ under each state so that we can determine the value of d_k^j without incurring additional time expense. Hence, the overall space complexity is $O(M^2N^2)$ while the time complexity is $O(NM)$.

B. Payment Sub-contracts

In our scheme, the payment sub-contracts are artificially designed to obtain **Incentive Compatibility** and **Individual Rationality** of our scheme. However, we will merely show the sketch of the payment sub-contracts in STS here. More formal analysis of the two economic properties together with the payment sub-contracts will be given in section VII.

The payment p_{ik} in STS is given as the following:

$$p_{ik} = a_k \theta_{ik} q_{ik} - \frac{\theta_{ik} q_{ik}}{\sum_{n \in \mathcal{N}} \theta_{nk} q_{nk}} Q_{SU_k} - c_{ik}. \quad (14)$$

c_{ik} in (14) is a non-negative term of which the function and setting will be demonstrated later in this subsection. Taking (7) into (14), we derive:

$$p_{ik} = a_k \theta_{ik} q_{ik} \frac{(2-b)R_{network} + U_o}{2R_{network}} - c_{ik}. \quad (15)$$

(15) shows that we can properly set b and c_{ik} to ensure $p_{ik} \geq 0$. Particularly, we can adjust c_{ik} so that $p_{ik} = 0$ when $\theta_{ik} = 0$. Taking (14) into (1), we can rewrite SU_k's utility function by:

$$U_{SU_k} = \sum_{i \in \mathcal{N}} c_{ik}. \quad (16)$$

(16) indicates that $U_{SU_k} \geq 0$ and that U_{SU_k} has no bearing with a_k, D_k and q_{ik} $(\forall i \in \mathcal{N})$ if we set c_{ik} to be independent of those parameters when $\theta_{ik} \neq 0$. In fact, this independence is an important consideration in enforcing SUs' truthful submission of their private information (see section VII). Taking (15) and (7) into (2), we can rewrite PU_i's utility function by:

$$U_{PU_i} = \sum_{k \in \mathcal{M}} \{a_k \theta_{ik} q_{ik}[(1-b) + \frac{U_o}{R_{network}}] - c_{ik}\}. \quad (17)$$

Similarly, by properly setting b and c_{ik}, it's easy to guarantee $U_{PU_i} \geq 0$.

Next, we conclude the strategy of setting c_{ik} in STS, and the strategy of setting b will be demonstrated in section VII-A.

• ***Strategy of Setting*** c_{ik}

If $\theta_{ik} = 0$, then we set $c_{ik} = 0$ to make $p_{ik} = 0$.

If $\theta_{ik} \neq 0$, we first define:

$$W := \min_{\theta_{ik} \neq 0, i \in \mathcal{N}, k \in \mathcal{M}} \{a_k \theta_{ik} q_{ik}\}.$$

From (16) and (17), it's obvious that we can set

$$0 \leq c_{ik} \leq W[(1-b) + \frac{U_o}{R_{network}}]$$

in order to ensure that $U_{SU_k} \geq 0$ and $U_{PU_i} \geq 0$. Alternatively, the above inequality can be written as

$$c_{ik} = gW[(1-b) + \frac{U_o}{R_{network}}], \quad (18)$$

where $g \in [0, 1]$ is a random number generated by the leader. From an SU_k's perspective, its a_k, D_k and q_{ik} $(\forall i \in \mathcal{N})$ won't influence the value of W since it is not aware of other SUs' information. Hence (18) shows that c_{ik} is independent of those parameters when $\theta_{ik} \neq 0$, and thus U_{SU_k} is also independent of SU_k's submitted information.

V. Long-Term Spectrum Sharing

The above short-term spectrum sharing only focuses distributing spectra precisely within one time slot, where the spectrum environment is static. However, as is mentioned in the introduction, SUs may demand long-period spectrum usage in the spectrum sharing, which covers multiple slots and aims at maintaining the consistency of the spectrum usage as well as the tradeoff between the congestion and handover costs in the long run. To fully satisfy SUs' heterogenous spectrum requirements, we will extend the Random Leader model to the long-term spectrum sharing in the following two sections.

A. Model Extension

Consider the physical model in section II-A with the following modifications and additions. In LTS, the whole sharing process covers T time slots (each time slot is denoted by t, $t \in \mathcal{T} := \{1, 2, \cdots, T\}$). Besides, we assume that each SU requires spectrum usage for much longer time than T slots and thus relax the demand constraint (i.e., D_k , $\forall k \in \mathcal{M}$) in STS. For simplicity of analysis, every SU can only access one PU's channel in each time slot (i.e., $\alpha_k = 1$, $\forall k \in \mathcal{M}$).

