Multicast Capacity for VANETs with Directional Antenna and Delay Constraint

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***Abstract*—Vehicular Ad Hoc Networks (VANETs) with base stations are called hybrid VANET, where base stations are deployed to improve the throughput capacity. In this paper, we study the multicast throughput capacity for hybrid wireless (VANET) with a directional antenna on each vehicle and the end-to-end delay is constrained. In the hybrid VANET, there are n mobile vehicles (or nodes) distributed in a unit area with m strategically deployed base stations connected using high-bandwidth wire links. There are n_s multicast sessions and each multicast session has one source which transmits identical data to its associated p destinations. We investigate the multicast throughput capacity for two mobility models with two mobility scales, respectively, while each vehicular node is equipped with a directional antenna and with a tolerant delay D. That is, a source node transmits to its p destinations only with the help of normal nodes within D consecutive time slots. Otherwise, the transmission will be performed with in the infrastructure mode, i.e., with the help of base stations. We demonstrate that the one dimensional i.i.d. slow mobility pattern catch the main feature of VANETs. And we find that the multicast throughput capacity of the hybrid wireless VANET greatly depends on the delay constraint D, the number of base stations m, and the beamwidth of directional antenna θ. In the order of magnitude, we obtain the closed form of the multicast throughput capacity of the hybrid directional VANET, where the impact of D, m and θ on the multicast throughput capacity is analyzed. Moreover, we derive the lower bound of the muticast throughput using a similar raptor coding approach as in [10].**

I. Introduction

Vehicular Networks (VANETs) are attracting increasing attention from major car manufacturers, governmental organizations and the academic community, which aim to provide information exchange service for vehicles on the road. In (VANET), vehicles are equipped with antennae which can transmit and receive data between each other. Literatures are abound in studying the vehicle-to-vehicle communication. In [1], Ding et al. study the connectivity of vehicle networks. Zheng et al. show that the vehicular connectivity could be dramatically improved by deploying access points (AP) in the network [2], [3]. Zhu et al. [4] give an experiment study on the inter-contact time in urban VANET. Benslimane et al. [5] investigate the VANET-UMTS integrated network for heterogeneous wireless networks. Recently, Jeong et al. [6] propose a trajectory-based statistical forwarding scheme for the multi-hop data delivery from infrastructure to moving vehicles in VANETs. As more and more vehicles are equipped with wireless communication devices, large scale base stations for VANETs are expected to be common road infrastructures in the near future. A fundamental and important problem is: What is the throughput capacity for such hybrid VANETs?

In the seminal work of Gupta and Kumar [7], they study the asymptotic capacity for large scale wireless ad hoc networks and show that the per-node throughput capacity is $O(\frac{1}{\sqrt{n}})$[1], for static networks, which decreases to zero as n tends to infinity. Later, Grossglauser and Tse [8] show that per-node capacity of $\Theta(1)$ is achievable when the nodes are mobile, which is much better than that of static networks at the cost of excessive packet delays. The VANET is a special case MANETs with applications in vehicular networks. Many works have been done to investigate the capacity of MANETs under a range of mobility models [9], [32], [31], [10], [11], [12]. Other works attempt to improve the capacity by introducing base stations as infrastructure support [27], [13], [18], [29]. The capacity of VANETs is also investigated. In [14], Pishro-Nik et al. give an asymptotic study based on the framework of [7]. Then, Nekoni et al. [15] analyze the capacity of VANETs with infrastructure. Nekoni et al. also consider the scaling law for distance limited communications in VANETs [16]. However, neither of them considers the presence of the directional antenna and the end-to-end delay constraint. Recently, Yan et al. [17] give a theoretical study of the throughput of mobile content distribution in VANETs by using symbol level network coding. Most of these works assumes unicast traffic flows and study the unicast capacity. Currently, as the demand of information sharing increases rapidly, multicast flows become popular in some realistic applications. For example, the vehicle near an accident can multicast accident information to some specific vehicles. The video needs to be sent to all participants in a wireless video conference. Few researchers have studied the multicast capacity of wireless networks under different conditions and applications, such as mobility, hybrid networks [19], [20], [21], [26], [22], [23].

Besides the capacity, delay is also an important performance metric of VANETs, considering the mobility character and the demand of practical applications. There are many studies taking the throughput capacity (with either unicast or multicast) under delay constraints into consideration. In [28], Pei et al. study the unicast throughput capacity using an L-maximum-

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[1]In this paper, for two functions $f(n)$ and $g(n)$, we denote $f(n) = O(g(n))$ if there is a positive constant c such that $f(n) \leq cg(n)$ for n large enough; $f(n) = \Omega(g(n))$ if $g(n) = O(f(n))$; $f(n) = \Theta(g(n))$ if both $f(n) = O(g(n))$ and $f(n) = \Omega(g(n))$ hold. In addition, $f(n) \sim g(n)$ if $\lim_{n\to\infty} f(n)/g(n) = 1$, and $f(n) = o(g(n))$ if $\lim_{n\to\infty} f(n)/g(n) = 0$.

hop routing strategy. Zhou and Ying [21] study the 2D-i.i.d. mobility model and provide a multicast throughput capacity under their network models. Specifically, they examine a network consisting of n_s multicast sessions, each of which had one source and p destinations. They show that the capacity per multicast session was $O\left(\min\left\{1, (\log p)(\log(n_s p))\sqrt{\frac{D}{n_s}}\right\}\right)$ given delay constraint D. Wang et al. [24] give a global perspective on the multicast capacity and delay tradeoffs in MANETs for 1D-i.i.d. mobility model and 2D-i.i.d. mobility model, respectively. In [25], Skordylis et al. study the delay-bounded routing for VANETs, where the focus is on the routing problem instead of capacity analysis. For hybrid VANETs, to our best knowledge, the multicast throughput capacity with delay constraint is still an open question.

In this paper, by equipping each vehicular node with a directional antenna, we study the hybrid multicast throughput capacity of a hybrid VANET under delay constraint D. In our network model, a source node will transmit to its destinations in ad hoc mode if the transmissions can be transmitted to the destinations in D time slots. They can transmit successfully if and only if each source destination node covers each other by their directional antenna beams. Otherwise, if the transmissions cannot be finished in D time slots, it will be carried out in the infrastructure mode.

Based on the network model and the assumptions, we first analyze the two dimensional i.i.d. mobility model with fast mobile vehicles. Then, we obtain the throughput capacity of the VANET for two dimensional i.i.d. mobility model with slow mobiles. As we known, one dimensional mobility model with slow mobile vehicles is the most realistic model for wireless VANETs. Base on the results obtained in the two dimensional i.i.d. mobility model, we derive the throughput capacity of one dimensional i.i.d. mobility model for both fast and slow mobile vehicles. In all the above situations, we give the closed form formula in order of magnitude. We find that the base station plays an important role in the throughput capacity of the hybrid VANET. When the number of base stations is fewer, then there is a tradeoff between the delay constraint and the beamwidth of the directional antenna on the multicast throughput capacity of the hybrid VANET. Also, we analyze the impact of D, m and θ on the multicast throughput capacity of the hybrid VANET and give some interesting results which could then serve as further directions to the VANETs design.

The main contributions of this paper can be summarized as follows:

(1) We present an asymptotic study of the multicast capacity for the hybrid VANETs, while each vehicle node equipped with a directional antenna and the end-to-end delay constraint has to be satisfied. Then, we obtain the closed form formula of the multicast capacity in order of magnitude.

(2) We analyze the impact of two mobility models and two mobility time scales on multicast capacity of the VANET. Based on former analysis on two dimensional model and fast mobility pattern, for the more realistic one dimensional mobility model and slow mobility pattern, we obtain the capacity results for hybrid VANETs, which is not considered in the the state-of-the-art research, especially under delay constraint.

(3) We analyze the impact of the base stations, the beamwidth of the directional antenna, and delay constraint on the multicast capacity. Our findings provide insights on network architecture and system design in VANETs.

The rest of the paper is organized as follows. In Section II, we give the definitions and notations. In Section III, we give the system model and assumptions. In Section IV, we give the intuitive results. In Section V, we analyze the multicast throughput upper bound of the two mobility models and two mobility time scales. In Section VI, we analyze the achievable lower bound. Finally, we conclude the paper in Section VII.

II. DEFINITIONS AND NOTATIONS

For the convenience of analysis and proof, we first present some definitions. We use the definition of aggregate multicast throughput capacity of the whole network instead of the per-node multicast throughput capacity. Note that the per-node multicast throughput capacity is simply $1/n$ of the aggregate multicast throughput capacity of the whole network. In this paper, we adopt the asymptotic notations defined in [7]. We now define the feasible aggregate multicast throughput and the aggregate multicast throughput capacity of the hybrid network model. To facilitate the reading, we list the major notations used in our model and analysis in Table I.

Feasible Aggregate Multicast Throughput: For a hybrid VANET of n vehicular nodes and m base stations, we say that the aggregate multicast throughput, denoted by $\Lambda(n, m)$, is feasible if there exists a spatial and temporal scheduling scheme[2] that yields an aggregate network throughput of $\Lambda(n, m)$ bits/sec.

Aggregate Multicast Throughput Capacity of a Network: Aggregate Multicast Throughput capacity is the guaranteed rate that can be achieved for all multicast sessions. The aggregate multicast throughput capacity of hybrid wireless networks is said to be of order $O(f(n, m))$ bits per second if there is a deterministic constant $c_1 < +\infty$ such that

$$\liminf_{n\to+\infty} P(\Lambda(n, m) = c_1 f(n, m) \text{ is feasible }) < 1,$$

and is of order $\Theta(f(n, m))$ bits per second if there are deterministic constants $0 < c_2 < c_3 < +\infty$ such that

$$\liminf_{n\to+\infty} P(\Lambda(n, m) = c_2 f(n, m) \text{ is feasible }) = 1,$$

$$\liminf_{n\to+\infty} P(\Lambda(n, m) = c_3 f(n, m) \text{ is feasible }) < 1.$$

Delay constraint: We assume a hard delay constraint D in this paper. A packet is said to be successfully multicast if all p destinations receive the packet within D time slots after the source sends out the packet.

[2] it means that there exist a scheme to schedule transmissions and choice of routes between source nodes and their destinations in ad hoc mode and/or infrastructure mode. And the overall data transfer from source nodes to their destinations can achieve such a rate.

