On the Performance of Interference Cancellation in D2D-enabled Cellular Networks

Chuan Ma, Weijie Wu, Ying Cui, Xinbing Wang

Abstract—Device-to-device (D2D) communication underlaying cellular networks is a promising technology to improve network resource utilization. In D2D-enabled cellular networks, the interference among spectrum-sharing links is more severer than that in traditional cellular networks, which motivates the adoption of interference cancellation (IC) techniques at the receivers. However, to date, how IC can affect the performance of D2D-enabled cellular networks is still unknown. In this paper, we present an analytical framework for studying the performance of two IC methods, unconditional interference cancellation (UIC) and successive interference cancellation (SIC), in large-scale D2D-enabled cellular networks using the tools from stochastic geometry. To facilitate the interference analysis, we propose the approach of stochastic equivalence of the interference, which converts the two-tier interference (interference from the cellular tier and D2D tier) to an equivalent single-tier interference. Based on the proposed stochastic equivalence models, we derive the general expressions for the successful transmission probabilities of both cellular uplinks and D2D links in the networks where UIC and SIC are respectively applied. We demonstrate how these IC methods affect the network performance by both analytical and numerical results.

Index Terms—D2D communication, cellular network, interference cancellation, stochastic equivalence, stochastic geometry.

I. INTRODUCTION

Recently, there has been a rapid increase in the demand of local area services and proximity services (ProSe) among the highly-capable user equipments (UEs) in cellular networks. In this context, a new technology called device-to-device (D2D) communication, which enables direct communication between UEs that are in proximity, has been proposed and has strongly appealed to both academia [2], [3] and industry [4], [5]. The integration of D2D communication to cellular networks holds the promise of many types of advantages [3]: allowing for high-rate low-delay low-power transmission for proximity services, increasing frequency reuse factor and network capacity, facilitating new types of peer-to-peer services, etc.

However, the introduction of D2D communication also brings a number of technical challenges, such as peer device discovery, mode selection and interference management. Interference management is a major issue in D2D-enabled cellular networks, since D2D links share the same spectrum resource with regular cellular links and the interference among the spectrum-sharing links severely hampers the performance of the network. To guarantee reliable communications in D2D-enabled cellular networks, extensive research has been undertaken on the design of effective interference management schemes. Most proposed schemes can be classified into three categories: (1) Interference avoidance: orthogonal time-frequency resource allocation schemes are adopted to avoid interference between D2D and cellular links [6]; (2) Interference coordination: intelligent power control and link scheduling schemes are employed to mitigate the interference between D2D and cellular links [7]–[9]; and (3) Interference cancellation: advanced signal processing techniques are applied at cellular and/or D2D links to cancel interfering signals [10], [11]. In this paper, we focus on the topic of interference cancellation in D2D-enabled cellular networks.

Interference cancellation (IC) is regarded as a promising technique to reduce interference and improve network capacity. In interference cancellation techniques, the interfering signals can be regenerated and subsequently canceled from the desired signal [12]. In this paper, we focus on the performance of two interference cancellation methods. One is unconditional interference cancellation (UIC), which is a simplified IC method with the assumption that the interference from the strong interferers whose received powers are greater than a certain threshold can be completely (unconditionally) canceled. The other is successive interference cancellation (SIC), which is one of the best known IC techniques. The key advantage of SIC compared to other IC techniques is that the SIC receiver is architecturally similar to traditional non-SIC receivers in terms of hardware complexity and cost [13], as it uses the same decoder to decode the composite signal at different stages and neither complicated decoders nor multiple antennas are required. It is also known that SIC can achieve the Shannon capacity region boundaries for both the broadcast and multiple access networks. As such, SIC has been widely studied and recently implemented in commercial wireless systems such as IEEE 802.15.4.

A. Contributions and Organization

To date, most analytical results on IC are for ad hoc and cellular networks. It is still unknown how IC can improve the performance of large-scale D2D-enabled cellular networks, in which the interference among spectrum-sharing links is more severer than that in traditional ad hoc and cellular networks. In this paper, we present an analytical framework via stochastic geometry to quantify the benefit of IC in large-scale D2D-enabled cellular networks. The main contributions of this paper are summarized as follows.

1) We first provide an analytical framework to model a large-scale D2D-enabled cellular network without IC capa-
bilities, and derive the general expressions for the successful transmission probabilities of cellular uplinks and D2D links. The derived results are the baseline for evaluating the performances of IC methods.

(2) To simplify the interference analysis in the network, we propose the approach of stochastic equivalence of the interference. By this approach, the two-tier interference (interference from the cellular tier and D2D tier) can be represented by an equivalent single-tier interference that maintains the same stochastic characteristics as the two-tier interference.

(3) Based on the stochastic equivalence models, we derive the general expressions for the successful transmission probabilities of cellular uplinks and D2D links in the networks where UIC and SIC are respectively applied. We demonstrate the effect of these two IC methods by both analytical and numerical results.

The rest of this paper is organized as follows. Section II describes the system model. Sections III and IV respectively analyze the network performances with UIC and SIC capabilities and present the numerical results. Section VI concludes the paper. A summary of the notations used in this paper is given in Table I.

B. Related Work

Interference cancellation for wireless networks. Very recently, there is a growing interest to exploit IC, especially SIC, at the physical layer to improve network performances at upper layers. In [10], Min et al. designed an interference cancellation scheme that exploits a retransmission of the interference from the base station to reduce the outage probability. In [14], Gelal et al. proposed a topology control framework for exploiting the benefits of multi-packet reception using IC. In [15], Jiang et al. combines interference cancellation and interference avoidance to improve the throughput of a multi-hop network. In [16], Xu et al. developed a decentralized power allocation scheme to achieve the maximum throughput for random access systems with SIC receivers. In [17], Lv et al. presented optimal link scheduling schemes for ad hoc networks with SIC capabilities. In [18], Mollanoori and Ghaderi derived the optimal decoding order of the concurrent transmissions in networks supporting SIC. These works do not take into account the spatial distribution of users and the results apply to fixed networks (or a snapshot of networks). However, in this paper, we consider a stochastic network and analyze the IC performance in such a network using tools from stochastic geometry.