In LTS, we extend the *spectrum allocation matrix* to a two-dimensional matrix function of t (denoted by $\mathbf{\Theta}(t)$, $\forall t \in \mathcal{T}$).

Its element $\theta_{ik}(t) \in \{0, 1\}$ indicates whether PU_i's channel is allocated to SU_k in the t-th time slot (0 for no allocation and 1 otherwise). In LTS, PUs' spectra can also be reused as in STS, but we won't further allocate a PU's spectrum specifically to every SU within a time slot for simplicity, i.e., the spectrum resources are equally distributed to all the SUs on the same channel in a slot. Hence, the number of SUs on a channel together with q_{ik} implies the quality of that channel, and we use $\lambda_i(t)$ to indicate the number of SUs on PU_i's channel in the t-th time slot. Obviously, the larger $\lambda_i(t)$ is, the inferior the channel quality is. Note that the *payment matrix* $\mathbf{P}$ is still two-dimensional, which is not a function of t.

B. Spectrum Mobility

In CRNs, high-priority PUs often reclaim their licensed channels, which causes the *spectrum mobility* for SUs. In our scheme, the spectrum mobility model is based on the *Markov Process*, which has been widely exploited to simulate PUs' activity model in the spectrum mobility management [14], [15]. For simplicity, we assume the Markov process can be established based on the current observation of PUs' channel states and state transition probabilities. Here, the state of PU_i's channel in the t-th time slot is given as $S_i^{PU}(t) \in \{0, 1\}$ (0 for the idle state and 1 otherwise). In our model, PU_i's state transition probabilities are denoted by $\{\xi_i^{0,0}, \xi_i^{0,1}, \xi_i^{1,0}, \xi_i^{1,1}\}$ and $\xi_i^{x,y}$ indicates the probability of PU_i's state transition from state x to y. We assume those probabilities are common knowledge obtained from PUs' spectrum occupancy statistics. These statistics are available in the database of spectrum regulators (e.g., FCC). When PUs appear on their licensed channels, SUs can choose to either handover to another idle channel or "wait" in the channel, i.e., SUs can cease their transmissions but don't reconfigure their tuning frequency, for the possibility that those PUs will release their channel in the future. For simplicity, we assume SUs won't suffer any utility loss but gain no utility while waiting in the channel.

Generally, spectrum handover can help SUs maintain the consistency in the spectrum usage but imposes some negative effects on SUs at the same time. SUs have to spare extra time and additional cost to execute spectrum handover, which is referred as *handover time* and *handover cost*, respectively.

- **Handover Time**: SUs must reconfigure their operating frequency when switching to another spectrum band, which leads to significant delay before they can proceed transmitting on new spectra. During the handover process, SUs cannot gain any utilities. In our model, we use τ_k to denote SU_k's time consumption per handover and assume that handover can happen immediately. For example, if SU_k executes handover in the t_0-th slot, then the handover process covers $t_0,\ t_0+1,\ ,\cdots,\ t_0+\tau_k-1$.
- **Handover Cost**: During the spectrum handover process, SUs might have to consume extra energy to reconfigure, sense PUs' spectra, etc. For the tractability of analysis, we put those costs together as a general handover cost denoted by π_k, i.e., every time an SU handovers to another channel, it will lose utility amounting to π_k.

Based on the above discussions, we can divide SUs' states in LTS into three major categories, shown in the followings.

SU State 1 (Transmitting State): An SU can normally transmit its own data on the allocated channel and gain utility from the spectrum usage.

SU State 2 (Handover State): An SU is executing the handover and cannot be allocated any channels (note that SU_k needs τ_k time slots to complete one handover). In terms of the utility obtained in this state, we can further divide *SU state 2* into two sub-states.

SU State 2.1: An SU is initializing the spectrum handover (i.e., the SU is in the first time slot of the handover process) and suffers handover cost.

SU State 2.2: An SU is in the sequential slots that the handover process covers after the initializing slot and gains zero utility.

SU State 3 (Waiting State): An SU chooses to wait in the reclaimed channel and gains zero utility.

In the rest of paper, we will use 0-or-1 variables $S_k^{SU,1}(t)$, $S_k^{SU,2.1}(t)$, $S_k^{SU,2.2}(t)$, $S_k^{SU,3}(t)$ to denote SU_k's state in the t-th time slot. For example, $S_k^{SU,1}(t_0) = 1$ means that SU_k is of State 1 in the t_0-th slot and 0 otherwise.