W	The total bandwidth of the VANET, which means the rate of data transfer in the VANET, measured in bits per second
W_1	The ad hoc mode bandwidth of the VANET
W_2	The uplink bandwidth of infrastructure mode of the VANET
W_3	The downlink bandwidth of infrastructure mode of the VANET
n	The number of vehicular nodes in the network
m	The number of base stations in the network
D	The maximum tolerant delay of the ad hoc mode transmission
θ	The beamwidth of a directional antenna
r_i	The transmission range of vehicular node i
X_i	vehicular node i and its location
$\Lambda^d[T]$	The number of bits that are successfully delivered to destinations up to time T when the packets are directly transmitted from source to their destinations
$\Lambda^r[T]$	The number of bits that are successfully delivered to destinations up to time T when the packets have to be transmitted from relays to their destinations
$\Lambda[T]$	The number of bits that are successfully delivered to destinations up to time T.
$\Lambda(n,m)$	The aggregate multicast throughput of the whole hybrid VANET with n vehicular nodes and m base stations
$\Lambda_a(n,m)$	The aggregate throughput of the ad hoc mode transmission
$\Lambda_i(n,m)$	The aggregate throughput of the infrastructure mode transmission
$\lambda(n,m)$	Per-node multicast throughput of the whole hybrid VANET with n vehicular nodes and m base stations
$\lambda_a(n,m)$	Per-node multicast throughput of the ad hoc mode transmission
$\lambda i(n,m)$	Per-node multicat throughput of the infrastructure mode transmission
P	The probability of an event
E	The expectation of a random variable
$\lvert \cdot \rvert$	The Eucledian distance

TABLE I
NOTATIONS

III. SYSTEM MODEL

We consider a hybrid VANET of n mobile ad hoc nodes (vehicles), overlaid with a cellular architecture of m base stations on a planar torus of unit area, as shown in Figure 1. There are n_s multicast sessions, where each session consists of one source and p destinations. In our system model, we assume every vehicular node can be source or destination in only one session. And every node can be relay node for any multicast sessions. Thus, there are $n = n_s(1+p)$ mobile nodes (vehicles) in the network. In particular, a hybrid VANET consists of two layers, an ad hoc layer and an infrastructure layer. In the ad hoc layer, we assume that n wireless nodes are uniformly and independently distributed on the unit area, which can communicate with other vehicular nodes or the base stations within their transmission ranges. The m stations are regularly deployed at the top of ad hoc layer. The unit area is divided into equal-sized squares of area $\frac{1}{m}$. We call the squares "cell" and one base station is placed at the center of each cell. The base stations only serve as relays to forward the traffic from wireless nodes in the ad hoc layer, i.e., they can neither serve as data sources nor as destinations. Moreover, the base stations are also assumed to be connected to each other by wired line or wireless channels (using frequency different from the frequency used between ordinary ad hoc wireless nodes). The links between base stations have very high bandwidth so that there are no bottlenecks associated with the base stations.

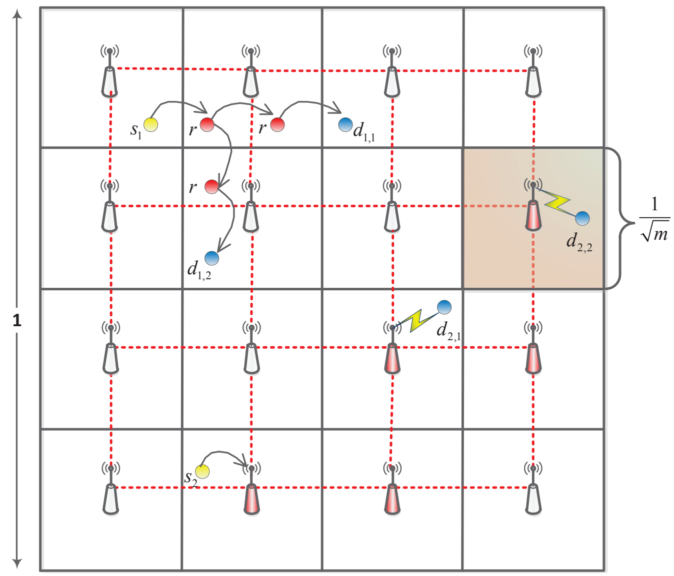

Fig. 1. Basic network model.

We also assume that the base stations have no power constraint so that the data can be relayed from base stations to their own cell nodes directly in the down link transmissions.

For the convenience of capacity analysis in different traffic mode (ad hoc mode and infrastructure mode), and also in order that the adopted model here is closely related to practical systems, we employ bandwidth partitioning and consider the ad hoc mode traffic and infrastructure mode traffic separately. This is because bandwidth partitioning can be implemented with the multi-channel capability provided by current radio devices, and different traffic types have different scaling behaviors.

A. Directional Antenna Model

In the study of VANETs, there are many types of directional antenna models. In this paper, we consider the directional antenna model as shown in Figure 2. We approximate the directional antenna as a circular sector with angle θ and radius equal to the transmission/reception range r. The angle of the sector approximates the beamwidth of the antenna [30]. In reality, the directional antenna pattern consists of a mainlobe which is the direction of maximum radiation or reception and several smaller backlobes arising due to inefficiencies in antenna design. For simplicity, we ignore discussing backlobes in this paper.

In our directional antenna model, the directional antenna gain is within the specific angle θ, which is the beamwidth of the antenna. The gain outside the beamwidth is assumed to be zero. We assume that each antenna is steerable, i.e., each node can point its antenna in any desired direction. Nodes can use their antennas for directional transmission and/or directional reception. That is, at any time, the antenna beam can only be placed to a certain direction. Thus, the probability that the beam covers a direction is $\theta/2\pi$. We also assume each node is equipped with only one directional antenna.

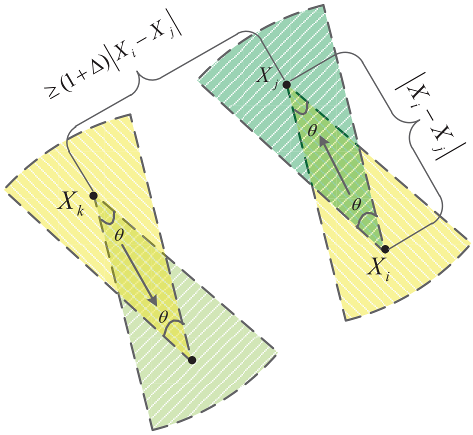

Fig. 2. Directional antenna and communication model.

It is generally well known that modeling a real directional antenna is complex, this is the main reason for using a simple directional antenna in our model. In this paper, as we will show in the following sections, the interference area of the nodes in the unit torus is the only characteristic that decided the results. Simplifying the shape of the antenna pattern will not change this property and using a complex model will not result in a fundamental change in this work, in particular on the throughput capacity analysis.

B. communication model

We adopt the protocol model introduced in [7] to study the impact of the wireless interference in hybrid VANETs. Let r_i denote the transmission radius of node i, then the following conditions should be hold for a successful transmission:

1) The receiver is within the transmission range of the transmitter, i.e.,

$$|X_i - X_j| \leq r_i.$$

2) Every other transmitter $X_k (k \neq i)$ simultaneously transmitting does not cause interference to node j, i.e.,

$$\begin{cases} |X_k - X_j| \geq (1+\Delta)|X_i - X_j| \\ \text{or } X_k' s \text{ beam does not cover node } X_j \end{cases}$$

Here, $\Delta > 0$ represent the guard zone, which is constant that does not depend on n.

where X_i not only denotes the location of a node but refers to the node itself. Figure 2 also shows that a transmission from node X_k will not cause interference to X_i's transmission since the antenna beam of X_k does not cover receiver X_j.

The above conditions mean that when node X_i transmits to node X_j, the transmission is successful if the antenna beam of the two nodes X_i, X_j will cover each other and no nodes within the region covered by X_j's antenna beam will interfere with X_j's reception as shown in Figure 2.

C. Transmission Model

There are two transmission modes in hybrid VANETs: ad hoc mode and infrastructure mode. In an ad hoc mode, packets

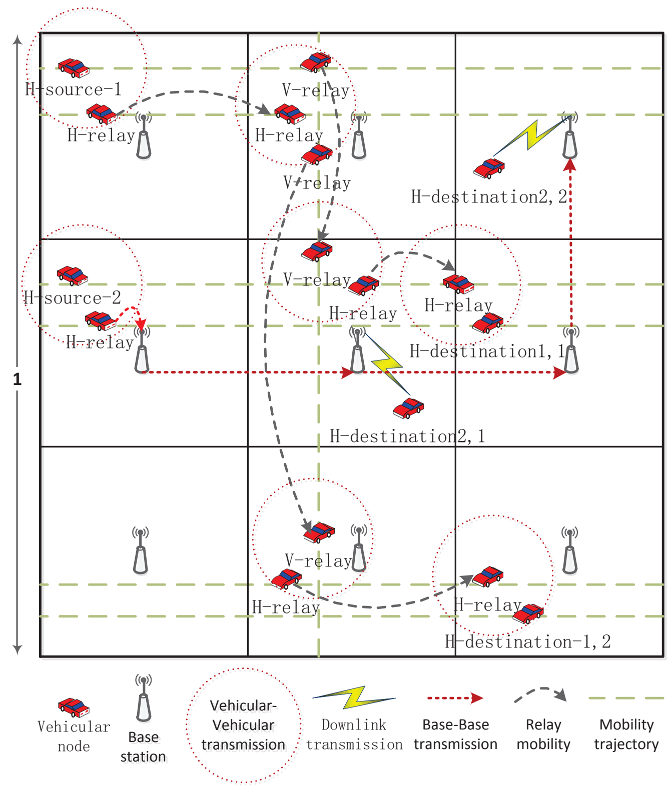

Fig. 3. Transmission and mobility model.

are forwarded from the source node to the destination nodes and can only be relayed by the normal nodes, i.e., without the help of base stations. While in an infrastructure mode, packets are transmitted from the source to the base station, and then forwarded to the destinations. The transmission and receiving processes are illustrated in Figure 1 and Figure 3.

In this paper, we consider a *D-delay constraint* ($D \geq 1$) transmission strategy. If a destination can be reached within D consecutive time slots from a source under the directional transmission, then the packets can be transmitted from this source to the destinations in an ad hoc mode. Otherwise, packets are transmitted in the infrastructure mode.