Stochastic geometry for wireless networks. As a mathematical tool to study random spatial patterns, stochastic geometry can be used to model and analyze the interference, connectivity and coverage in large-scale wireless networks [19]. Most of the literature in the area of modeling networks via stochastic geometry focus on ad hoc [20], [21] and cellular [22], [23] networks. Recently, stochastic geometry has also been employed to model D2D-enabled cellular networks [24]–[26]. In [24]–[26], the cellular and D2D networks were modeled by independent PPPs, and their SINR distributions were derived without considering interference cancellation techniques. The stochastic geometry-based analysis of IC has been presented in literature [27]–[31]. In [27], [28], simplified IC models were given by assuming that all signals from transmitters within a specific radius can all be completely canceled. Exact SIC models were investigated for ad hoc network in [29], [30] and for cellular networks in [31]. Different from [27]–[31], in this paper, we focus on the analysis of the effect of IC for D2D-enabled cellular networks. Since the stochastic characteristics of heterogeneous networks (comprising D2D users and cellular users) are much more complex than those of homogeneous networks (comprising only ad hoc users or cellular users), the analysis of IC performance in this paper is more challenging than that in [27]–[31].

II. SYSTEM MODEL

In this section, we elaborate on the network model and describe the IC methods.

A. Network Model

We consider a spectrum-sharing D2D-enabled cellular network consisting of both cellular users and D2D users over a large two-dimensional space, and focus on the uplink transmission for cellular users. The cellular users are assumed to be spatially distributed as a homogeneous Poisson point process (PPP) \( \Phi_c \) with density \( \lambda_c \), and an independent collection of base stations (BSs) is assumed to be located according to some independent stationary point process \( \Phi_b \). We assume that each cellular user is associated with its nearest base station, and each base station has only one active uplink cellular user scheduled. Under such assumptions, each base station can be considered to be uniformly distributed in the Voronoi cell of its associated cellular user, as shown in Fig.1. It is noted that the orthogonal scheduling policy leads to coupling between the locations of cellular users and those of base stations. Nevertheless, it has been shown that the dependence introduced by coupling has negligible effects on the performance analysis [23], [31]. Therefore, for analytical

<table>
<thead>
<tr>
<th>Notation</th>
<th>Description</th>
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<tbody>
<tr>
<td>( \Phi_c )</td>
<td>Poisson point process of cellular users (density ( \lambda_c ))</td>
</tr>
<tr>
<td>( \Phi_d )</td>
<td>Poisson point process of D2D transmitters (density ( \lambda_d ))</td>
</tr>
<tr>
<td>( \Phi_{c-intf}^{eq} )</td>
<td>Equivalent Poisson point process of the interferers for cellular links (density ( \lambda_{c-intf}^{eq} ))</td>
</tr>
<tr>
<td>( \Phi_{d-intf}^{eq} )</td>
<td>Equivalent Poisson point process of the interference for D2D links (density ( \lambda_{d-intf}^{eq} ))</td>
</tr>
<tr>
<td>( P_c )</td>
<td>Transmission power of cellular users</td>
</tr>
<tr>
<td>( P_d )</td>
<td>Transmission power of D2D users</td>
</tr>
<tr>
<td>( \alpha )</td>
<td>Path loss exponent (( \delta = \frac{2}{\alpha} ))</td>
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<tr>
<td>( T )</td>
<td>SIR threshold for successful transmission</td>
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<tr>
<td>( p_c )</td>
<td>Successful transmission prob. of cellular links without IC</td>
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<tr>
<td>( p_d )</td>
<td>Successful transmission prob. of D2D links without IC</td>
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<tr>
<td>( p_{d-c}^{c} )</td>
<td>Successful transmission prob. of cellular links with UIC</td>
</tr>
<tr>
<td>( p_{d-d}^{c} )</td>
<td>Successful transmission prob. of D2D links with UIC</td>
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<tr>
<td>( p_{c-SIC}^{c} )</td>
<td>Successful transmission prob. of cellular links with infinite and (finite) ( N )-level SIC</td>
</tr>
<tr>
<td>( p_{d-M-SIC}^{d} )</td>
<td>Successful transmission prob. of D2D links with infinite and (finite) ( M )-level SIC</td>
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</table>
tractability, we assume that the point processes of cellular users and base stations are independent.

The D2D transmitters in the network are assumed to be distributed according to a homogeneous PPP $\Phi_d$ with density $\lambda_d$. For a given D2D transmitter, its associated receiver is assumed to be located at a distance $l$ away with isotropic direction, where $l$ is Rayleigh distributed:

$$f_l(l) = 2\pi \lambda_d l e^{-\pi \lambda_d l^2}.$$  (1)

This Rayleigh distribution assumption is of practical interest and is employed in many works [24]–[26]. Other distributions of $l$ can be easily incorporated into the framework.