Now we can define the throughput that SU_k obtains on PU_i's channel in the t-th time slot as $\eta_{ik}(t)$, given by:

$$\eta_{ik}(t) = \begin{cases} a_k \frac{\theta_{ik}(t) q_{ik}}{\lambda_i(t)} & if\ S_i^{PU}(t) = 0 \\ 0 & if\ S_i^{PU}(t) = 1 \end{cases} \tag{19}$$

In (19), a_k has the same meaning as that in STS and $\frac{q_{ik}}{\lambda_i(t)}$ reflects PU_i's spectrum quality perceived by SU_k in the t-th slot mentioned in section V-A. Note that $\eta_{ik}(t)$ varies with PUs' channel states and can be regarded as a stochastic process due to the randomness of PU_i's channel state.

However, we hold that merely *spectrum allocation matrix* $\mathbf{\Theta}(t)$ cannot fully characterize an SU's behaviors in LTS since $\mathbf{\Theta}(t)$ contains no information about handover issues. As a result, it's necessary to design a *handover sub-contracts* for SUs at the same time, and we introduce *handover vector function* $\mathbf{H}(t) := \{h_1(t), h_2(t), \cdots, h_M(t)\}$ to denote this sub-contract, where $h_k(t) \in \{0, 1\}$ indicating whether SU_k should execute handover in the t-th slot (1 for handover and 0 otherwise). $\mathbf{\Theta}(t)$ and $\mathbf{H}(t)$ together can offer sufficient information for SUs about which channel they are allocated at a certain time, when they should execute handover and what the handover destination is.

C. Utility Function

SU_k's utility function in LTS is shown as:

$$U_{SU_k} = \sum_{t\in\mathcal{T}} \sum_{i\in\mathcal{N}} \eta_{ik}(t) - \sum_{i\in\mathcal{N}} p_{ik} - \sum_{t\in\mathcal{T}} h_k(t)\pi_k - Q_{SU_k}. \tag{20}$$

Here, $\sum_{t\in\mathcal{T}} \sum_{i\in\mathcal{N}} \eta_{ik}(t)$ denotes SU_k's total throughput obtained in LTS and $\sum_{t\in\mathcal{T}} h_k(t)\pi_k$ is SU_k's total handover cost. The implication of other terms is similar to that in STS.

PUs' and the leader's utility functions are the same as that in STS, shown in (2) and (3).

D. Detailed Procedures

The procedures of LTS with a Random Leader can be derived from STS with some modifications and extensions.

• ***Stage I***: A leader is first elected from all PUs. SUs sense the current channel states of all PUs and then submit their private information as well as PUs' states to the leader.

Stage I in LTS is almost the same as that in STS. The slight difference is that SUs must sense the current channel states of all PUs to observe the initial states of the Markov Process.

• ***Stage II***: The leader designs the socially optimal spectrum allocation sub-contracts as well as the handover sub-contracts by solving **NETWORK OPT** in LTS and determines the payment sub-contracts accordingly.

Before demonstrating **NETWORK OPT** in LTS, we first define the *network throughput* $R_{network}$ in LTS as:

$$R_{network} = \sum_{t\in\mathcal{T}}\sum_{i\in\mathcal{N}}\sum_{k\in\mathcal{M}} \eta_{ik}(t) - \sum_{k\in\mathcal{M}}\sum_{t\in\mathcal{T}} h_k(t)\pi_k. \quad (21)$$

$R_{network}$ means the total throughput gained in LTS minus the cost consumed in the CRN during the sharing process. Since $\eta_{ik}(t)$ is a stochastic process, $R_{network}$ becomes a random variable.

Next, we will discuss some constraints when the leader designs the socially optimal spectrum allocation sub-contracts and handover sub-contracts.

Constraint 1: $\sum_{i\in\mathcal{N}}\theta_{ik}(t) = 0$ $(t = t_0, t_0+1, \cdots, t_0+\tau_k-1,\ \forall k\in\mathcal{M})$ must be satisfied if $h_k(t_0) = 1$.

This constraint implies that SUs cannot be allocated any channels when executing handovers.

Constraint 2: For SU_k $(\forall k\in\mathcal{M})$, $\sum_{i\in\mathcal{N}}\theta_{ik}(t_0+\tau_k) = 1$ must be satisfied when $h_k(t_0) = 1$.

This constraint demonstrates that spectrum allocation sub-contracts need to inform SUs of what channels they should handover to when they finish handovers.

Constraint 3: For SU_k $(\forall k\in\mathcal{M})$, $\theta_{ik}(t) = 0$ $(t = t_0 + 1, t_0+2\cdots, t_0+\tau_k)$ must be satisfied when $\sum_{n\in\mathcal{N}}\theta_{nk}(t_0) \neq 0,\ \theta_{ik}(t_0) = 0$, and $h_k(t_0) = 0$.

