On the Performance of Successive Interference Cancellation in D2D-enabled Cellular Networks

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Apr. 28, 2015

Outline

1. Introduction
2. System model
3. Network performance without SIC
4. Network performance with SIC
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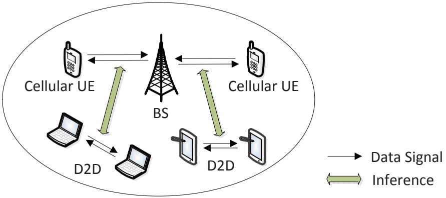
Device-to-Device (D2D) communication

D2D Communication: enabling direct communication between users that are in proximity.

- Gains: increase capacity, extend coverage, facilitate new types of proximity services, ...
- Challenges: device and service discovery, mode selection, interference management, ...



Interference management in D2D-enabled cellular networks

Existing works:

- Interference avoidance: orthogonal resource allocation schemes
- Interference coordination: intelligent scheduling schemes
- Interference cancellation: advanced signal processing techniques, e.g., superposition coding, rate splitting, dirty paper coding, beamforming, ...

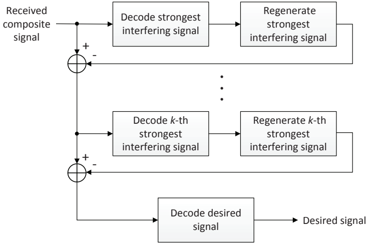
Our work:

- Interference cancellation: successive interference cancellation (SIC)
- Study how SIC affects the performance of large-scale D2D-enabled cellular networks
- Tools: stochastic geometry

Successive interference cancellation (SIC)

Basic concept:

- Regenerate the interfering signals and subsequently cancel them from the received composite signal
- Improve the SIR of the desired signal



Key advantages:

- SIC receiver is architecturally similar to non-SIC receivers
- Achieve Shannon capacity region boundaries for BC & MAC

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System model

Cellular link: uplink, nearest-BS association

- Cellular users: PPP, $\Phi_c(\lambda_c)$, P_c
- Base stations: Φ_b

D2D link:

- D2D Txs: PPP, $\Phi_d(\lambda_d)$, P_d
- D2D Rxs: distance l (Rayleigh distribution)

Channel:

- Path loss: path loss exponent $\alpha > 2$ $\left(\delta = \frac{2}{\alpha}\right)$
- Fading: Rayleigh fading, $h \sim \exp(1)$

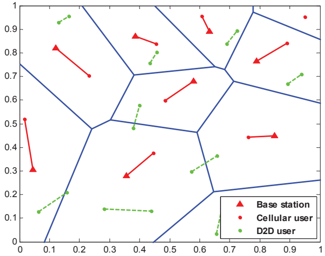

Figure: Network model

Outline

Successful transmission probability of cellular links

Conduct the analysis on a typical cellular link:

- Typical BS: located at the origin
- Associated cellular user: located at a random distance r away

$$f_r(r) = 2\pi\lambda_c r e^{-\pi\lambda_c r^2} \quad \text{(nearest-BS association)}$$

- Received SIR:

$$\mathrm{SIR}_c = \frac{P_c g_0 r^{-\alpha}}{\sum_{x_i \in \Phi_c \setminus \{x_0\}} P_c g_i \left\|x_i\right\|^{-\alpha} + \sum_{y_i \in \Phi_d} P_d h_i \left\|y_i\right\|^{-\alpha}}$$

- Successful transmission probability: (T is SIR threshold)

$$p_c \triangleq \mathbb{P}\left[\mathrm{SIR}_c > T\right]$$

Contd...

Theorem 1

The successful transmission probability of cellular links without SIC capability is

$$p_c = \frac{\lambda_c}{\lambda_c\left(\mu+1\right)+\lambda_d\left(\frac{P_d}{P_c}\right)^{\delta}\nu},$$

where

$$\mu = \frac{\delta}{1-\delta}T\cdot {}_2F_1\left(1,1-\delta;2-\delta;-T\right),$$
$$\nu = T^{\delta}\Gamma\left(1-\delta\right)\Gamma\left(1+\delta\right),$$

and ${}_2F_1\left(\cdot\right),\Gamma\left(\cdot\right)$ are respectively the Hypergeometric function and Gamma function.