The transmission powers are assumed to be $P_c$ at uplink cellular users and $P_d$ at D2D transmitters respectively. We adopt a unified channel model that comprises standard path loss and Rayleigh fading for both cellular and D2D links: given transmission power $P$ of the transmitter located at $x_i$, the received power at the receiver located at $x_j$ can be expressed as $Ph \parallel x_i - x_j \parallel^{-\alpha}$, where $h$ is the fading factor following an exponential distribution with unit mean, i.e., $h \sim \exp(1)$, and $\alpha > 2$ is the path loss exponent. In this paper we use $\delta$ to denote $\frac{2}{\alpha}$ for brevity of expressions. As interference dominates noise in most modern cellular networks, we consider the network to be interference-limited.

B. IC Methods

In this paper, we focus on the performance of two IC methods, unconditional interference cancellation (UIC) and successive interference cancellation (SIC).

Unconditional interference cancellation (UIC) is simpliﬁed interference cancellation method with the assumption that the interference from the strong interferers whose received powers are greater than a certain threshold can be completely (unconditionally) canceled [28]. UIC does not refer to a specific IC technique, but rather uses a cancellation-power-threshold based approximation to model the effect of IC techniques. This model allows for analytical tractability and provides a general insights on IC performance.

Successive interference cancellation (SIC) is a promising interference cancellation technique that has been widely studied for wireless networks. The basic concept of SIC is to regenerate the interfering signals and subsequently cancel them from the received composite signal to improve the signal-to-interference ratio (SIR) of the desired signal. Specifically, the SIC receiver first decodes the strongest interfering signal by treating other signals as noise. Then it regenerates the analog signal from the decoded signal and cancels it from the received signal. After this stage, the remaining signal is free from the interference of the strongest interfering signal. Then, the SIC receiver proceeds to decode, regenerate and cancel the second strongest interfering signaling from the remaining signal and so forth, until the desired signal can be decoded. The schematic diagram of SIC process is shown in Fig.2.

In the following part of this paper, we first analyze the performance of the network without IC capabilities as baseline results, and then study how UIC and SIC affect the network performance.

III. NETWORK PERFORMANCE WITHOUT IC

In this section, we consider the scenario that neither the cellular receivers (BSs) nor the D2D receivers have IC capabilities, and derive the successful transmission probabilities of both cellular and D2D links. We also propose a stochastic equivalence model of the interference in the network, which is essential for analyzing the performance of IC techniques in later sections.

A. Successful Transmission Probability of Cellular Links

Without loss of generality, we conduct the analysis on a typical cellular link that comprises a typical BS located at the origin and its associated cellular user located at a random distance $r$ away. Under the nearest-BS association policy, the random variable $r$ can be shown to be Rayleigh distributed and its probability density function (pdf) follows [23]:

$$f_r(r) = 2\pi \lambda_c r e^{-\pi \lambda_c r^2}.$$  (2)

Denote the fading factor of the typical cellular link by $g_0$, which is i.i.d exponential with $g_0 \sim \exp(1)$. Then, the received
SIR of the typical cellular link, i.e., the received SIR at the typical BS, can be expressed as
\[
\text{SIR}_c = \frac{P_c g_0 r^{-\alpha}}{I_c},
\]
where
\[
I_c = \sum_{x_i \in \mathcal{F}_c \setminus \{x_0\}} P_c g_i \|x_i\|^{-\alpha} + \sum_{y_i \in \mathcal{F}_d} P_d h_i \|y_i\|^{-\alpha}
\]
is the cumulative interference from all other cellular users (except the typical cellular user \(x_0\)) that are located at \(x_i\) with fading factor \(g_i\) and D2D transmitters that are located at \(y_i\) with fading factor \(h_i\). \(^1\)

The successful transmission probability of cellular links can be defined as
\[
p_c \triangleq \mathbb{P}[\text{SIR}_c > T],
\]
where \(T\) is the SIR threshold. The expression of \(p_c\) is given by the following theorem.

**Theorem 1.** The successful transmission probability of cellular links without IC capability is
\[
p_c = \frac{\lambda_c}{\lambda_c (\mu + 1) + \lambda_d (P_c^\delta \nu)},
\]
where
\[
\mu = \frac{\delta}{1 - \delta} T \cdot 2 F_1 (1, 1 - \delta; 2 - \delta; -T),
\]
\[
\nu = T^\delta \Gamma (1 - \delta) \Gamma (1 + \delta),
\]
and \(2 F_1 (\cdot), \Gamma (\cdot)\) are respectively the Hypergeometric function and Gamma function.

**Proof:** Starting from the definitions of \(p_c\), we have
\[
p_c \triangleq \mathbb{P}[\text{SIR}_c > T]
\]
\[
= \mathbb{E}_{r, I_c} [\mathbb{P}_{g_0} [\text{SIR}_c > T]]
\]
\[
= \mathbb{E}_{r, I_c} \left[\mathbb{P}_{g_0} \left[g_0 > P_c^{-1} T r^\alpha I_c\right]\right]
\]
\[
(=) \mathbb{E}_{r, I_c} \left[\mathbb{P}_{g_0} \left[g_0 > P_c^{-1} T r^\alpha I_c\right]\right]
\]
\[
= \mathbb{E}_{r, I_c} \left[\mathbb{L}_{I_c} (P_c^{-1} T r^\alpha)\right]
\]
\[
= \int_0^\infty \mathbb{L}_{I_c} (P_c^{-1} T r^\alpha) \cdot f_r (r) \, dr.
\]

\((a)\) follows from the Rayleigh distribution assumption of channel fading. In \((b)\), \(\mathbb{L}_{I_c} (\cdot)\) denotes the Laplace transform of \(I_c\). Let \(I_{c-e} = I_c - I_d,\) where \(I_{c-e} = \sum_{x_i \in \mathcal{F}_c \setminus \{x_0\}} P_c g_i \|x_i\|^{-\alpha}\) and \(I_{c-d} = \sum_{y_i \in \mathcal{F}_d} P_d h_i \|y_i\|^{-\alpha}\) denote the interference from cellular links and D2D links respectively. Then it is straightforward to get
\[
\mathbb{L}_{I_c} (s) = \mathbb{L}_{I_{c-e}} (s) \cdot \mathbb{L}_{I_{c-d}} (s).
\]