This constraint indicates that an SU cannot change its accessing channels without spectrum handover, which implies the possible advantages brought by the handover. Through the handover, in spite of the handover time τ_k and the handover cost π_k, SU_k is able to access a channel with better channel conditions at $t_0+\tau_k$, which might help obtain more network throughput in the new channel.

Besides, since an SU can only access one PU's channel simultaneously, we have another constraint.

Constraint 4:

$$\sum_{i\in\mathcal{N}}\theta_{ik}(t) \le 1 \quad \forall k\in\mathcal{M},\ \forall t\in\mathcal{T}.$$

Now we can formulate **NETWORK OPT** in LTS as:
NETWORK OPT (LTS):

$$(\mathbf{\Theta}^*(t), \mathbf{H}^*(t)) = \arg\max_{\mathbf{\Theta}(t),\mathbf{H}(t)} \mathbb{E}\{R_{network}\} \quad (22)$$

$$s.t \qquad Constraint\ 1-4.$$

We will also use the Dynamic Programming algorithm to solve **NETWORK OPT** of LTS in section VI.

• ***Stage III***: The leader collects its extra utility and assigns those sub-contracts to all participants. Then all participants carry out their contracts.

Here, we first quantify Q_{SU_k} and Q_{PU_i} in LTS:

$$Q_{PU_i} = \frac{\sum_{t\in\mathcal{T}}\sum_{k\in\mathcal{M}}\eta_{ik}(t)}{2A}(bA - U_o); \quad (23)$$

$$Q_{SU_k} = \frac{\sum_{t\in\mathcal{T}}\sum_{i\in\mathcal{N}}\eta_{ik}(t)}{2A}(bA - U_o) - b\sum_{t\in\mathcal{T}} h_k(t)\pi_k, \quad (24)$$

where

$$A := \sum_{t\in\mathcal{T}}\sum_{i\in\mathcal{N}}\sum_{k\in\mathcal{M}}\eta_{ik}(t).$$

The implication of Q_{SU_k} and Q_{PU_i} in STS (see section III for details) also applies to LTS except that we minus an additional term $b\sum_{t\in\mathcal{T}} h_k(t)\pi_k$ in Q_{SU_k} so that the economic properties of our scheme are still maintained in LTS (see section VII).

Taking (23) and (24) into (3), we can rewrite the leader's utility function in LTS by:

$$U_{leader} = bR_{network}. \quad (25)$$

As in STS, the leader's utility in LTS is also proportional to the network throughput, which motivates it to maximize the network throughput in order to optimize its own revenue.

It should be mentioned that the implementation of our scheme in LTS is similar to that in STS except that spectrum sensing should be carried out immediately after the election and that the sharing period in LTS will be T slots instead of 1 in STS.

VI. Optimal Contracts Of Long-Term Spectrum Sharing

A. Optimal Spectrum Allocation and Spectrum Handover Sub-contracts

In this subsection, we focus on designing an algorithm which can solve **NETWORK OPT** in LTS so as to derive the optimal spectrum allocation and handover sub-contracts. Still, we use **Dynamic Programming** as the basis of our algorithm. We divide the whole dynamic programming procedures into $T\times N\times M$ stages, and each stage is denoted by $stage^t_{i,k}$, which means that the algorithm is programming SU_k and PU_i in the t-th slot. The *strategy variable* at $stage^t_{i,k}$ is given by the pair $(\theta^t_{i,k}, h^t_{i,k})$, where $\theta^t_{i,k} := \theta_{ik}(t) \in \{0,1\}$ denoting whether to allocate PU_i's channel to SU_k in the t-th slot and $h^t_{i,k}\in\{0,1\}$ indicating whether SU_k executes handover from PU_i's channel in the t-th slot. The *state variable* $s^t_{i,k}$ in LTS denotes the number of SUs that have accessed PU_i's channel before $stage^t_{i,k}$. Besides, similar to STS, we reduce $stage^t_{i,k}$ to $stage_j$ $(j = 1, 2, \cdots, T\times N\times M)$ for the simplicity of denotation and each j has a corresponding i, k and t, shown by:

$$j = (T-t)\times N\times M + (i-1)\times M + k. \quad (26)$$

The *stage value function* is denoted by $v^t_{i,k}(\theta^t_{i,k}, h^t_{i,k}, s^t_{i,k})$, which is summarized in Table I (due to the space limitation, we relegate the explanation of Table I to our technical report [25]). Based on this table, the *expected stage value function* $\mathbb{E}\{v^t_{i,k}\}$ can be derived, as is shown in (27). $Pr^{idle}_i(t)$ in (27)