Moreover, we assume a total available bandwidth of W bits/sec, which can be carried over multiple sub-channels with different frequency bands (for example multiple orthogonal channels). We divide the wireless channel so that ad hoc mode transmissions and infrastructure mode transmissions go through different sub-channels. We further divide the sub-channel for infrastructure mode transmissions into uplink and downlink parts, according to the direction of the transmissions relative to the base station.

The bandwidth assigned to intra-cell, uplink, and downlink sub-channels are W_1, W_2, and W_3, respectively. The transmission rates should sum to W, i.e., $W_1 + W_2 + W_3 = W$[3]. Since the uplink has the same amount of traffic as the downlink, we

[3]Notice that here the bandwidth means the maximum rate of data transfer that a transmission can support, measured in bits per second. And W_1, W_2, W_3 are constant which have no relation with the number of nodes n and base stations m.

have $W_2 = W_3$. Thus, $W = W_1 + 2W_2$.

D. Mobility Model

We consider the following mobility models in this paper.

(1) **Two-dimensional i.i.d. mobility model:** Our two dimensional i.i.d. mobility model is defined as follows:

  (i) At each time slot, the nodes are uniformly, randomly positioned in the unit square.

  (ii) The node positions are independent of each other, and independent from time slot to time slot. So the nodes are totally reshuffled at each time slot.

(2) **One-dimensional i.i.d. mobility model:** Our one dimensional i.i.d. mobility model is defined as follows:

  (i) Reasonably, we assume the number of mobile nodes n and source nodes n_s are both even numbers. As shown in Figure 3, among the mobile nodes, $n/2$ nodes (including $n_s/2$ source nodes), named H-nodes, move horizontally; and the other $n/2$ nodes (including the other $n_s/2$ source nodes), named V-nodes, move vertically.

  (ii) Let (x_i, y_i) denote the position of node i. If node i is an H-node, y_i is fixed and x_i is a value randomly uniformly chosen from $[0, 1]$. We also assume that H-nodes are evenly distributed vertically, so y_i takes values $2/n, 4/n, \cdots, 1$. V-nodes have similar properties.

  (iii) Assume that source and destinations in the same multicast session are the same type of nodes. Also assume that node i is an H-node if i is odd, and a V-node if i is even.

  (iv) The orbit distance of two H(V)-nodes is defined to be the vertical (horizontal) distance of the two nodes.

For VANETs, we first investigate the results on two dimensional i.i.d. mobility model. Then, based on the derived results, we consider the mobility trajectories as one dimensional lines. We adopt the one dimensional i.i.d. mobility to model the mobility character of vehicular nodes. Although the real mobility pattern of vehicular nodes are not exactly one dimensional i.i.d. mobility model, we find that it can catch the essential mobility character of vehicular nodes. The results on one dimensional i.i.d. mobility model provide theoretical framework for other mobility models.

Time scale of mobility: Two time scales of mobility are considered in this paper.

- **Fast mobility:** The mobility of nodes is at the same time scale as the transmission of packets, so in each time slot, only one-hop transmission is allowed and $W_2(W_3)$ is a constant independent of n.
- **Slow mobility:** The mobility of nodes is much slower than the transmission of packets, i.e., multi-hop transmissions may happen within a single time slot and $W_2(W_3) = \Omega(n)$. Thus, for slow mobility, the packet size should be scaled as $W_2/h(n)$ for $h(n) = O(n)$ to guarantee $h(n)$-hops transmission in a time slot.

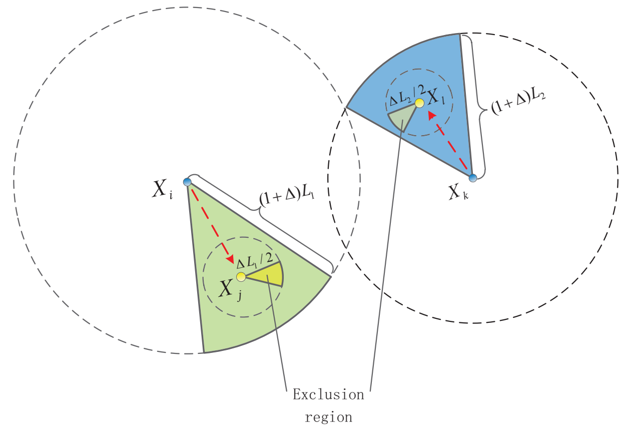

Fig. 4. The exclusion region of directional transmit-receiving.

IV. Intuitive Analysis

In this section, we present the main results and the key intuition of this paper. In the study of VANETs, the mobiles may assume moving according some mobility models. In this paper, we consider four type of mobility models, i.e., two dimensional i.i.d. fast mobility model, two dimensional i.i.d. slow mobility model, one dimensional i.i.d. fast mobility model, and one dimensional i.i.d. slow mobility model. In the VANETs, the mobiles mobile speed is far smaller than the packets transmissions, that is, the mobility patterns are more like the slow mobility model. For the convenient of analysis, we first analyze the i.i.d. fast mobility model, then we switch to the more general slow i.i.d. mobility model. We adopt the virtual channel idea proposed in [10] to analysis heuristically the system.

In our system, we assume that all the mobiles are equipped with directional antenna which can send and receive signals in half-duplex mode. There are two transmission modes in our system, ad hoc mode and infrastructure mode.

First, we consider the two dimensional i.i.d. mobility model with fast mobility mobiles.

Ad Hoc Mode: In the ad hoc mode, packets are transmitted from sources to their destinations only using ordinary ad hoc node, without the help of base stations. In general, a successful delivery in the ad hoc mode consists of three phases, which can be considered as three virtual channels.

- **Reliable broadcasting channel**, the packet is transmitted from the source to some relay node;
- **Unreliable relay channel**, the relay node moves to the neighborhood of one of the p destinations of the packet;
- **Reliable receiving channel**, the relay sends the packet to its destinations.

In all the above three phases, the vehicle nodes are transmitting and receiving massage using directional antenna, and the three phases can be thought as virtual channels.

- Reliable broadcasting channel: For the purpose of successful transmission, on the aspect of transmission view, the exclusion regions of successful transmissions should be disjoint with each other, as shown in Figure 4. We assume all sources use a common transmission radius

L_1 for sending out the information. Then, consider the utilization of directional antenna, the exclusion region[4] has an area of $\theta\pi(L_1)^2$. Here we omit the constant Δ for simplicity, which have no influence on the final results. Thus, we can know the number of simultaneous transmitting at one time slot is at most $\frac{1}{\theta\pi(L_1)^2}$. And each source has P_1 fraction of time to transmit on average, where

$$P_1 = \frac{1}{\theta\pi(L_1)^2 n_s}.$$

Follow the above analysis, it is easy to know that the throughput of each broadcasting channel is

$$\frac{W_1}{\theta\pi(L_1)^2 n_s}.$$

On average, each packet will be received by $\theta\pi(L_1)^2 n$ nodes in the covered sectors, and has $\theta\pi(L_1)^2 n$ duplicated copies in the network.

- Unreliable relay channel: The transmission radius of the relays is assumed as L_2. Then, for the transmissions from relays to destinations, the probability that a duplicated packet does not fall into the covered area of a specific one of its p destinations during D consecutive time slots is

$$P_2 = (1 - \theta\pi L_2^2)^D.$$

As analyzed above, each source packet will have $\theta\pi(L_1)^2 n$ copies after sent out from the source. So the probability that none of the duplicated packets fall into the covered area of the p destinations during D consecutive time slots is

$$P_3 = (1 - \theta\pi L_2^2)^{\theta D\pi(L_1)^2 n}.$$

- Reliable receiving channel: Consider the transmissions from relays to destinations. When a packet is being transmitted from a relay, it is delivered to all the destinations that distributed in the area covered by the transmission range of the relay. Only one of the deliveries is named as target delivery, and the rest as free-ride deliveries. Note that all covered area of the receiving nodes that associated with the successful target deliveries should be disjoint from each other. Also, as for a successful target-delivery, it will consumes an area of $\theta\pi(L_2)^2$. So the number of target deliveries at one time slot is no more than

$$\frac{W_1}{\theta\pi(L_2)^2}.$$

Furthermore, on average, there are

$$(p-1)\theta\pi(L_2)^2$$

free-ride deliveries along with each target delivery. Then, we can have

$$\frac{W_1(1+(p-1)\theta\pi(L_2)^2)}{\theta\pi(L_2)^2}$$

deliveries at each time slot. We can derive the throughput per multicast session as

$$\frac{W_1(1+(p-1)\theta\pi(L_2)^2)}{\theta n_s p\pi(L_2)^2} = \frac{W_1}{\theta n_s p\pi(L_2)^2} + \frac{W_1(p-1)}{n_s p}$$

bit per time slot, which is because the same multicast session request identical information.

Let λ denote the multicast capacity, which is defined as the maximum throughput per multicast session. Based on the above virtual channel analysis, we can conclude heuristically that

$$\lambda = \max\ \min\{\left(1 - (1-\theta\pi L_2^2)^{\theta D\pi(L_1)^2 n}\right)\frac{W_1}{\theta\pi(L_1)^2 n_s}, \frac{W_1}{\theta n_s p\pi(L_2)^2} + \frac{W_1(p-1)}{n_s p}\}$$

Next we turn to the two dimensional i.i.d. mobility model with slow mobiles. As described in [7], we can extend the unicast results to our multicast results with directional antenna. We also use the virtual channel model as used in the fast mobility model, which is different just for the reliable broadcasting channel. We can conclude that the throughout of each broadcasting channel is

$$\frac{W_1}{\pi L_1\sqrt{n_s}}$$

Then, we have the heuristically result

$$\lambda = \max\ \min\{\left(1 - (1-\theta\pi L_2^2)^{\theta D\pi(L_1)^2 n}\right)\frac{W_1}{\theta\pi L_1\sqrt{n_s}}, \frac{W_1}{\theta n_s p\pi(L_2)^2} + \frac{W_1(p-1)}{n_s p}\}$$

We next consider the one dimensional i.i.d. mobility model. For the one dimensional i.i.d. mobility model, each mobile move along a line. If the orbits of relay nodes and destination nodes are vertical to each other, the probability that two nodes are in the same square with side length $2L$ is $1-(1-4\theta^2 L^2)^{Dp}$. If the orbits of relay nodes and destination nodes are parallel to each other, then we have $1-(1-2\theta L)^{Dp}$. Recall that we assume a unit area, thus $2L < 1$, and we have for the reliable relay channel

$$1 - (1 - 2\theta L)^{Dp}.$$

Then, for one dimensional i.i.d. fast mobility model, we have

$$\lambda = \max\ \min\{\left(1 - (1-2\theta L)^{Dp}\right)\frac{W_1}{\theta\pi L_1\sqrt{n_s}}, \frac{W_1}{\theta n_s p\pi(L_2)^2} + \frac{W_1(p-1)}{n_s p}\}$$

For the one dimensional i.i.d. mobility model with slow mobility model, the mainly difference from fast mobility is the reliable broadcasting channel. We have the throughout of each broadcasting channel is

$$\frac{W_1}{\pi L_1\sqrt{n_s}}$$

[4]More description of exclusion region is in Appendix C

Then, we have the heuristically result

$$\lambda = \max\ \min\{\left(1-(1-2\theta L)^{Dp}\right)\frac{W_1}{\pi L_1\sqrt{n_s}}, \frac{W_1}{\theta n_s p\pi (L_2)^2}+\frac{W_1(p-1)}{n_s p}\}$$

Infrastructure Mode: In the infrastructure mode, the base stations are deployed regularly on the network with unit area, which split the whole network into cells with each base station in the center of a cell. Packets are transmitted from source nodes to their nearest base stations and then to the destinations with the help of the base stations which distributed within the same cell of the destinations. As shown in Figure 1, the transmission from source to base station (uplink) can employ multihop mode and the transmission from base stations to the destinations (downlink) employ single hop mode. In our paper, we assume that the transmission are switched between ad hoc and infrastructure mode, i.e., a packet can be transmitted by the ad hoc mode to its destinations during D consecutive time slots, otherwise, it will be transmitted with the help of base stations directly.