Successful transmission probability of D2D links

Conduct the analysis on a typical D2D link:

- Typical D2D transmitter: located at the origin
- Associated D2D receiver: located at a random distance l away

$$f_l(l) = 2\pi\lambda_d l e^{-\pi\lambda_d l^2}$$

- Received SIR:

$$\mathrm{SIR}_d = \frac{P_d h_0 l^{-\alpha}}{\sum_{y_i\in\Phi_d\setminus\{y_0\}} P_d h_i \|y_i\|^{-\alpha} + \sum_{x_i\in\Phi_c} P_c g_i \|x_i\|^{-\alpha}}$$

- Successful transmission probability: (T is SIR threshold)

$$p_d \triangleq \mathbb{P}[\mathrm{SIR}_d > T]$$

Contd...

Theorem 2

The successful transmission probability of D2 links without SIC capability is

$$p_d = \frac{\lambda_d}{\lambda_d (\nu + 1) + \lambda_c \left(\frac{P_c}{P_d}\right)^\delta \nu},$$

where

$$\nu = T^\delta \Gamma(1-\delta)\Gamma(1+\delta).$$

Remark. In Theorem 1 and 2,

- μ, ν represent the effect of the interference from the cellular links and D2D links respectively
- $(P_d/P_c)^\delta$, $(P_c/P_d)^\delta$ can be regarded as the conversion factors of powers

Stochastic equivalence of interference

Motivation:

- Trivial to analyze the two-tier (cellular-tier and D2D-tier) interference
- Difficult to obtain the analytical results of $p_c^{\mathrm{SIC}}, p_d^{\mathrm{SIC}}$ based on the two-tier interference model

Method:

- Equate the two-tier interference by a single-tier interference
- Have the same stochastic characteristics in terms of successful transmission probability

Contd...

Interferers for the typical cellular link:

- Original:

$$\Phi_{c-intf} = (\Phi_c \setminus \{x_0\}) \bigcup \Phi_d$$

- Equivalence:

$$\Phi^{\mathrm{eq}}_{c-intf} \setminus \{x_0\}$$

with density $\lambda^{\mathrm{eq}}_{c-intf}$ and transmission power P_c

Lemma 1

The density of the equivalent interferers for cellular links is

$$\lambda^{\mathrm{eq}}_{c-intf} = \lambda_c + \lambda_d \left(\frac{P_d}{P_c}\right)^{\delta} \frac{\nu}{\mu}.$$

Contd...

Interferers for the typical D2D link:

- Original:

$$\Phi_{d-intf} = (\Phi_d \setminus \{y_0\}) \bigcup \Phi_c$$

- Equivalence:

$$\Phi^{\mathrm{eq}}_{d-intf} \setminus \{y_0\}$$

density $\lambda^{\mathrm{eq}}_{d-intf}$ and transmission power P_d

Lemma 2

The density of the equivalent interferers for D2D links is

$$\lambda^{\mathrm{eq}}_{d-intf} = \lambda_d + \lambda_c \left(\frac{P_c}{P_d}\right)^{\delta}.$$

Numerical results

The system parameters are set as $\alpha = 4$, $P_c = P_d = 1$, $\lambda_c = 0.01$.

Result 1 (p_d, p_d v.s. T): the analytical results of the proposed stochastic equivalence models are in quite good agreement with the corresponding simulation results.

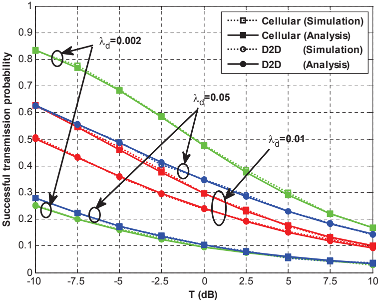

Figure: Successful transmission probabilities without IC

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Successful transmission probability of cellular links

Conduct the analysis on a typical cellular link:

- Typical BS: located at the origin
- Typical cellular user: located at a distance r away (Rayleigh)
- Equivalent interferers: ordered by received power at the typical BS such that $P_c g_i \|x_i\|^{-\alpha} > P_c g_j \|x_j\|^{-\alpha}, \forall\, i < j$
- Successful transmission probability:

$$p_c^{\mathrm{SIC}} = p_c + \sum_{n=1}^{\infty} \left(\prod_{i=1}^{n} p_{c-intf}^{(i)} \right) \left(\prod_{i=0}^{n-1} \left(1 - p_c^{(i)}\right) \right) p_c^{(n)}$$

$p_c^{(n)}$: the successful transmission probability of the typical cellular link given that n strongest equivalent interferers are canceled

$p_{c-intf}^{(n)}$: the probability of canceling (decoding) n-th strongest equivalent interferer given that all $n-1$ stronger equivalent interferers are canceled

Contd...

(1) Given that n strongest equivalent interferers are canceled,

- Received SIR at the typical BS:

$$\mathrm{SIR}_c^{(n)} = \frac{P_c g_0 r^{-\alpha}}{\sum_{x_i \in \Phi_{c-intf}^{\mathrm{eq}} \backslash \{x_0, x_1, \ldots x_n\}} P_c g_i \left\| x_i \right\|^{-\alpha}}$$

- Successful transmission probability of the typical cellular link:

$$p_c^{(n)} \triangleq \mathbb{P}\left[\mathrm{SIR}_c^{(n)} > T\right]$$

Lemma 3

$$p_c^{(n)} = \int_0^\infty \int_0^\infty \frac{2\left(\lambda_{c-intf}^{\mathrm{eq}} \pi d_n^2\right)^n}{d_n \Gamma(n)} e^{-\lambda_{c-intf}^{\mathrm{eq}} \pi \xi(r, d_n)} f_r(r) \, \mathrm{d}d_n \mathrm{d}r,$$

where $\xi(r, d_n) = \frac{\delta}{1-\delta} T r^\alpha d_n^{2-\alpha} {}_2F_1\left(1, 1-\delta; 2-\delta; -\frac{T r^\alpha}{d_n^\alpha}\right) + d_n^2$.

Contd...

(2) Given that all $n-1$ stronger equivalent interferers are canceled,

- Received SIR of n-th strongest interferer at the typical BS:

$$\mathrm{SIR}_{c-intf}^{(n)} = \frac{P_c g_n \left\|x_n\right\|^{-\alpha}}{\sum_{x_i \in \Phi_{c-intf}^{\mathrm{eq}} \setminus \{x_n, x_1, \ldots x_{n-1}\}} P_c g_i \left\|x_i\right\|^{-\alpha}}$$

- Probability of canceling n-th strongest equivalent interferer:

$$p_{c-intf}^{(n)} \triangleq \mathbb{P}\left[\mathrm{SIR}_{c-intf}^{(n)} > T\right]$$

Lemma 4

$$p_{c-intf}^{(n)} = \frac{1}{(\mu+1)^n}.$$

Contd...

Thoerem 3

The successful transmission probability of cellular links with infinite SIC capability is

$$p_c^{\mathrm{SIC}} = p_c + \sum_{n=1}^{\infty} \left(\prod_{i=1}^{n} p_{c-intf}^{(i)} \right) \left(\prod_{i=0}^{n-1} \left(1 - p_c^{(i)}\right) \right) p_c^{(n)}.$$

Proof.

The event of successful transmission with n-level SIC:

$$E_0 : \mathrm{SIR}_c^{(0)} > T,$$

$$E_n : \left(\bigcap_{i=1}^{n} \mathrm{SIR}_{c-intf}^{(i)} > T \right) \cap \left(\bigcap_{i=0}^{n-1} \mathrm{SIR}_c^{(i)} < T \right) \cap \left(\mathrm{SIR}_c^{(n)} > T \right).$$

Therefore, $p_c^{\mathrm{SIC}} = \sum_{n=0}^{\infty} \mathbb{P}[E_n]$. □

Successful transmission probability of D2D links

Lemma 5

Given that n strongest equivalent interferers are canceled, the successful transmission probability of the typical D2D link is

$$p_d^{(n)} = \int_0^\infty \int_0^\infty \frac{2\left(\lambda_{d-intf}^{\mathrm{eq}} \pi k_n^2\right)^n}{k_n \Gamma(n)} e^{-\lambda_{d-intf}^{\mathrm{eq}} \pi \kappa(l, k_n)} f_l(l)\, \mathrm{d}k_n \mathrm{d}l,$$

where $\kappa(l, k_n) = \frac{\delta}{1-\delta} T l^\alpha k_n^{2-\alpha} {}_2F_1\left(1, 1-\delta; 2-\delta; -\frac{T l^\alpha}{k_n^\alpha}\right) + k_n^2$.