The Laplace transform of \(I_{c-e}\) is given by
\[
\mathbb{L}_{I_{c-e}} (s) = \mathbb{E} \left[\exp \left(-s \sum_{x_i \in \mathcal{F}_c \setminus \{x_0\}} P_c g_i \|x_i\|^{-\alpha}\right)\right]
\]

\(^1\)To distinguish different links, in this paper we use \(g \sim \exp (1), h \sim \exp (1)\) to represent the fading factors of links related to cellular transmitters (cellular users) and D2D transmitters respectively. It is noted that there is no essential distinction between these two symbols.

\[^2\]It is noted that the translations do not change the distribution of PPP \([34]\).
Theorem 2. The successful transmission probability of D2D links without IC capability is

\[ p_d = \frac{\lambda_d}{\lambda_d (\nu + 1) + \lambda_c \left( \frac{P_d}{P_c} \right)^\delta \nu} \]  

where \( \nu \) is given in (8).

Proof: Starting from the definitions of \( p_d \), we have

\[ p_d \triangleq \mathbb{P} \left\{ \text{SIR}_d > T \right\} \]

\[ = \mathbb{E}_{L_d} \left[ \mathbb{P}_{\mathbb{B}_0} \left( \text{SIR}_d > T \right) \right] \]

\[ = \mathbb{E}_{L_d} \left[ \left( P_d^{-1} T \right)^\alpha \right] \]

\[ = \int_0^\infty L_d \left( P_d^{-1} T \right)^\alpha \cdot f_l (l) \, dl. \]  

Following approaches similar to those in previous proofs and Slivnyak’s theorem [32]: \( \mathbb{P}^{\text{eq}} = \mathbb{P} \), we have

\[ L_d(s) = \mathbb{E}_{L_d} \left[ \exp \left( -s L_d \right) \right] \]

\[ = \mathbb{E} \left[ \exp \left( -s \sum_{y_i \in \Phi_d \backslash \{y_0\}} P_d h_i \| y_i \|^{-\alpha} \right) \right] \]

\[ \times \mathbb{E} \left[ \exp \left( -s \sum_{x_i \in \Phi_c} P_c g_i \| x_i \|^{-\alpha} \right) \right] \]

\[ = \exp \left( -\lambda_d \pi (s P_d)^\delta \Gamma (1 - \delta) \Gamma (1 + \delta) \right) \]

\[ \times \exp \left( -\lambda_c \pi (s P_c)^\delta \Gamma (1 - \delta) \Gamma (1 + \delta) \right). \]  

Therefore,

\[ L_d \left( P_d^{-1} T \right)^\alpha = \exp \left( -\pi \left[ (s P_d)^\delta \Gamma (1 + \delta) \right] \frac{\nu T^2}{\alpha^2} \right), \]  

where \( \nu \) is given in (8). Then by plugging (1) (20) into (18), we complete the proof.

Remark 1. Via the expressions of \( p_c \) and \( p_d \) shown in Theorem 1 and 2, we can observe that \( \mu, \nu \) represent the effect of the interference from the cellular links and D2D links respectively, and \( (P_d/P_c)^\delta, (P_c/P_d)^\delta \) can be regarded as the conversion factors of powers.

C. Stochastic Equivalence of Interference

By (4) (15), the cumulative interference at each link is generated by two-tier interferers, i.e., cellular-tier and D2D-tier interferers. The analysis of such two-tier interference is tedious, as shown in the derivations of Theorem 1 and 2. Therefore, to simplify the analysis and facilitate the performance evaluation of IC techniques in later sections, we propose an approach to equate the two-tier interference by a single-tier interference that has the same stochastic characteristics (in terms of successful transmission probability) as the two-tier interference.

We first study the stochastic equivalence of the interference for cellular links. By (4), the interferers for the typical cellular link constitute \( \Phi_{c-intf} = (\Phi_c \setminus \{x_0\}) \cup \Phi_d \). We represent \( \Phi_{c-intf} \) by an equivalent PPP \( \Phi_{c-intf}^{\text{eq}} \) \( \setminus \{x_0\} \) with density \( \lambda_{c-intf}^{\text{eq}} \) and transmission power \( P_c \). Then, the equivalent interference at the typical cellular link can be expressed as

\[ I_c^{\text{eq}} = \sum_{x_i \in \Phi_{c-intf}^{\text{eq}} \setminus \{x_0\}} P_c g_i \| x_i \|^{-\alpha}, \]

which has the same stochastic characteristics as \( I_c \). The equivalent interferer density for the typical cellular link can be obtained from the following lemma.

Lemma 1. The density of the equivalent interferers for cellular links is

\[ \lambda_{c-intf}^{\text{eq}} = \lambda_c + \lambda_d \left( \frac{P_d}{P_c} \right)^\delta \frac{\nu}{\mu}, \]

where \( \mu, \nu \) are given in (7), (8) respectively.