V. Upper Bound Analysis

For a large scale VANET, the upper bound of its capacity is important both in theory and application. In this section, base on the asymptotic analysis as in [7], we derive the upper bound multicast capacity for hybrid VANET with directional antenna and delay constraint. We proof our results for the two mobility models and two mobility time scales as introduce above. In the following sections, in order to simplify the redundancy of the formula, we let beamwidth of the directional antenna $\theta/2\pi \in (0,1)$, which represent a probability and do not have fundamental change on the results.

A. Two Dimensional I.I.D. Fast Mobility Model

In this section, the upper bound on the multicast capacity of the hybrid VANET with two dimensional i.i.d. mobility model and fast mobiles is presented under the conditions of directional antennas and delay constraints.

Let $\Lambda^d[T]$ be the number of bits that are successfully delivered to destinations up to time T when the packets are directly transmitted from source to their destinations. And $\Lambda^r[T]$ the number of bits that are successfully delivered to destinations up to time T when the packets have to be transmitted from relays to their destinations. Then, we have $\Lambda[T] = \Lambda^d[T] + \Lambda^r[T]$, which the number of bits that are successfully delivered to destinations up to time T. We also let $B[T]$ denote the bits delivered by target deliveries up to time T. Let $H(j,\gamma,t,\theta)$ denote the number of destinations that belong to the same multicast session as node j (destination node) and their distance between j is less than γ at time t.

Lemma 1: There exist $\kappa > 0$, independent of n_s, p and θ, such that for any $\gamma \in (0,1]$

$$E[Z_{\gamma,\kappa,\theta}[T]] \le \frac{T}{(\theta n_s p)^2}$$

where

$$Z_{\gamma,\kappa,\theta}[T] = \sum_{t=1}^{T}\sum_{j} 1_{H(j,\gamma,t,\theta)} \ge \kappa(1+\theta p\gamma^2)\log(\theta n_s p)$$

Proof: The proof is presented in Appendix A. ■

Lemma 2: Consider the 2D-i.i.d. mobility and the protocol model, using the directional model, the following inequality holds:

$$E[\Lambda[T]] \le 5\kappa\log(\theta n_s p)E[B[T]] + \frac{16\kappa W_1 T}{\Delta^2}p(\log p)\log(\theta n_s p).$$

Proof: The proof is presented in Appendix B. ■

Lemma 3: For the 2D-i.i.d. fast mobility and the protocol model, using the directional antenna model, when the packets are directly transmitted from source to their destinations, we have

$$E[\Lambda^d[T]] \le 5\kappa\log(\theta n_s p)\left(\frac{1}{1-\theta/2\pi}W_1 T\sqrt{\frac{32}{\Delta^2}}\sqrt{n_s p}\right) + \frac{16\kappa W_1 T}{\Delta^2}p(\log p)\log(\theta n_s p).$$

Proof: First, we consider the sources that directly send packets to their destinations using directional antenna transmissions. We first bound the total number of targeted deliveries. Let s_i denote the source of multicast session i, $d_{i,j}$ denote the j^{th} destinations of multicast session i, and $D(s_i,t)$ the distance between source s_i and its nearest destination, i.e.,

$$D(s_i,t) = \min_{1\le j\le p} dist(s_i,d_{i,j})(t).$$

Then, based on the geometry probability construction, we have

$$Pr(D(s_i,t)\le L, \text{covered by} s_i) \le 1-(1-\theta^2\pi L^2)^p,$$

which implies

$$E\left[\sum_{t=1}^{T}\sum_{i=1}^{n_s} 1_{D(s_i,t)\le L, covered by s_i}\right] \le \theta^2 T n_s p\pi L^2.$$

Since at most W_1 bits a source can send during each transmission, we have

$$\begin{aligned} E[B[T]] =& E\left[\sum_{B=1}^{B[T]} 1_{\alpha_B\le L}\right] + E\left[\sum_{B=1}^{B[T]} 1_{\alpha_B > L}\right] \\ \le& W_1 E\left[\sum_{t=1}^{T}\sum_{i=1}^{n_s} 1_{D(s_i,t)\le L,\text{ covered by} s_i}\right] \\ &+ E\left[\sum_{B=1}^{B[T]} 1_{\alpha_B > L}\right] \\ \le& \theta^2 T n_s p\pi L^2 + E\left[\sum_{B=1}^{B[T]} 1_{\alpha_B > L}\right]. \end{aligned}$$

Then, using the Cauchy-Schwarz inequality, we have

$$\left(\sum_{B=1}^{B[T]} \alpha_B\right)^2 \leq \left(\sum_{B=1}^{B[T]} 1\right)\left(\sum_{B=1}^{B[T]} (\alpha B)^2\right) \\ \leq B[T]\frac{4WT}{\theta\pi\Delta^2},$$

and we can obtain that

$$\begin{aligned}\sqrt{\frac{4W_1T}{\theta\pi\Delta^2}}\sqrt{E[B[T]]} \geq& E\left[\sqrt{\frac{4W_1T}{\theta\pi\Delta^2}}B[t]\right] \\ \geq& E\left[\sum_{B=1}^{B[T]} \alpha_B\right] \\ \geq& LE\left[\sum_{B=1}^{B[T]} 1_{\alpha_B > L}\right] \\ \geq& L(E[B[T]] - \theta^2 W_1 T n_s p \pi L^2),\end{aligned}$$

where the first inequality follows the Jensen's inequality. Now let $L = \sqrt{\frac{EB[T]}{2\theta W_1 T n_s p\pi}}$, we can obtain that

$$\frac{1}{1-\theta/2\pi} WT\sqrt{\frac{32}{\Delta^2}}\sqrt{n_s p} \geq E[B[T]].$$

By submitting into the bound on $\Lambda_1[T]$ in Lemma 2, we have

$$\begin{aligned}E[\Lambda^d[T]] \leq& 5\kappa\log(\theta n_s p)\left(\frac{1}{1-\theta/2\pi}W_1T\sqrt{\frac{32}{\Delta^2}}\sqrt{n_s p}\right) \\ &+\frac{16\kappa W_1 T}{\Delta^2} p(\log p)\log(\theta n_s p).\end{aligned}$$

which conclude the proof. ■

Lemma 4: For the 2D-i.i.d. mobility and the protocol model, using the directional antenna model, when the packets have to be transmitted from relays to their destinations, we have

$$\begin{aligned}E[\Lambda^r[T]] \leq& 5\kappa\log(\theta n_s p)\left(\sqrt{\frac{32}{\Delta^2}}\frac{1}{1-\theta/2\pi}W_1T\sqrt{(p+1)n_s D}\right) \\ &+\frac{16\kappa W_1 T}{\Delta^2} p(\log p)\log(\theta n_s p).\end{aligned}$$

Proof: Consider the transmissions from relays to the destinations, under the condition of delay constraint D and directional antenna θ, denote by $H(B)$ the minimum distance between the relay carrying bit B and any of the p destinations. We have

$$Pr(H(B) \leq L) \leq 1 - (1-\theta^2\pi L^2)^{Dp},$$

which implies

$$E\left[\sum_{b\in\mathcal{R}[T]} 1_{H(B)\leq L}\right] \leq n_s(p+1)W_1 T\pi L^2 Dp,$$

and

$$\begin{aligned}E[B[T]] =& E\left[\sum_{B=1}^{B[T]} 1_{\alpha_B \leq L}\right] + E\left[\sum_{B=1}^{B[T]} 1_{\alpha_B > L}\right] \\ \leq& E\left[\sum_{B=1}^{B[T]} 1_{H(B)\leq L}\right] + E\left[\sum_{B=1}^{B[T]} 1_{\alpha_B > L}\right] \\ \leq& \theta^2 n_s(p+1)WT\pi L^2 Dp + E\left[\sum_{B=1}^{B[T]} 1_{\alpha_B > L}\right].\end{aligned}$$

Then, using the Cauchy-Schwarz inequality, we can obtain

$$\left(\sum_{B=1}^{B[T]} \alpha_B\right)^2 \leq \left(\sum_{B=1}^{B[T]} 1\right)\left(\sum_{B=1}^{B[T]} (\alpha_B)^2\right) \\ \leq B[T]\frac{4W_1T}{\theta\pi\Delta^2},$$

which implies that

$$\begin{aligned}\sqrt{\frac{4W_1T}{\theta\pi\Delta^2}}\sqrt{E[B[T]]} \geq& E\left[\sqrt{\frac{4W_1T}{\theta\pi\Delta^2}}B[t]\right] \\ \geq& E\left[\sum_{B=1}^{B[T]} \alpha_B\right] \\ \geq& LE\left[\sum_{B=1}^{B[T]} 1_{\alpha_B > L}\right] \\ \geq& L(E[B[T]] - \theta^2 W_1 T n_s(p+1)\pi L^2 Dp),\end{aligned}$$

where the first inequality follows from the Jensen's inequality and the last inequality follows from the upper bound of $E[B[T]]$. Since the inequality holds for any $L > 0$. By choosing $L = \sqrt{\frac{E[B[T]]}{2W_1 T\theta\pi n_s(p+1)pD}}$, we have

$$\sqrt{\frac{32}{\Delta^2}}\frac{1}{1-\theta/2\pi}W_1T\sqrt{\theta(p+1)n_s D} \geq E[B[T]].$$