Lemma 6

Given that all $n-1$ stronger equivalent interferers are canceled, the probability of canceling n-th strongest equivalent interferer for the typical D2D link is

$$p_{d-intf}^{(n)} = \frac{1}{(\mu+1)^n}.$$

Contd...

Thoerem 4

The successful transmission probability of D2D links with infinite SIC capability is

$$p_d^{\mathrm{SIC}} = p_d + \sum_{n=1}^{\infty} \left(\prod_{i=1}^{n} p_{d-intf}^{(i)} \right) \left(\prod_{i=0}^{n-1} \left(1 - p_d^{(i)} \right) \right) p_d^{(n)}.$$

Corollary 1

The successful transmission probability of D2D links with (finite) M-level ($M \geq 1$) SIC capability is

$$p_d^{M-\mathrm{SIC}} = p_d + \sum_{n=1}^{M} \left(\prod_{i=1}^{n} p_{d-intf}^{(i)} \right) \left(\prod_{i=0}^{n-1} \left(1 - p_d^{(i)} \right) \right) p_d^{(n)}.$$

Discussions

The approach of stochastic equivalence:

Cons:

- The stochastic equivalence models are obtained based on the non-SIC scenario.
- When applied to the SIC scenario, these models can only produce approximated values of the interference.

Pros:

- This approach provides analytical expressions of the interference in SIC networks $\rightarrow$ facilitating performance evaluation of SIC (and design of corresponding scheduling schemes).
- Numerical results show that the gaps between the analytical and simulation results are small $\rightarrow$ good approximation.

Numerical results

Result 1 ($p_c^{\mathrm{SIC}}, p_d^{\mathrm{SIC}}$ v.s. T):

- The analytical results can be regarded as lower bounds on p^{SIC}
- 2-level SIC can improve p^{SIC} by almost 50%
- When the SIC level > 2, SIC cannot further improve p^{SIC}

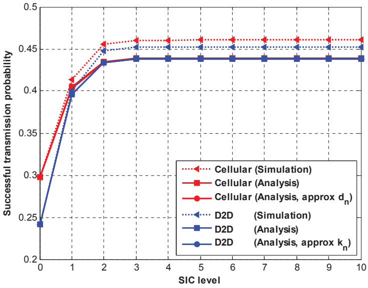

Figure: Successful transmission probabilities with SIC

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Result 2 (p_c^{SIC} v.s. λ_d):

- Increasing λ_d leads to a decrease in p_c^{SIC}
- SIC can compensate part of the performance loss of p_c^{SIC}:
  1-level SIC can compensate $\lambda_d = 0.0035$ (point A), and
  2-level SIC can compensate $\lambda_d = 0.0045$ (point B)

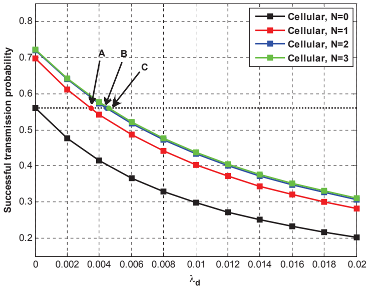

Figure: Successful transmission probability of cellular links with SIC

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Conclusions

- Study the performance of SIC in large-scale D2D-enabled cellular networks using the tools from stochastic geometry.
- Derive the successful transmission probabilities of the network without SIC as the baseline results.
- Propose the approach of stochastic equivalence of the interference.
- Derive the successful transmission probabilities of cellular and D2D links with infinite and finite SIC capabilities based on the stochastic equivalence models.
- Validate the SIC gains by analytical and numerical results.

Thank You !

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