TABLE I
STAGE VALUE FUNCTION UNDER VARIOUS CONDITIONS

	$S_i^{PU}(t)$	$\theta_{i,k}^t$	$h_{i,k}^t$	$v_{i,k}^t$
$S_k^{SU,2.2}(t)=1$ or $d_k(t)=1$	0	0	0	0
	1	0	0	0
$S_k^{SU,2.2}(t)=0$ and $d_k(t)=0$	0	0	0	0
	0	0	1	$-\pi_k$
	0	1	0	$\sum_{n=1}^{k} \frac{a_n q_{in}}{s_{i,k}^t+1} - \sum_{n=1}^{k-1} \frac{a_n q_{in}}{s_{i,k}^t}$
	1	0	0	0
	1	0	1	$-\pi_k$
	1	1	0	0

is the probability that PU_i's channel is idle in the t-th slot, which can be calculated from $\{\xi_{0,0}^i, \xi_{0,1}^i, \xi_{1,0}^i, \xi_{1,1}^i\}$ and PU_i's initial state in the Markov process.

Then the *optimal value function* $f_j(s_j)$ which denotes the optimal network throughput gained from $stage_j$ to $stage_{T\times N\times M}$ under s_j is shown by (28).

$$\begin{cases} f_j(s_j) = \max_{\theta_j, h_j}\{f_{j+1}(s_{j+1}) + \mathbb{E}\{v_{i,k}^t\}\} \\ f_{T\times N\times M+1}(s_{T\times N\times M+1}) = 0 \end{cases} \tag{28}$$

In addition, we give the state transfer equation in LTS as:

$$s_{j+1} = \begin{cases} s_j + \theta_{i,k}^t & if\ k \neq M \\ 0 & if\ k = M \end{cases} \tag{29}$$

Due to the space limitation, we omit the explanation of (28) and (29) here. Readers can refer to [25] for the details.

On the basis of (27), (28) and (29), we design the dynamic programming algorithm for spectrum allocation and handover in LTS, given by **Algorithm 2**. Similar to the discussion in STS, the time complexity of Algorithm 2 is $O(TNM^2)$ and the space complexity is $O(T^2N^2M^3)$, which are also both pseudo-polynomial.

Algorithm 2 Socially Optimal Spectrum Allocation and Spectrum Handover in LTS

$f_{1,1}^0(0) = 0$;
$Path_{1,1,0}^{\theta}(0) = 0$;
$Path_{1,1,0}^{h}(0) = 0$;
for $t = 1:T$, $i = N:1$, $k = M:1$, $s_{i,k}^t = 0:k-1$ **do**
  Record $f_{i,k}^t(s_{i,k}^t)$ according to (27), (28) and (29);
  Record $Path_{i,k,t}^{\theta}(s_{i,k}^t)$ and $Path_{i,k,t}^{h}(s_{i,k}^t)$;
end for
for each $t = T:1$, $i = 1:N$ **do**
  $s_{i,1}^t = 0$; $h_k^*(t) = 0$;
  for $k = 1:M$ **do**
    $\theta_{ik}^*(t) = Path_{i,k,t}^{\theta}(s_{i,k}^t)$;
    $h_k^*(t) = h_k^*(t) + Path_{i,k,t}^{h}(s_{i,k}^t)$;
    $s_{i,k+1}^t = s_{i,k}^t + Path_{i,k,t}^{\theta}(s_{i,k}^t)$;
  end for
end for
END.

B. Payment Sub-contracts

The philosophy behind the payment sub-contracts in LTS inherits directly from STS. Hence we only provide a brief sketch of the payment design in LTS here and omit some repeated explanations.

The payment p_{ik} in LTS is expressed by

$$p_{ik} = \sum_{t\in\mathcal{T}} \eta_{ik}(t) - \frac{\sum_{t\in\mathcal{T}} \eta_{ik}(t)(Q_{SU_k} + \sum_{t\in\mathcal{T}} h_k(t)\pi_k)}{\sum_{t\in\mathcal{T}}\sum_{n\in\mathcal{N}} \eta_{nk}(t)} - c_{ik}, \tag{30}$$

where c_{ik} also denotes the additional term used for ensuring $U_{PU_i} \geq 0$ ($\forall i \in \mathcal{N}$) and $U_{SU_k} \geq 0$ ($\forall k \in \mathcal{M}$).