And finally we have

$$\begin{aligned}&E[\Lambda^r[T]] \\ &\leq 5\kappa\log(\theta n_s p)\left(\sqrt{\frac{32}{\Delta^2}}\frac{1}{1-\theta/2\pi}W_1T\sqrt{(p+1)n_s D}\right) \\ &+\frac{16\kappa W_1 T}{\Delta^2} p(\log p)\log(\theta n_s p).\end{aligned}$$

which concludes the proof. ■

Theorem 1: The ad hoc mode multicast capacity with delay constraint under the 2D-i.i.d. mobility an protocol model is

$$\lambda_a(n,m) = O\left((\log p)(\log(\theta n_s p))\frac{1}{1-\theta/2\pi}\sqrt{\frac{D}{n_s}}\right).$$

Proof: Based on the results of Lemma 1 and Lemma 2, we have

$$
\begin{aligned}
&E[\Lambda[T]] \\
&\leq 5\kappa \log(\theta n_s p) \frac{1}{1-\theta/2\pi} \left(W_1 T \sqrt{\frac{32}{\Delta^2}} \sqrt{(p+1) n_s D} + \sqrt{n_s p} \right) \\
&+ \frac{16\kappa W_1 T}{\Delta^2} p (\log p) \log(\theta n_s p).
\end{aligned}
$$

In the order case, the throughput by using relay case dominates the throughput without relay. Then,

$$
E[\Lambda[T]/n_s p] \leq O\left((n_s p T \log p)(\log(\theta n_s p)) \frac{1}{1-\theta/2\pi} \sqrt{\frac{D}{n_s}} \right).
$$

From the definition of the multicast capacity, we have $\lambda_a(n,m) \leq \frac{\Lambda[T]}{T n_s p}$ because a successful multicast requires p successful deliveries, which concludes the proof of the theorem.

■

B. Two Dimensional I.I.D. Slow Mobility Model

In this section, we study the capacity of two dimensional i.i.d. mobility model with slow mobiles. We first present the upper bound of the multicast capacity, and then propose the scheme which can achieve a capacity close to the upper bound up to logarithmic factors.

Under slow mobility model, the mobility of nodes is much slower than the transmission of packets, i.e., multihop transmissions may happen within a single time-slot. Let $H(B)$ the number of hops bit B travels in the time slot that bit B delivered to its destinations. And let $L(B)$ the Euclidean distance bit B travels in above time slot. We also let α_B^h the transmission radius used in hop h for $1 \leq h \leq H(B)$.

Lemma 5: For any mobility model, the following inequalities holds for nodes with directional antenna and under the protocol model:

$$
\sum_{B=1}^{B[T]} \sum_{h=1}^{H(B)} 1 \leq n_s(p+1) W_1 T \tag{1}
$$

$$
\sum_{B=1}^{B[T]} \sum_{h=1}^{H(B)} \frac{\Delta^2}{16} (\alpha_B^h)^2 \leq \frac{W_1 T}{\theta \pi} \tag{2}
$$

We also give two lemmas on the throughput for packets delivered from source to their destinations directly and from relays to their destinations, respectively.

Lemma 6: For the 2D-i.i.d. slow mobility and the protocol model, using the directional antenna model, when the packets are directly transmitted from source to their destinations, we have

$$
\begin{aligned}
&E[\Lambda^d[T]] \\
\leq& 5\kappa \log(\theta n_s p) \left(\frac{(4\sqrt{2})^{2/3}}{\Delta^{2/3}} \frac{W_1 T}{(1-\theta/2\pi)^{2/3}} \sqrt[2/3]{n_s p} \right) \\
+& \frac{16\kappa W T}{\Delta^2} p (\log p) \log(\theta n_s p).
\end{aligned}
$$

Proof: By the Cauchy-Schwarz inequality, we have

$$
\begin{aligned}
\left(\sum_{B=1}^{B[T]} \sum_{h=1}^{H(B)} \alpha_B^h \right)^2 \leq& \left(\sum_{B=1}^{B[T]} \sum_{h=1}^{H(B)} 1 \right) \left(\sum_{B=1}^{B[T]} \sum_{h=1}^{H(B)} (\alpha_B^h)^2 \right) \\
\leq& n_s(p+1) W_1 T \sum_{B=1}^{B[T]} \sum_{h=1}^{H(B)} (\alpha_B^h)^2 \\
\leq& n_s(p+1) W_1 T \frac{16 W_1 T}{\theta \pi \Delta^2},
\end{aligned}
$$

By Lemma 5, and $\sum_{h=1}^{H(B)} \alpha_B^h \geq L(B)$, we have

$$
\frac{4 W_1 T}{\Delta} \sqrt{\frac{n_s(p+1)}{\theta \pi}} \geq \sum_{B=1}^{B[T]} L(B). \tag{3}
$$

Similar as the proof of lemma 3, we have

$$
\begin{aligned}
E[B[T]] =& E\left[\sum_{B=1}^{B[T]} 1_{L(B) \leq L} \right] + E\left[\sum_{B=1}^{B[T]} 1_{L(B) > L} \right] \\
\leq& W_1 E\left[\sum_{t=1}^{T} \sum_{i=1}^{n_s} 1_{D(s_i,t) \leq L, \text{ covered by} s_i} \right] \\
+& E\left[\sum_{B=1}^{B[T]} 1_{L(B) > L} \right] \\
\leq& \theta^2 T n_s p \pi L^2 + E\left[\sum_{B=1}^{B[T]} 1_{L(B) > L} \right].
\end{aligned}
$$

which implies

$$
E[B[T]] - E\left[\sum_{B=1}^{B[T]} 1_{L(B) > L} \right] \leq \theta^2 W_1 T n_s p \pi L^2 \tag{4}
$$

Then, we can conclude that

$$
\begin{aligned}
E\left[\frac{4 W_1 T}{\Delta} \sqrt{\frac{n_s(p+1)}{\theta \pi}} \right] =& \frac{4 W_1 T}{\Delta} \sqrt{\frac{n_s(p+1)}{\theta \pi}} \\
\geq& E\left[\sum_{B=1}^{B[T]} L(B) \right] \\
\geq& L E\left[\sum_{B=1}^{B[T]} 1_{L(B) > L} \right] \\
\geq& L(E[B[T]] - \theta^2 W_1 T n_s p \pi L^2),
\end{aligned}
$$

where the first inequality follows the Jensen's inequality. Now let $L = \sqrt{\frac{E B[T]}{2\theta W_1 T n_s p \pi}}$, we can obtain that

$$
\frac{(4\sqrt{2})^{2/3}}{\Delta^{2/3}} \frac{W_1 T}{(1-\theta/2\pi)^{2/3}} \sqrt[2/3]{n_s p} \geq E[B[T]].
$$

By submitting into the bound on $\Lambda_1[T]$ in Lemma 2, we have

$$
\begin{aligned}
E[\Lambda^d[T]] \leq& 5\kappa \log(\theta n_s p) \left(\frac{(4\sqrt{2})^{2/3}}{\Delta^{2/3}} \frac{W_1 T}{(1-\theta/2\pi)^{2/3}} \sqrt[2/3]{n_s p} \right) \\
&+ \frac{16\kappa W T}{\Delta^2} p (\log p) \log(\theta n_s p).
\end{aligned}
$$

which concludes the proof. ■

Lemma 7: For the 2D-i.i.d. mobility and the protocol model, using the directional antenna model, when the packets have to be transmitted from relays to their destinations, we have

$$E[\Lambda^r[T]] \leq 5\kappa \log(\theta n_s p)\left(\frac{(4\sqrt{2})^{2/3}}{\Delta^{2/3}}\frac{W_1 T}{(1-\theta/2\pi)^{2/3}}\sqrt[2/3]{n_s p D}\right) + \frac{16\kappa W T}{\Delta^2} p(\log p)\log(\theta n_s p).$$

Proof: The proof can be finished by combining Lemma 4 and Lemma 6. ■

Combine Lemma 6 and Lemma 7, we have the following theorem:

Theorem 2: Consider the two dimensional i.i.d. mobility model with fast mobility mobiles, under the conditions that every node equipped with directional antenna and the delay constraint D, we have

$$E[\Lambda[T]] \leq 5\kappa \log(\theta n_s p)\left[\frac{(4\sqrt{2})^{2/3}}{\Delta^{2/3}}\frac{W_1 T}{(1-\theta/2\pi)^{2/3}}(\sqrt[2/3]{n_s p D}+1)\right] + \frac{16\kappa W T}{\Delta^2} p(\log p)\log(\theta n_s p).$$

C. One Dimensional I.I.D. Fast Mobility Model

For VANETs, the mobiles moving orbits are usually one dimensional. We use the one dimensional i.i.d. mobility model to characterize the mobile node. We use the simple i.i.d. mobility model as a representation of the mobility model, which can caught the main feature of a mobile node, and can extend to other mobility models, such as random walk, random way point. In this section, we study one dimensional i.i.d. mobility model with fast mobiles.