Proof: Considering \( I_c^{\text{eq}} \) has the same stochastic characteristics as \( I_c \), we have

\[ L_{I_c} \left( P_c^{-1} T \right)^\alpha = L_{I_c^{\text{eq}}} \left( P_c^{-1} T \right)^\alpha. \]

The Laplace transform of \( I_c^{\text{eq}} \) is obtained as

\[ L_{I_c^{\text{eq}}} (s) = \mathbb{E}_{I_c^{\text{eq}}} \left[ \exp \left( -s I_c^{\text{eq}} \right) \right] \]

\[ = \mathbb{E} \left[ \exp \left( -s \sum_{x_i \in \Phi_{c-intf}^{\text{eq}} \setminus \{x_0\}} P_c g_i \| x_i \|^{-\alpha} \right) \right] \]

\[ = \exp \left( -\lambda_{c-intf}^{\text{eq}} \pi \left( s P_c \right)^\delta \Gamma (1 - \delta) \Gamma (1 + \delta) \right) \]

\[ \times \exp \left( -\lambda_c \pi \left( s P_c \right)^\delta \Gamma (1 - \delta) \Gamma (1 + \delta) \right). \]  

Therefore,

\[ L_{I_c^{\text{eq}}} \left( P_c^{-1} T \right)^\alpha = \exp \left( -\lambda_{c-intf}^{\text{eq}} \pi \mu \mu^2 \right), \]  

where \( \mu \) is given in (7). Then by plugging (13) (25) into (23), we complete the proof.

It is noted that \( p_c \) can be obtained as \( \int_0^\infty L_{I_c^{\text{eq}}} \left( P_c^{-1} T \right)^\alpha \cdot f_r (r) \, dr \), and by (24), we have

\[ p_c = \frac{\lambda_c}{\lambda_c \mu + \lambda_c}, \]

which is consistent with the result of Theorem 1.

We next study the stochastic equivalence of the interference for D2D links. By (15), the interferers for the typical D2D
link constitute \( \Phi_{d-intf} = (\Phi_d \setminus \{y_0\}) \cup \Phi_c \). We represent \( \Phi_{d-intf} \) by an equivalent PPP \( \Phi_{d-intf}^{eq} \setminus \{y_0\} \) with density \( \lambda_{d-intf}^{eq} \) and transmission power \( P_d \). Then, the equivalent interference at the typical D2D link can be expressed as

\[
I_d^{eq} = \sum_{y_i \in \Phi_{d-intf}^{eq} \setminus \{y_0\}} P_d h_i \|y_i\|^{-\alpha}.
\]  

(27)

The equivalent interferer density for the typical D2D link can be obtained from the following lemma.

**Lemma 2.** The density of the equivalent interferers for D2D links is

\[
\lambda_{d-intf}^{eq} = \lambda_d + \lambda_c \left( \frac{P_c}{P_d} \right)^{\delta}.
\]

(28)

**Proof:** Starting from the Laplace transform of \( I_d^{eq} \),

\[
\mathcal{L}_{I_d^{eq}}(s) = \mathbb{E}[e^{-s I_d^{eq}}] = \mathbb{E} \left[ \exp \left( -s \sum_{y_i \in \Phi_{d-intf}^{eq} \setminus \{y_0\}} P_d h_i \|y_i\|^{-\alpha} \right) \right] = \exp \left( -\lambda_{d-intf}^{eq} \pi (sP_d)^{\delta} \left( 1 - \delta \right) (1 + \delta) \right).
\]

(29)

Therefore,

\[
\mathcal{L}_{I_d^{eq}}(P_d^{-1}T_l^{\alpha}) = \exp \left( -\lambda_{d-intf}^{eq} \pi \nu T_l^{\alpha} \right),
\]

(30)

where \( \nu \) is given in (8). Then by letting \( \mathcal{L}_{I_d^{eq}}(P_d^{-1}T_l^{\alpha}) = \mathcal{L}_{I_d}(P_d^{-1}T_l^{\alpha}) \), we complete the proof.

It is noted that \( p_d \) can be obtained as \( \int_0^{\infty} \mathcal{L}_{I_d^{eq}}(P_d^{-1}T_l^{\alpha}) \cdot f_l(l) \, dl \), and by (30), we have

\[
P_d = \frac{\lambda_d}{\lambda_{d-intf}^{eq} \nu + \lambda_d},
\]

(31)

which is consistent with the result of Theorem 2.

**D. Numerical Results**

Here we provide some numerical results to validate the proposed stochastic equivalence models and corresponding analytical results. The system parameters are set as \( \alpha = 4 \), \( P_c = P_d = 1 \), and \( \lambda_c = 0.01 \).

Fig. 3 shows the successful transmission probabilities of cellular and D2D links without IC capabilities. As can be observed from the figure, the analytical results of the proposed stochastic equivalence models are in quite good agreement with the corresponding simulation results. This fact confirms that the proposed models closely match the practical D2D-enabled cellular networks. In addition, as shown in the figure, when \( \lambda_c = \lambda_d \), cellular links have a higher successful transmission probability than D2D links. The reason is as follows: when \( \lambda_c = \lambda_d \), both the mean signal powers and mean inter-type (cellular-D2D) interference powers of cellular and D2D links are equal; however, the mean intra-type (cellular-cellular or D2D-D2D) interference power of cellular links is smaller than that of D2D links, since for each cellular receiver (BS), the cellular interferers are located at farther distances away than the associated cellular user (due to the nearest-BS association policy), while for each D2D receiver, the D2D interferers can be located at any distances away (due to the random distribution of D2D users).

**IV. NETWORK PERFORMANCE WITH UIC**

In this section, we analyze how UIC affects the successful transmission probabilities in D2D-enabled cellular networks. The analysis is based on the stochastic equivalence models proposed in subsection III.C.