Taking (30) into (20), we can rewrite SU_k's utility function in LTS by:

$$U_{SU_k} = \sum_{i\in\mathcal{N}} c_{ik},$$

which is the same as (16) in STS. Besides, taking (24) into (30), we can rewrite p_{ik} in LTS by:

$$p_{ik} = (\frac{2-b}{2} + \frac{U_o}{2A}) \sum_{t\in\mathcal{T}} \eta_{ik}(t) - c_{ik}. \tag{31}$$

If we further take (31) into (2), PU_i's utility in LTS can be equally written by (32).

$$U_{PU_i} = \sum_{k\in\mathcal{M}} [(1 - b + \frac{U_o}{2A}) \sum_{t\in\mathcal{T}} \eta_{ik}(t) - c_{ik}]. \tag{32}$$

From (31) and (32), we can observe that $p_{ik} \geq 0$ and $U_{PU_i} \geq 0$ can be ensured in LTS if we set appropriate c_{ik} and b. The strategy of setting c_{ik} in LTS is almost the same as that in STS, given as the following.

When $\sum_{t\in\mathcal{T}} \eta_{ik}(t) = 0$, we set $c_{ik} = 0$ otherwise c_{ik} is given by:

$$c_{ik} = gV(1 - b + \frac{U_o}{2A}), \tag{33}$$

where $g \in [0,1]$ is set in advance by the leader and V is defined as:

$$V := \min_{\sum_{t\in\mathcal{T}} \eta_{ik}(t)\neq 0, i\in\mathcal{N}, k\in\mathcal{M}} \{\sum_{t\in\mathcal{T}} \eta_{ik}(t)\}.$$

The strategy of setting b in LTS will be further shown in section VII-A.

VII. Analysis of Economic Robustness

In this section, we will discuss the economic robustness of our scheme, which includes **Incentive Compatibility** and **Individual Rationality**.

A. Incentive Compatibility

Definition 1 (Incentive Compatibility): A mechanism is Incentive Compatible if every rational participant's best strategy is to follow the rules of the mechanism.

Based on the definition, we have the following economic properties involved with Incentive Compatibility.

$$\mathbb{E}\{v_{i,k}^t\} = \begin{cases} 0 & if\ (\theta_{i,k}^t, h_{i,k}^t) = (0,0) \\ -\pi_k & if\ (\theta_{i,k}^t, h_{i,k}^t) = (0,1) \\ Pr_i^{idle}(t)(\sum_{n=1}^{k} \frac{a_n q_{in}}{s_{i,k}^t+1} - \sum_{n=1}^{k-1} \frac{a_n q_{in}}{s_{i,k}^t}) & if\ (\theta_{i,k}^t, h_{i,k}^t) = (1,0) \end{cases} \quad (27)$$

Economic Proposition 1: The leader has the incentive to maximize the throughput of the whole network in **STS** when

$$1 - \frac{gW}{\max_{k\in\mathcal{M}}\{a_k\theta_{i^*k}q_{i^*k}\}} \leq b \leq 1.$$

Economic Proposition 2: The leader has the incentive to maximize the network throughput in **LTS** when

$$1 - \frac{gV}{\max_{k\in\mathcal{M},t\in\mathcal{T}}\{\eta_{i^*k}(t)\}} \leq b \leq 1.$$

Economic Proposition 3: All participants have no incentives to submit their private information to the leader untruthfully in both STS and LTS.

The proofs to *Economic Proposition 1-3* are quite lengthy and we present them in [25] due to the space limitation.

In the rest of this subsection, we will discuss two other incentive problems in our model and design corresponding mechanisms to address them.

• ***Issue 1:*** How does the leader collect its extra payment from the followers honestly? How do followers pay the extra payment to the leader honestly?

The two problems can be addressed together by using the *half-open* mechanism. We first show this mechanism in STS.

Firstly, the leader informs each PU (e.g., PU_i) of vector $\mathbf{A} = (a_1, a_2, \cdots, a_M)^T$, vector $\mathbf{q}_{PU_i} = \{q_{i1}, q_{i2}, \cdots, q_{iM}\}$, the optimal network throughput $R^*_{network}$, the optimal spectrum allocation vector $\mathbf{\Theta}^*_{PU_i} = \{\theta^*_{i1}, \theta^*_{i2}, \cdots, \theta^*_{iM}\}$ and the leader's payment vector $\mathbf{P}_{PU_{i^*}} = (p_{i^*1}, p_{i^*2}, \cdots, p_{i^*M})^T$ through the CCC. Similarly, $\mathbf{\Theta}^*_{SU_k}$, $\mathbf{P}_{PU_{i^*}}$ and the network throughput is transmitted to SU_k ($\forall k \in \mathcal{M}$). Secondly, all followers pay their extra payment to the leader according to the information revealed in the first step. Finally, the leader announces the *payment sub-contracts* to followers through the CCC, and then all participants carry out their contracts.