Lemma 8: Consider the one dimensional i.i.d. mobility model with fast mobiles, under the directional antenna and delay constraint, we have

$$E[\Lambda[T]] \leq 5\kappa \log(\theta n_s p)\left(W_1 T\left(\frac{\theta}{\pi}\right)^{1/3}\left[\frac{8 n_s p(p+1)D}{\Delta}\right]^{2/3}\right) + \frac{16\kappa W_1 T}{\Delta^2} p(\log p)\log(\theta n_s p). \quad (5)$$

Proof: Consider the transmissions from relays to the destinations, under the condition of delay constraint D and directional antenna θ, recall that $H(B)$ denotes the minimum distance between the relay carrying bit B and any of the p destinations. In the case of one dimensional mobility model, each mobile move along a line. If the orbits of relay nodes and destination nodes are vertical to each other, then $H(B)$ holds only if at some time slot t, the two nodes are in the same square with side length $2L$. In this scenario, we have

$$Pr(H(B) \leq L) \leq 1 - (1 - 4\theta^2 L^2)^{Dp}.$$

If the orbits of relay nodes and destination nodes are parallel to each other, then we have

$$Pr(H(B) \leq L) \leq 1 - (1 - 2\theta L)^{Dp}.$$

Note that we consider unit area, thus $2L \leq 1$, and we can conclude that

$$Pr(H(B) \leq L) \leq 1 - (1 - 2\theta L)^{Dp} \leq 2\theta L D p.$$

which implies

$$E\left[\sum_{b\in\mathcal{R}[T]} 1_{H(B)\leq L}\right] \leq n_s(p+1)W_1 T 2\theta L D p,$$

and

$$\begin{aligned} E[B[T]] =& E\left[\sum_{B=1}^{B[T]} 1_{\alpha_B \leq L}\right] + E\left[\sum_{B=1}^{B[T]} 1_{\alpha_B > L}\right] \\ \leq& E\left[\sum_{B=1}^{B[T]} 1_{H(B)\leq L}\right] + E\left[\sum_{B=1}^{B[T]} 1_{\alpha_B > L}\right] \\ \leq& n_s(p+1)W_1 T 2\theta L D p + E\left[\sum_{B=1}^{B[T]} 1_{\alpha_B > L}\right]. \end{aligned}$$

Then, using the Cauchy-Schwarz inequality, we can obtain

$$\begin{aligned} \left(\sum_{B=1}^{B[T]} \alpha B\right)^2 \leq& \left(\sum_{B=1}^{B[T]} 1\right)\left(\sum_{B=1}^{B[T]} (\alpha_B)^2\right) \\ \leq& B[T]\frac{4W_1 T}{\theta\pi\Delta^2}, \end{aligned}$$

which implies that

$$\begin{aligned} \sqrt{\frac{4W_1 T}{\theta\pi\Delta^2}}\sqrt{E[B[T]]} \geq& E\left[\sqrt{\frac{4W_1 T}{\theta\pi\Delta^2}B[t]}\right] \\ \geq& E\left[\sum_{B=1}^{B[T]} \alpha_B\right] \\ \geq& LE\left[\sum_{B=1}^{B[T]} 1_{\alpha_B > L}\right] \\ \geq& L(E[B[T]] - n_s(p+1)W_1 T 2\theta L D p), \end{aligned}$$

where the first inequality follows from the Jensen's inequality and the last inequality follows from the upper bound of $E[B[T]]$. Since the inequality holds for any $L > 0$. By choosing $L = \frac{E[B[T]]}{2\theta n_s(p+1)W_1 T D p L}$, we have

$$E[B[T]] \leq W_1 T\left(\frac{\theta}{\pi}\right)^{1/3}\left[\frac{8 n_s p(p+1)D}{\Delta}\right]^{2/3}$$

And finally we have

$$E[\Lambda[T]] \leq 5\kappa \log(\theta n_s p)\left(W_1 T\left(\frac{\theta}{\pi}\right)^{1/3}\left[\frac{8 n_s p(p+1)D}{\Delta}\right]^{2/3}\right) + \frac{16\kappa W_1 T}{\Delta^2} p(\log p)\log(\theta n_s p).$$

which concludes the proof. ■

D. One Dimensional I.I.D. Slow Mobility Model

Lemma 9: For the one dimensional i.i.d. slow mobility and the protocol model, using the directional antenna model, when the packets are directly transmitted from source to their destinations, we have

$$\begin{aligned}&E[\Lambda^d[T]]\\&\leq 5\kappa\log(\theta n_s p)\left\{(\theta/\pi)^{1/4}(p/\Delta)^{1/2}4W_1T\left[n_s(p+1)\right]^{3/4}\right\}\\&+\frac{16\kappa WT}{\Delta^2}p(\log p)\log(\theta n_s p).\end{aligned}$$

Proof: Recall that for one dimensional i.i.d. slow mobility model, lemma 5 holds, and the orbits of the mobiles are the same as the fast mobility model. By the Cauchy-Schwarz inequality, we have

$$\begin{aligned}\left(\sum_{B=1}^{B[T]}\sum_{h=1}^{H(B)}\alpha_B^h\right)^2 &\leq \left(\sum_{B=1}^{B[T]}\sum_{h=1}^{H(B)}1\right)\left(\sum_{B=1}^{B[T]}\sum_{h=1}^{H(B)}(\alpha_B^h)^2\right)\\&\leq n_s(p+1)W_1T\sum_{B=1}^{B[T]}\sum_{h=1}^{H(B)}(\alpha_B^h)^2\\&\leq n_s(p+1)W_1T\frac{16W_1T}{\theta\pi\Delta^2},\end{aligned}$$

By Lemma 5, and $\sum_{h=1}^{H(B)}\alpha_B^h \geq L(B)$, we have

$$\frac{4W_1T}{\Delta}\sqrt{\frac{n_s(p+1)}{\theta\pi}} \geq \sum_{B=1}^{B[T]}L(B). \quad (6)$$

Similar as the proof of lemma 6, we have

$$\begin{aligned}E[B[T]] =& E\left[\sum_{B=1}^{B[T]}1_{L(B)\leq L}\right]+E\left[\sum_{B=1}^{B[T]}1_{L(B)>L}\right]\\&\leq W_1E\left[\sum_{t=1}^{T}\sum_{i=1}^{n_s}1_{D(s_i,t)\leq L,\text{ covered by} s_i}\right]\\&+E\left[\sum_{B=1}^{B[T]}1_{L(B)>L}\right]\\&\leq n_s(p+1)W_1T2\theta Lp+E\left[\sum_{B=1}^{B[T]}1_{L(B)>L}\right].\end{aligned}$$

which implies

$$E[B[T]]-E\left[\sum_{B=1}^{B[T]}1_{L(B)>L}\right]\leq n_s(p+1)W_1T2\theta Lp \quad (7)$$

Then, we can conclude that

$$\begin{aligned}E\left[\frac{4W_1T}{\Delta}\sqrt{\frac{n_s(p+1)}{\theta\pi}}\right] &= \frac{4W_1T}{\Delta}\sqrt{\frac{n_s(p+1)}{\theta\pi}}\\&\geq E\left[\sum_{B=1}^{B[T]}L(B)\right]\\&\geq LE\left[\sum_{B=1}^{B[T]}1_{L(B)>L}\right]\\&\geq L(E[B[T]]-n_s(p+1)W_1T2\theta Lp),\end{aligned}$$

where the first inequality follows the Jensen's inequality. Now let $L=\frac{E[B[T]]}{2\theta n_s(p+1)W_1TpL}$, we can obtain that

$$E[B[T]]\leq\left(\frac{\theta}{\pi}\right)^{1/4}\left(\frac{p}{\Delta}\right)^{1/2}4W_1T\left[n_s(p+1)\right]^{3/4}.$$

By submitting into the bound on $\Lambda_1[T]$ in Lemma 1, we have

$$\begin{aligned}&E[\Lambda^d[T]]\\&\leq 5\kappa\log(\theta n_s p)\left\{(\theta/\pi)^{1/4}(p/\Delta)^{1/2}4W_1T\left[n_s(p+1)\right]^{3/4}\right\}\\&+\frac{16\kappa WT}{\Delta^2}p(\log p)\log(\theta n_s p).\end{aligned}$$

which concludes the proof. ∎

Lemma 10: For the 1D-i.i.d. mobility and the protocol model, using the directional antenna model, when the packets have to be transmitted from relays to their destinations, we have

$$\begin{aligned}&E[\Lambda^r[T]]\\&\leq 5\kappa\log(\theta n_s p)\left((\theta/\pi)^{1/4}(p/\Delta)^{1/2}4W_1T\left[n_s(p+1)D\right]^{3/4}\right)\\&+\frac{16\kappa WT}{\Delta^2}p(\log p)\log(\theta n_s p).\end{aligned}$$

Combine Lemma 9 and Lemma 10, we have the following theorem:

Theorem 3: Consider the one dimensional i.i.d. mobility model with fast mobility mobiles, under the conditions that every node equipped with directional antenna and the delay constraint D, we have

$$\begin{aligned}E[\Lambda[T]] \leq& 5\kappa\log(\theta n_s p)\{(\theta/\pi)^{1/4}(p/\Delta)^{1/2}4W_1T\\&\left[n_s(p+1)(D+1)\right]^{3/4}\}\\&+\frac{16\kappa WT}{\Delta^2}p(\log p)\log(\theta n_s p).\end{aligned}$$

E. Impact of Base Stations

Consider the capacity contributed by transmissions in the infrastructure mode, recall that the unit area is tessellated into m equal-sized squares of side length $\frac{1}{\sqrt{m}}$, and place one base station at the center of each squares. For the infrastructure mode transmission, the routing scheme consists tree phases: uplink phase, BS-to-BS phase and downlink phase. During the uplink phase, source nodes in each cell transmit their packets to the base station associated with the cell. Then, the base station receiving the packets from sources to base stations in the same cell of their destinations. At last, during

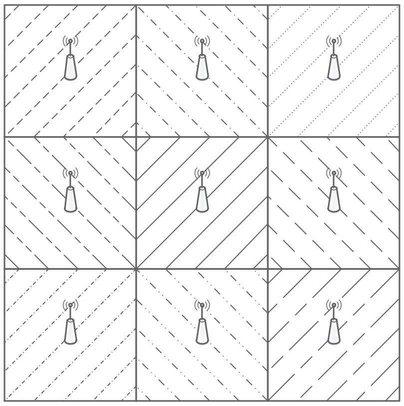

Fig. 5. Illustration of base station frequency reuse

the downlink phase, each base station broadcasts the packets to the destination nodes in their cells.

Recall that the each cell employs a frequency reuse policy the avoid mutual interference from adjacent cells. As shown in Figure 5, for the tessellation of the network, which shown that a 9-cell frequency reuse pattern can be adopted. We know that the bandwidth of uplink and downlink for infrastructure mode transmission is W_2. Thus, the throughput capacity per cell is upper bounded by W_2 and lower bounded by $\frac{1}{9}W_2$.

Theorem 4: Under the conditions of delay constraint and directional antenna, the throughput capacity of the network contributed by infrastructure mode transmissions is

$$\lambda_i(n,m) = \frac{\Lambda_i(n,m)}{n_s p} = \Theta\left(\frac{m}{n_s p}W_2\right)$$

Proof: The result is easy to illustrate by the deployment of the base stations and the definition of infrastructure mode capacity. ∎

Theorem 5: Under the conditions of delay constraint and directional antenna, the multicast throughput capacity of the VANET is

$$\lambda(n,m) = \lambda_a(n,m) + \lambda_i(n,m),$$

where $\lambda_a(n,m) = \frac{\Lambda_a(n,m)}{n_s p}$ is the multicast capacity of ad hoc node transmission and $\lambda_i(n,m) = \frac{\Lambda_i(n,m)}{n_s p}$ is the multicast capacity of infrastructure node.