**A. Successful Transmission Probability of Cellular Links**

We conduct the analysis on a typical cellular link that comprises a typical UIC-capable BS located at the origin and its associated cellular user located at \( x_0 \), where \( \|x_0\| = r \). According to Lemma 1, the interferers for the typical cellular link constitute an equivalent PPP \( \Phi_{c-intf}^{eq} \setminus \{x_0\} \) with density \( \lambda_{c-intf}^{eq} \) and transmission power \( P_c \). By applying UIC, the typical BS can completely cancel the interference from the strong interferers whose received powers are greater than a threshold \( \chi \). Hence, the equivalent cumulative interference at the typical cellular link can be expressed as

\[
I_c^{eq}(x) = \sum_{x_i \in \Phi_{c-intf}^{eq} \setminus \{x_0\}} P_c g_i \|x_i\|^{-\alpha} \cdot \Delta_{x_i},
\]

(32)

where

\[
\Delta_{x_i} = \begin{cases} 0, & \text{if } P_c g_i \|x_i\|^{-\alpha} > \chi, \\ 1, & \text{else} \end{cases}
\]

(33)

Then, the received SIR of the typical cellular link is

\[
\text{SIR}^{\text{UIC}}_c = \frac{P_c g_0 r^{-\alpha}}{I_c^{eq}(x)},
\]

(34)

and the successful transmission probability of the typical cellular links is defined as

\[
p_c^{\text{UIC}} \triangleq \mathbb{P} \left[ \text{SIR}^{\text{UIC}}_c > T \right].
\]

(35)

The expression of \( p_c^{\text{UIC}} \) is given by the following theorem.
The Laplace transform of \( p_{\text{UIC}} \) is
\[
p_{\text{UIC}} = \int_0^\infty e^{-\lambda \tau \alpha} \pi_{\text{intf}}(r) \cdot f_r(r) \, dr,
\]
where \( f_r(r) \) is given in (2) and
\[
\kappa_1(r) = \frac{\delta}{1 - \delta} T r^2 F_1(1, 1 - \delta; 2 - \delta; -T),
\]
\[
\kappa_2(r) = e^{-P_{c}^{-1} T r^2} \sum_{n=0}^{\infty} \frac{(-1)^{n+1}(\chi P_{c}^{-1})^n}{n!} \frac{r^{2 + n \alpha}}{1 + n \alpha} \times 2 F_1(1, -\delta - n; -\delta - n + 1; -T),
\]
\[
\kappa_3(r) = \delta (\chi^{-1} P_{c})^\delta \Gamma(\delta, r^\alpha \chi P_{c}^{-1}).
\]

Proof: Starting from the definitions of \( p_{\text{UIC}} \), we have
\[
p_{\text{UIC}} \triangleq \mathbb{P}\left[ \text{SIR}_{\text{UIC}} > T \right] = \mathbb{E}_r \left[ \mathcal{L}_{\text{e}}(r) \left( P_{c}^{-1} T r^\alpha \right) \right] = \int_0^\infty \mathcal{L}_{\text{e}}(r) \cdot f_r(r) \, dr.
\]

The Laplace transform of \( I_{\text{e}}^\text{eq}(x) \) is given by
\[
\mathcal{L}_{\text{e}}(r) \triangleq \mathbb{E}_e \left[ \exp(-s I_{\text{e}}^\text{eq}(r)) \right] = \mathbb{E}_Q \mathbb{E}_x \left[ \prod_{i=1}^{Q_e} \mathbb{E}_x \left[ e^{-s P_{c} g_i |x_i|^{-\alpha} \cdot \Delta x_i} \right] \right] = e^{- \sum_{i=1}^{Q_e} \mathbb{E}_x \left[ e^{-s P_{c} g_i |x_i|^{-\alpha} \cdot \Delta x_i} \right]} \cdot e^{- \sum_{i=1}^{Q_e} \mathbb{E}_x \left[ e^{-s P_{c} g_i |x_i|^{-\alpha} \cdot \Delta x_i} \right]}.
\]

Since \( g \sim \exp(1) \), the expectation term in (41) is obtained as
\[
\mathbb{E}_x \left[ e^{-s P_{c} g_i |x_i|^{-\alpha} \cdot \Delta x_i} \right] = \int_0^\infty e^{-s P_{c} g_i |x_i|^{-\alpha} \cdot \Delta x_i} \cdot e^{-g_i} \, dg_i = \int_0^\infty e^{-s P_{c} g_i |x_i|^{-\alpha} \cdot \Delta x_i} \cdot e^{-g_i} \, dg_i + \int_{P_{c}^{-1} |x_i|^\alpha}^{\infty} e^{-g_i} \, dg_i = 1 - e^{-s P_{c}^{-1} |x_i|^\alpha} + e^{-s P_{c}^{-1} |x_i|^\alpha}.
\]

Substituting (42) in (41), we get
\[
\mathcal{L}_{\text{e}}(r) = e^{- \sum_{i=1}^{Q_e} \mathbb{E}_x \left[ e^{-s P_{c} g_i |x_i|^{-\alpha} \cdot \Delta x_i} \right]} \cdot e^{- \sum_{i=1}^{Q_e} \mathbb{E}_x \left[ e^{-s P_{c} g_i |x_i|^{-\alpha} \cdot \Delta x_i} \right]} \cdot e^{- \sum_{i=1}^{Q_e} \mathbb{E}_x \left[ e^{-s P_{c} g_i |x_i|^{-\alpha} \cdot \Delta x_i} \right]} \cdot e^{- \sum_{i=1}^{Q_e} \mathbb{E}_x \left[ e^{-s P_{c} g_i |x_i|^{-\alpha} \cdot \Delta x_i} \right]} \cdot e^{- \sum_{i=1}^{Q_e} \mathbb{E}_x \left[ e^{-s P_{c} g_i |x_i|^{-\alpha} \cdot \Delta x_i} \right]}.
\]