In LTS, this mechanism only differs from STS in the information revelation in the first step. The optimal network throughput $R^*_{network}$, the optimal payoff vector $\vec{\eta}^*_{PU_i}(t) = \{\eta^*_{i1}(t), \eta^*_{i2}(t), \cdots, \eta^*_{iM}(t)\}$ ($\forall t \in \mathcal{T}$) and the leader's payment $\mathbf{P}_{PU_{i^*}}$ are revealed to PU_i ($\forall i \in \mathcal{N}$) while $\vec{\eta}^*_{SU_k}(t)$ ($\forall t \in \mathcal{T}$), $h^*_k(t)$ ($\forall t \in \mathcal{T}$) and the optimal network throughput are delivered to SU_k ($\forall k \in \mathcal{M}$) through the CCC.

This mechanism is *half-open* because the leader only opens a part of the contracts (i.e., it hides the payment sub-contracts) before it receives all the extra payment from other participants. With the information opened in the first step, all followers are able to calculate the extra payment they should pay to the leader. Therefore, the leader cannot casually collect its extra payment. However, followers cannot fully carry out their contracts merely with the information opened in the first step so they have to pay honestly to the leader otherwise the leader won't reveal the rest of the contracts.

• ***Issue 2:*** How do participants follow their assigned contracts honestly?

We design the *gradual payment* mechanism to handle this problem. When one SU accesses a certain PU, the SU doesn't give all the payment to the PU but gradually pays. For example, in STS, we divide the SU's contracted time ϕ for spectrum usage by X parts (In LTS, it would be convenient to divide the spectrum allocation by T parts, i.e., divide by a single time slot), and the payment is p. Every time when the SU uses the PU's spectrum for a period of ϕ/X, it pays the PU p/X. After the PU allocates enough time for the SU's occupying the spectrum, the PU receives all the payment from the SU. Once the SU does not pay enough to the PU after every ϕ/X, the PU would forfeit the SU's right to use the spectrum, which brings the SU less utility than its following the contracts. If the PU does not lease enough time to the SU, the PU would also lose utility because it receives less payment from the SU. Therefore, all participants will follow the contracts honestly for fear of losing utilities.

B. Individual Rationality

Definition 2 (Individual Rationality): A mechanism satisfies the Individual Rationality constraints if every participant receives non-negative utility, i.e.,

$$U_{PU_i} \geq 0, \ \forall i \in \mathcal{N},$$
$$U_{SU_k} \geq 0, \ \forall k \in \mathcal{M},$$
$$U_{leader} \geq 0.$$

Based on the definition of Individual Rationality, we have the following economic property.

Economic Proposition 4: Our scheme is Individually Rational for all participants in both STS and LTS.

This property can be directly obtained from the discussion of the payment sub-contracts since we have shown that our design of the payment ensures $U_{PU_i} \geq 0$ ($\forall i \in \mathcal{N}$) and $U_{SU_k} \geq 0$ ($\forall k \in \mathcal{M}$). Furthermore, from *Economic Proposition 1-2*, we have $U_{leader} \geq U_o \geq 0$. Hence, our scheme is individually rational for all participants in both STS and LTS.

VIII. Simulation

A. Short-Term Spectrum Sharing

In the following, we use MATLAB as the simulation tool. In our simulation of STS, we assume that D_k, L_i and q_{ik} ($\forall i \in \mathcal{N}, k \in \mathcal{M}$) are uniformly distributed in $[0, 1]$ and set a_k to be SU_k's transmission rate distributing uniformly between $[1, 1.5]$Mbps. Besides, α_k ($\forall k \in \mathcal{M}$) is a random integer following the uniform distribution within $[1, 5]$.

We first simulate the *spectrum allocation sub-contracts* using **Algorithm 1** and each data point is the average result of 50 experiments. In Figure 1, we focus on the relationship between the *network throughput* and the number of PUs and SUs (i.e., N and M). Figure 1 shows that when N is relatively small, the average network throughput increases with N almost linearly. However, this increase gradually slows down and finally stops when N is raised. If we set larger M, the increase can continue for a while but stops eventually, which shows that although the spectrum resources are sufficient, the network throughput cannot increase infinitely due to the *demand limitation* of SUs.

Next, we simulate the *payment sub-contracts* and set $g = 0.6,\ M = 50,\ N = 40$ here. Figure 2 shows the comparison among participants' average utilities varying with b. Firstly, all participants' utilities are non-negative when $b \leq 1$, which shows the Individual Rationality of SUs and PUs. Secondly, there's an interval where the leader's utility exceeds its original utility, which coincides with *Economic Proposition 1* and demonstrates the incentive to be the leader. Finally, larger b brings the leader more utility but decreases other participants' utilities, so we need to set b to be a proper value between $1 - \frac{gW}{\max_{k \in \mathcal{M}}\{a_k \theta_{i^*k} q_{i^*k}\}} \leq b \leq 1$ in the practical situation.