Then we have

(i) for two dimensional i.i.d. fast mobility model:

$$\lambda(n,m) = O\left((\log p)(\log(\theta n_s p))\frac{1}{1-\theta/2\pi}\sqrt{\frac{D}{n_s}}\right) + \Theta\left(\frac{m}{n_s p}W_2\right)$$

(ii) for two dimensional i.i.d. slow mobility model:

$$\lambda(n,m) = O\left[(\log p)(\log(\theta n_s p))\frac{1}{(1-\theta/2)^{2/3}}\frac{D^{2/3}}{(n_s p)^{1/3}}\right] + \Theta\left(\frac{m}{n_s p}W_2\right)$$

(iii) for one dimensional i.i.d. fast mobility model:

$$\lambda(n,m) = O\left[(\log p)(\log(\theta n_s p))\left(\frac{\theta}{\pi}\right)^{1/3}\frac{[(p+1)D]^{2/3}}{(n_s p)^{1/3}}\right] + \Theta\left(\frac{m}{n_s p}W_2\right)$$

(iv) for one dimensional i.i.d. slow mobility model:

$$\lambda(n,m) = O\left[(\log p)(\log(\theta n_s p))\left(\frac{\theta}{\pi}\right)^{1/4}\frac{[(p+1)D]^{2/3}}{(n_s)^{1/4}p^{1/2}}\right] + \Theta\left(\frac{m}{n_s p}W_2\right).$$

Consider the effect of delay on the throughput capacity, we have

$$\lambda(n,m) = \begin{cases} \Theta(\sqrt[4]{\theta}) + \Theta\left(\frac{m}{n_s p}W_2\right), \\ \quad \text{if } D = \Omega\left(\frac{n_s}{(\log p)^2(\log(\theta n_s p))^2}\right); \\ \Theta\left(\frac{m}{n_s p}W_2\right), \\ \quad \text{if } D = O\left(\sqrt[3]{\frac{n_s}{(\log p)^2(\log(\theta n_s p))^2}}\right); \\ O\left((\log p)(\log(\theta n_s p))\sqrt[4]{\frac{\theta}{n_s}}D^{2/3}\right) + \Theta\left(\frac{m}{n_s p}W_2\right), \\ \quad \text{otherwise.} \end{cases}$$

Case 1: $D = \Omega\left(\frac{n_s}{(\log p)^2(\log(\theta n_s p))^2}\right)$.
We have $\lambda(n,m) = \Omega(\sqrt[4]{\theta}) + \Theta(\frac{m}{n_s p}W_2)$.

- If $m = \Omega(n_s p) = \Omega(n)$, then we can have higher throughput when $W_1 = 0$, i.e., $W_2 = W/2$, and $\lambda_{max}(n,m) = \Theta(\sqrt[4]{\theta}) + \Theta(1) = \Theta(1)$.
- If $m = o(n_s p) = o(n)$, then $\lambda(n,m) = \Theta(\sqrt[4]{\theta}) + \Theta\left(\frac{m}{n_s p}\right)$, the throughput is supported by the combination of ad hoc mode and infrastructure modes transmissions.
  - if $\theta = \Omega\left((\frac{m}{n_s p})^2\right)$, the mainly transmission is supported by the ad hoc mode, and hence, $\lambda(n,m) = \Theta(\sqrt{\theta})$, the throughput is increasing with the angle of the antenna θ.
  - if $\theta = O\left((\frac{m}{n_s p})^2\right)$, the mainly transmission is supported by the infrastructure mode, and hence, $\lambda(n,m) = \Theta\left(\frac{m}{n_s p}\right)$, which means the throughput tends to zero as the number of nodes n tends to infinity.

Case 2: $D = O\left(\sqrt[3]{\frac{n_s}{(\log p)^2(\log n_s p)^2}}\right)$.

We have $\lambda(n,m) = 0 + \Theta(\frac{m}{n_s p}W_2)$, it is easy to verify that under the delay constraint D, the information can be transmitted in the ad hoc mode is less than one bit, so the capacity is zero in this case. Thus, the throughput is mainly supported by the infrastructure mode.

- If $m = \Omega(n_s p) = \Omega(n)$, then we can have higher throughput when $W_1 = 0$, i.e., $W_2 = W/2$, and $\lambda_{n,m} = \Theta(1)$, which means although the tolerant delay is small, if the number of base stations is large enough, the throughput can also scale.
- If $m = O(n_s p) = O(n)$, then $\lambda(n,m) = 0$, which means that when the tolerant delay D and the number

TABLE II
MULTICAST THROUGHPUT CAPACITY IN HYBRID VANET WITH DIRECTIONAL ANTENNA AND DELAY CONSTRAINT

D	m	θ	Multicast Capacity
$\Omega\left(\frac{n_s}{(\log p)^2(\log(\theta n_s p))^2}\right)$	$\Omega(n_s p)$	$\in(0,2\pi)$	$\Theta(W_2)$
	$o(n_s p)$	$\Omega((\frac{m}{n_s p})^2)$	$\Theta(\sqrt[4]{\theta}) \to \Theta(W_1)$
		$O((\frac{m}{n_s p})^2)$	$\Theta(\frac{m}{n_s p}W_2)$
$O\left(\sqrt[3]{\frac{n_s}{(\log p)^2(\log n_s p)^2}}\right)$	$\Omega(n_s p)$	$\in(0,2\pi)$	$\Theta(W_2)$
	$o(n_s p)$	$\in(0,2\pi)$	$\to 0$
$\Omega\left(\frac{n_s}{(\log p)^2(\log(\theta n_s p))^2}\right)$ & $O\left(\sqrt[3]{\frac{n_s}{(\log p)^2(\log n_s p)^2}}\right)$	$(\Omega\left(p\log p\log(\theta n_s p)\sqrt{\theta n_s D}\right), O(n_s p))$	$\in(0,2\pi)$	$\Theta(\frac{m}{n_s p}W_2)$
	$\Omega(n_s p)$	$\in(0,2\pi)$	$\Theta(W_2)$
	$o\left(p\sqrt{\theta n_s D}\right)$	$\in(0,2\pi)$	$O\left((\log p)(\log(\theta n_s p))\sqrt[4]{\frac{\theta}{n_s}}D^{2/3}\right)$

of base stations m are both small, the throughput will tend to zero as the number of nodes n tends to infinity. Directional antenna has little effect on the total hybrid network capacity.

Case 3: $D = \Omega\left(\frac{n_s}{(\log p)^2(\log(\theta n_s p))^2}\right)$ & $D = O\left(\sqrt[3]{\frac{n_s}{(\log p)^2(\log n_s p)^2}}\right)$.

In this case, the tolerant delay is neither too large nor too small. We have

$$\lambda(n,m) = O\left((\log p)(\log(\theta n_s p))\sqrt[4]{\frac{\theta}{n_s}}D^{2/3}\right) + \Theta\left(\frac{m}{n_s p}W_2\right).$$

- If $m = \Omega\left((\log p)(\log(\theta n_s p))\sqrt[4]{\theta}\sqrt{n_s}D^{2/3}\right)$, then we can have higher throughput when $W_1 = 0$, i.e., $W_2 = W/2$, and $\lambda(n,m) = \Theta\left(\frac{m}{n_s p}W_2\right)$. In this case, the number of base stations is function of delay D and directional antenna θ, the throughput is dominated by the delay D, directional antenna can help to adjust the throughput and can not change the main trend. The capacity also tends to zero in a more flat way.
- If $m = O\left(p\sqrt[4]{\theta}\sqrt{n_s}D^{2/3}\right)$, then $\lambda(n,m) = O\left((\log p)(\log(\theta n_s p))\sqrt[4]{\frac{\theta}{n_s}}D^{2/3}\right)$, which shown that the capacity can not scale.

In summary, the number of base stations play an important role in the multicast throughput capacity for two dimensional i.i.d. fast mobility model. The cooperation of a directional antenna beamwidth θ and a tolerant delay D can also improve the throughput capacity if m is small. As shown in Table II, the network throughput capacity can achieve $\Theta(\sqrt[4]{\theta})$ when $m = \Omega(n_s p)$, which can tends to $\Theta(1)$ when the antenna beamwidth θ becomes 2π. We can improve the multicast throughout capacity by increasing the tolerant delay D and decreasing the directional antenna beamwidth θ if the number of base stations is small. But if the number of base stations and the maximum-hops are both small, the directional antenna can not provide multicast throughput capacity improvement. When the number of base stations is small and the tolerant delay D is moderately large, i.e., $\Omega\left(\frac{n_s}{(\log p)^2(\log(\theta n_s p))^2}\right)$ & $O\left(\sqrt[3]{\frac{n_s}{(\log p)^2(\log n_s p)^2}}\right)$, the directional antenna beamwidth θ can improve the multicast throughput, however, the throughput still tends to zero in a more flat way.

VI. Lower Bound Analysis

The achievable lower bound for the hybrid VANET with two mobility models and two mobility time scales can be given by the modified joint coding-scheduling algorithms which are introduce in [10]. In order to reduce the redundancy of similar proof, we just give a example analysis of the two dimensional i.i.d. fast mobility model.

1) Case 1: $n_s = \Theta(1)$: When $n_s = \Theta(1)$, we let the sources broadcast their packet to all the mobiles in the network in a round robin fashion. Under this simple algorithm, both throughput per multicast session and delay are $\Theta(1)$.

2) Case 2: $n_s = \Omega(1)$: In this case, using similar Joint Raptor Coding Scheduling Algorithm as in[10], we have the flowing theorem:

Theorem 3: Assume the delay constraint D is both $\Omega(\sqrt[3]{\theta n_s}\log\theta n_s p)$ and $O(\theta n_s)$. For sufficiently large n_s, at the end of each super time slot with length $2D$, every source successfully transmit $\Theta\left(D\sqrt[4]{\frac{\theta}{n_s}}D^{2/3}\right)$ packets to all p destinations with high probability.