In (43), the first integral term is obtained using the definite integral [33, 3.194.2]:
\[
\int_r^\infty \frac{s P_{c} v^{-\alpha}}{s P_{c} v^{-\alpha} + 1} \, dv = \frac{1}{\delta} \int_r^\infty \frac{x^{\frac{\alpha}{2} - 1}}{1 + x^{\frac{\alpha}{2} - 1} P_{c}^{-1} x} \, dx = \frac{1}{\delta} \frac{2}{21 - \delta} s P_{c} r^{\frac{\alpha}{2} - 2} F_1(1, 1 - \delta; 2 - \delta; -s P_{c}^{-1} r^\alpha),
\]
the second integral term is obtained using the definite integral [33, 3.194.2] and Taylor series expansions of exponential functions:
\[
\int_r^\infty \frac{e^{-s(x + P_{c}^{-1} v^\alpha)}}{s P_{c} v^{-\alpha} + 1} \, dv = \frac{1}{\alpha} \int_r^\infty \frac{e^{-s P_{c}^{-1} x}}{s P_{c} + x} \, dx = e^{-s x} \sum_{n=0}^{\infty} \frac{(-1)^{n+1}(\chi P_{c}^{-1})^n}{n!} \frac{r^{2 + n \alpha}}{2 + n \alpha} (2 F_1(1, -\delta - n; -\delta - n + 1; -s P_{c}^{-1} r^\alpha)),
\]
and the third integral term is obtained as
\[
\int_r^\infty e^{-s P_{c}^{-1} v^\alpha} \, dv = \frac{1}{\alpha} \int_r^\infty \frac{s^{\frac{\alpha}{2} - 1} e^{-s P_{c}^{-1} x}}{s P_{c} + x} \, dx = \frac{\delta}{2} \left( \chi^{-1} P_{c} \right)^\delta \Gamma(\delta, r^\alpha \chi P_{c}^{-1}).
\]

Then by plugging (44) – (46) into (43) and letting \( s = P_{c}^{-1} T r^\alpha \), we complete the proof.

In Theorem 3, \( \kappa_1(r), \kappa_2(r) \) represent the effect of the weak (uncanceled) interference and \( \kappa_3(r) \) represents the effect of the strong (canceled) interference.

B. Successful Transmission Probability of D2D Links

We conduct the analysis on a typical D2D link that comprises a typical UIC-capable D2D receiver located at the origin and a typical D2D transmitter located at \( y_0 \), where \( \|y_0\| = L \). According to Lemma 2, the interferers for the typical D2D link constitute an equivalent PPP \( \Phi_{d} \text{intf} \setminus \{y_0\} \) with density \( \lambda_{d} \text{intf} \) and transmission power \( P_d \). The equivalent cumulative interference at the typical D2D link can be expressed as
\[
I_d^\text{eq}(x) = \sum_{y_i \in \Phi_{d} \text{intf} \setminus \{y_0\}} P_d h_i \|y_i\|^{-\alpha} \cdot \Delta y_i,
\]
where
\[
\Delta y_i = \begin{cases} 0, & \text{if } P_d h_i \|y_i\|^{-\alpha} > \chi, \\ 1, & \text{else}. \end{cases}
\]

Then, the received SIR of the typical D2D link is
\[
\text{SIR}_{d}^\text{UIC} = \frac{P_d h_0 \|y_0\|^{-\alpha}}{I_d^\text{eq}(x)},
\]
and the successful transmission probability of the typical D2D link is defined as
\[
p_d^\text{UIC} \triangleq \mathbb{P}\left[ \text{SIR}_{d}^\text{UIC} > T \right].
\]

The expression of \( p_d^\text{UIC} \) is given by the following theorem.

Theorem 4. The successful transmission probability of D2D links with UIC is
\[
p_d^\text{UIC} = \int_0^\infty e^{-\lambda_{d} \text{intf} \pi_{\text{intf}}(r_1) + \tau_2(r_2) - \tau_3(r_3) \cdot f_1(l) \, dl,
\]
where \( f_1(l) \) is given in (1) and
\[
\tau_1(l) = \frac{\delta T^\delta l^2 \Gamma(\delta) \Gamma(1 - \delta)},
\]
\[
\tau_2(l) = \delta T^\delta l^2 H(1 + \delta) \Gamma(-\delta, P_{c}^{-1} \pi^{\alpha} T\chi),
\]
\[
\tau_3 = \delta (\chi^{-1} P_{c})^\delta \Gamma(\delta).
\]
**Proof:** Starting from the definitions of $p_{d}^{\text{UIC}}$, we have

$$p_{d}^{\text{UIC}} \triangleq \mathbb{P} \left[ \text{SIR}_{d}^{\text{UIC}} > T \right] = \mathbb{E}_{\omega} \left[ \mathcal{L}_{f_{d}(\omega)} \left( P_{d}^{-1} T \right) \right]$$

$$= \int_{0}^{\infty} \mathcal{L}_{f_{d}(\omega)} \left( P_{d}^{-1} T \right) \cdot f_{l} \left( l \right) \, dl. \tag{55}$$

The Laplace transform of $f_{d}(\omega)$ is given by

$$\mathcal{L}_{f_{d}(\omega)} \left( s \right) = \mathbb{E}_{\omega} \left[ \exp \left( -s f_{d}(\omega) \right) \right]$$

$$= \mathbb{E}_{\Phi_{d}-\text{intf}} \left[ \prod_{y_{i} \in \Phi_{d}-\text{intf} \setminus \{ y_{0} \}} \mathbb{E}_{\omega} \left[ e^{-s P_{d} h_{i} \| y_{i} \|^{-\alpha} \cdot \Delta_{yi}} \right] \right]$$

$$= e^{-x_{d}^{\text{intf}} \int_{0}^{\infty} \left( 1 - \mathbb{E}_{\omega} \left[ e^{-s P_{d} h_{i} \| y_{i} \|^{-\alpha} \cdot \Delta_{yi}} \right] \right) \, dy_{i}}. \tag{56}$$