B. Long-Term Spectrum Sharing

Since the *payment sub-contracts* in LTS is similar to that in STS, we only concentrate on the simulation of *spectrum allocation sub-contracts* and *spectrum handover sub-contracts* in LTS using **Algorithm 2**. For the simulation of LTS, a_k, π_k and τ_k ($\forall k \in \mathcal{M}$) are uniformly distributed in $[1, 1.5]$Mbps, $[0.01, 0.20]$Mb and $[1, 3]$, respectively. Besides, we set PUs' initial states to be "idle" or "busy" with the equal probability 0.5 and the transition probability $\xi_i^{0,0}$ and $\xi_i^{1,0}$ ($\forall i \in \mathcal{N}$) coincides with uniform distribution between $[0, 1]$ (note that $\xi_i^{0,1} = 1 - \xi_i^{0,0}$ and $\xi_i^{1,1} = 1 - \xi_i^{1,0}$). The value of q_{ik} is set to be 1 for all SUs and PUs in LTS. Each data point is derived from the average results in 50 experiments.

In LTS, we mainly investigate the influence of the sharing period of LTS (i.e., T). Figure 3 shows the expected network throughput varying with T. Intuitively, the more time slots that LTS covers, the more network throughput can be obtained, and such an increase shows no stopping sign. Besides, as is shown in Figure 4, the average number of handovers (handover times) in the CRN increases with T at the beginning since longer sharing period implies more opportunities for spectrum handover in the CRN. However, the handover times remain stable after T reaches a certain values. Approximately, this value is $T = 7$. The stable tendency of handover times with respect to T is due to the stability of the idle probabilities of PUs' channels. In the ergodic *Markov process*, the *absolute probability* (i.e., the idle probability of a channel in a certain time slot) will approximate a fix value. In our model, the variance of the idle probability is negligible after about 7 time slots, which means that the network reaches a relatively stable state after about 7 time slots. Thus, SUs in the CRN no longer need to execute handovers which are intended for their adapting to the spectrum mobility after 7 time slots.

IX. Conclusion

In this paper, we propose a distributed-centralized and incentive-aware spectrum sharing scheme by introducing a Random Leader. Our scheme can obtain the socially optimal throughput of the whole network as centralized mechanisms but can be implemented in any distributed spectrum markets without central control entities. Besides, our scheme enforces Incentive Compatibility and Individual Rationality, which ensures the economic robustness of our scheme. As an integrated design of both short-term and long-term spectrum sharing, our scheme fulfils SUs' heterogeneous spectrum requirements. In the short-term spectrum sharing, we optimally distribute PUs' spectra within exactly one time slot. In the long-term sharing, we jointly consider spectrum sharing and spectrum mobility management in one model, accounting for both SUs' and PUs' incentives and obtaining the socially optimal channel allocation and handover strategies, which further maximizes the expected network throughput.

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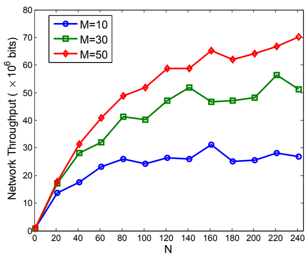

Fig. 1. Average network throughput varies with the number of PUs and SUs (N and M) in STS.

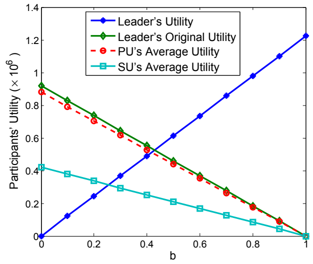

Fig. 2. Comparison among participants' average utilities with respect to b in STS.

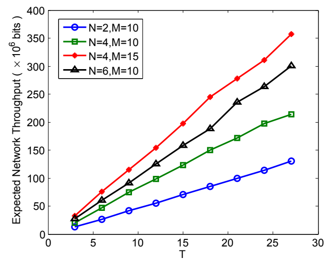

Fig. 3. Relationship between the expected network throughput and the LTS period T.

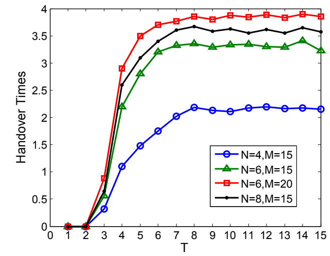

Fig. 4. Relationship between the average handover times and the LTS period T.

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