Lemma 6: The throughput per multicast session is

$$\Theta\left(D\sqrt[4]{\frac{\theta}{n_s}}D^{2/3}\right) \times \frac{1}{2D} = \Theta\left(\sqrt[4]{\frac{\theta}{n_s}}D^{2/3}\right).$$

VII. Conclusion

In this paper, we have studied the multicast throughput capacity in hybrid wireless VANETs with delay constraits and under the condition that each node is equipped with a directional antenna. We have investigated two mobility models of the wireless VANET, and two mobile time scales, which can catch the main character of VANETs. We have proved that with a delay tolerant threshold D, the per-session multicast throughput capacity in hybrid VANETs greatly depends on the maximum tolerant delay D, the number of base stations m and the beamwidth of directional antenna θ.

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APPENDIX A
PROOF OF LEMMA 1

In the multicast scenario, we know that there are p destinations in each multicast session. The probability that a mobile is within a distance of γ from node j is $\pi\gamma^2$. Thus, from the definition of geometry probability theory, we know that $H(j,\gamma,t,\theta)$ is a binomial random variable with $p-1$ trials and probability of a success $\theta\pi\gamma^2$, and then we have

$$E[H(j,\gamma,t,\theta)] = \theta(p-1)\pi\gamma^2. \tag{8}$$

By the Chernoff bound, we have

$$\begin{aligned} & Pr(H(j,\gamma,t,\theta) > \kappa(1+\theta p\gamma^2)\log(\theta n_s p)) \\ & \leq \left(\frac{e^\delta}{(1+\delta)^{1+\delta}}\right)^{\theta(p-1)\pi\gamma^2} \\ & \leq \left(\frac{e}{1+\delta}\right)^{\theta(p-1)\pi\gamma^2(1+\delta)} \\ & \leq \left(\frac{e}{1+\delta}\right)^{\kappa(1+\theta p\gamma^2)\log(\theta n_s p)} \\ & \leq \left(\frac{e}{\frac{\kappa(1+\theta p\gamma^2)\log(\theta n_s p)}{\theta(p-1)\pi\gamma^2}}\right)^{\kappa(1+\theta p\gamma^2)\log(\theta n_s p)} \\ & \leq e^{-\kappa(1+\theta p\gamma^2)\log(\theta n_s p)} \leq e^{\kappa\log(\theta n_s p)}, \end{aligned} \tag{9}$$

where

$$\delta = \frac{\kappa(1+\theta p\gamma^2)\log(\theta n_s p)}{\theta(p-1)\pi\gamma^2} - 1 \tag{10}$$

and $\delta > 0$. In fact, we can choose κ such that

$$\kappa(1+\theta p\gamma^2)\log(\theta n_s p) > \theta(p-1)\pi\gamma^2 \tag{11}$$

thus, $\frac{\theta(p-1)\pi\gamma^2}{\kappa(1+\theta p\gamma^2)\log(\theta n_s p)} < 1$ for $n_s p > 3$. Finally, we can conclude that there exists $\kappa > 0$, which is independent of n_s, p and θ, such that

$$\begin{aligned} E[Z_{\gamma,\kappa,\theta}[T]] & \leq E\left[\sum_{j\,:\,j \text{ is a destination}} \sum_{t=1}^{T} 1_{H(j,\gamma,t,\theta)\geq\kappa\gamma^2\log(\theta n_s p)}\right] \\ & = \sum_{t=1}^{T}\sum_{j} E[1_{H(j,\gamma,t,\theta)\leq\kappa\gamma^2\log(\theta n_s p)}] \\ & \leq n_s p T e^{-\kappa\log(\theta n_s p)} \\ & \leq \frac{T}{(\theta n_s p)^2}, \end{aligned} \tag{12}$$

where the last inequality holds for any $\kappa > 2$.

Appendix B
Proof of Lemma 2

First, we present some important inequalities that will be used in the follow analysis. Recall that $H(j, \gamma, t, \theta)$ denote the number of destinations that belong to the same multicast session as node j (destination node) and their distance between j is less than γ at time t. We also denote $\mathcal{R}[T]$ the number of bits that are carried by the mobile nodes other than their sources at time T, and α_B the transmission radius used to deliver bit B. The following inequalities hold for multicast scenarios when the nodes are equipped with directional antenna, which can be obtained by modifying the lemma in [21].

$$E\left(\sum_{t=1}^{T}\sum_{j} 1_{H(j,\gamma,t,\theta)\geq\kappa(1+\theta p\gamma^2)\log(\theta n_s p)}\right) \leq \frac{T}{(\theta n_s p)^2}$$

$$\Lambda[T] \leq n_s p W_1 T, |\mathcal{R}[T]| \leq n_s(p+1)W_1 T$$

$$\sum_{B=1}^{B[T]} \frac{\Delta^2}{16}(\alpha_B)^2 \leq \frac{W_1 T}{\theta\pi}$$

where $|\mathcal{R}[T]|$ is the cardinality of set $\mathcal{R}[T]$.

Then, we begin the proof of Lemma 2. Let B the number of target deliveries, and β_B denote the number of deliveries associated with target delivery B. We divide the target deliveries into classes according to α_B. We say a target delivery belonging to class (γ, m) if $2^{m-1} \leq \alpha_B < 2^m\gamma$, where $\gamma \in [0, 1]$. Then, we have

$$E[\Lambda[T]] \leq E\left[\sum_{B=1}^{B[T]} \beta_B 1_{\alpha_B<\gamma}\right] + \sum_{m=1}^{\lceil -\log_2\gamma\rceil} E\left[\sum_{B=1}^{B[T]} \beta_B 1_{2^{m-1}\gamma\leq\alpha_B<2^m\gamma}\right]$$

Note that $\beta_B > \kappa(1 + 4\theta p\gamma^2)\log(\theta n_s p)$ implies that

$$H(d_B, 2\gamma, t, \theta) \leq \kappa(1 + 4\theta p\gamma^2)\log(\theta n_s p) \tag{13}$$

where d_B is the destination receiving the target delivery, so we have

$$\begin{aligned}&\beta_B 1_{\alpha_B<\gamma}\\ &\leq \kappa(1+4\theta p\gamma^2)\log(\theta n_s p) 1^{\alpha_B<\gamma}_{H(d_B,2\gamma,t,\theta)\geq\kappa(1+4\theta p\gamma^2)\log(\theta n_s p)}\\ &+ \beta_B 1^{\alpha_B<\gamma}_{H(d_B,2\gamma,t,\theta)\geq\kappa(1+4\theta p\gamma^2)\log(\theta n_s p)}\end{aligned} \tag{14}$$

Thus, we can then have

$$\begin{aligned}&E\left[\sum_{b=1}^{B[T]} \beta_B 1^{\alpha_B<\gamma}_{H(d_B,2\gamma,t,\theta)\geq\kappa(1+4\theta p\gamma^2)\log(\theta n_s p)}\right]\\ &+ \sum_{m=1}^{\lceil -\log_2\gamma\rceil} E\left[\sum_{B=1}^{B[T]} \beta_B 1^{2^{m-1}\gamma\leq\alpha_B<2^m\gamma}_{H(d_B,2\gamma,t,\theta)\geq\kappa(1+4\theta^2 p\gamma^2 2^{2m+2})\log(\theta n_s p)}\right]\\ &\leq \frac{pW_1T}{\theta^2 n_s^2 p^2} = \frac{W_1 T}{n_s^2 p}\end{aligned} \tag{15}$$

Furthermore, for any $0 < \gamma < 1$, we have

$$\begin{aligned}&E[\Lambda[T]]\\ &\leq \frac{W_1T}{\theta^2 n_s^2 p} + \kappa\log(\theta n_s p)((1+4\theta^2 p\gamma^2)E\left[\sum_{B=1}^{B[T]} 1_{\alpha_B<\gamma}\right]\\ &+ \sum_{m=1}^{\lceil -\log_2\gamma\rceil}(1+\theta^2 p 2^{2m+2}\gamma^2)E\left[\sum_{B=1}^{B[T]} 1_{2^{m-1}\gamma\leq\alpha_B<2^m\gamma}\right]\\ &\leq \frac{W_1T}{\theta^2 n_s^2 p} + \kappa\log(\theta n_s p)(1+4\theta^2 p\gamma^2)E[B[T]]+\\ &\kappa\log(\theta n_s p)\sum_{m=1}^{\lceil -\log_2\gamma\rceil}\theta p 2^{2m+2}\gamma^2 \frac{W_1T}{\theta\pi\frac{2^{2m-2}\Delta^2\gamma^2}{4}}\\ &\leq \frac{W_1T}{\theta^2 n_s^2 p} + \kappa\log(\theta n_s p)(1+4\theta^2 p\gamma^2)E[B[T]]+\\ &\frac{64\kappa W_1 T}{\pi\Delta^2\log_2}(-\log\gamma)p\log(\theta n_s p)\\ &\leq 5\kappa\log(\theta n_s p)E[B[T]] + \frac{\kappa W_1 T}{\Delta^2}p(\log p)\log(\theta n_s p),\end{aligned} \tag{16}$$

which concludes the proof of Lemma 2.

Appendix C
Directional Exclusion Region

The two transmissions can succeed simultaneously if the distance between node X_j and node X_k is larger than $(1+\Delta)r_k$ and the distance between node X_i and node X_l is larger than $(1+\Delta)r_i$.

From the receiving node point of view, if every node is equipped with an Omni-directional antenna, and that node X_i transmits successfully to node X_j and node X_k transmit successfully to node X_l at the same time, then from the triangle inequality, as shown in Figure 4, we have,

$$\begin{aligned}|X_j - X_l| &\geq |X_j - X_k| - |X_k - X_l|\\ &\geq (1+\Delta)|X_i - X_j| - |X_k - X_l|.\end{aligned}$$

Similarly,

$$\begin{aligned}|X_l - X_j| &\geq |X_l - X_i| - |X_i - X_j|\\ &\geq (1+\Delta)|X_k - X_l| - |X_i - X_j|.\end{aligned}$$

Adding the two inequalities, we obtain

$$|X_l - X_j| \geq \frac{\Delta}{2}(|X_k - X_l| + |X_i - X_j|).$$

Because of $|X_k - X_l| \leq r(n)$ and $|X_i - X_j| \leq r(n)$, we have

$$\frac{\Delta}{2}(|X_k - X_l| + |X_i - X_j|) \leq \Delta r(n).$$

For the directional antenna mode, the interfering area and exclusion regions become small because of the beamwidth θ. That is, disks of radius $\Delta/2$ times the transmission range centered at the receivers are essentially disjoint.