Since $h \sim \exp(1)$, the expectation term in (56) is obtained as

$$\mathbb{E}_{\omega} \left[ e^{-s P_{d} h_{i} \| y_{i} \|^{-\alpha} \cdot \Delta_{yi}} \right] = \int_{0}^{\infty} e^{-s P_{d} h_{i} \| y_{i} \|^{-\alpha} \cdot \Delta_{yi}} \cdot e^{-h_{i}} \, dh_{i}$$

$$= \int_{0}^{\infty} P_{d}^{-\alpha} \| y_{i} \|^{-\alpha} e^{-s P_{d} h_{i} \| y_{i} \|^{-\alpha} \cdot h_{i}} + e^{-x_{d}^{\text{intf}}}$$

$$\frac{1 - e^{-\left( s P_{d} \| y_{i} \|^{-\alpha} + 1 \right) x_{d}^{\text{intf}} \| y_{i} \|^{-\alpha}}}{s P_{d} \| y_{i} \|^{-\alpha} + 1} + e^{-x_{d}^{\text{intf}}} \| y_{i} \|^{-\alpha}. \tag{57}$$

Substituting (57) in (56), we get

$$\mathcal{L}_{f_{d}(\omega)} \left( s \right) =$$

$$e^{-x_{d}^{\text{intf}} \int_{0}^{\infty} \left( 1 - \mathbb{E}_{\omega} \left[ e^{-s P_{d} h_{i} \| y_{i} \|^{-\alpha} \cdot \Delta_{yi}} \right] \right) \, dy_{i}}$$

$$e^{-x_{d}^{\text{intf}} \int_{0}^{\infty} \left( \frac{s P_{d} \| y_{i} \|^{-\alpha} + e^{-\left( s P_{d} \| y_{i} \|^{-\alpha} + 1 \right) x_{d}^{\text{intf}} \| y_{i} \|^{-\alpha}}}{s P_{d} \| y_{i} \|^{-\alpha} + 1} \right) \, dy_{i}}. \tag{58}$$

In (58), the first integral term is obtained using the definite integral [33, 3.241.4]:

$$\int_{0}^{\infty} \frac{s P_{d} v^{-\alpha}}{s P_{d} v^{-\alpha} + 1} \, dv = \int_{0}^{\infty} \frac{v}{1 + s^{-1} P_{d} v^{-\alpha}} \, dv$$

$$= \frac{\delta}{2} \left( s P_{d} \right)^{\delta} \left( \Gamma \left( 1 - \delta \right) \right). \tag{59}$$

The second integral term is obtained using the definite integral [33, 3.383.10]:

$$\int_{0}^{\infty} \frac{e^{-\left( s x + s P_{d} \right) v^{-\alpha}}}{s P_{d} v^{-\alpha} + 1} \, dv = \frac{1}{\alpha} \int_{0}^{\infty} \frac{e^{-s x} x^{\frac{\alpha}{2}} e^{-x P_{d}^{-1} x}}{s P_{d} + x} \, dx$$

$$= \frac{\delta}{2} \left( s P_{d} \right)^{\delta} \left( \Gamma \left( 1 - \delta, s x \right) \right). \tag{60}$$

And the third integral term is obtained as

$$\int_{0}^{\infty} e^{-x P_{d}^{-1} v^{-\alpha}} v^{-\alpha} \, dv = \frac{1}{\alpha} \int_{0}^{\infty} \frac{x^{\frac{\alpha}{2}} e^{-x P_{d}^{-1} x}}{s P_{d} + x} \, dx$$

$$= \frac{\delta}{2} \left( \chi^{-1} P_{d} \right)^{\delta} \left( \Gamma \left( \delta \right) \right). \tag{61}$$

Then by plugging (59) – (61) into (58) and letting $s = P_{d}^{-1} T$, we complete the proof.

In Theorem 4, $\tau_{1}(l)$, $\tau_{2}(l)$ represent the effect of the weak (uncanceled) interference and $\tau_{3}$ represents the effect of the strong (canceled) interference.

**Figure 4:** Successful transmission probabilities of cellular and D2D links with UIC. The system parameters are set as $\alpha = 4$, $P_{c} = P_{d} = 1$, $\lambda_{c} = \lambda_{d} = 0.01$, $T = 1$ (0 dB).

**C. Discussions and Numerical Results**

Now that we have developed expressions for the successful transmission probabilities of cellular and D2D links with UIC capabilities, based on the stochastic equivalence models proposed in subsection III.C. It is noted that the stochastic equivalence models are obtained based on the non-IC scenario, and thus when applied to UIC networks, these models do not necessarily produce exact results of the successful transmission probabilities. Revisiting the UIC process, we can make the following observations: for D2D links, the UIC process does not change the equivalent density of cellular interferers, and thus the stochastic equivalence model can produce exact values of the successful transmission probability of D2D links with UIC; however, for cellular links, the UIC process changes the equivalent density of D2D interferers, and thus the stochastic equivalence model can only produce approximated values of the successful transmission probability of cellular links with UIC. Here we provide some numerical results to compare the analytical results based on the stochastic equivalence models with the actual (simulation) results. The system parameters are set as $\alpha = 4$, $P_{c} = P_{d} = 1$, $\lambda_{c} = \lambda_{d} = 0.01$, $T = 1$.

Fig. 4 shows the successful transmission probabilities of cellular and D2D links with UIC capabilities, where $\chi_{0} = -27.96$ dB is the reference power threshold that equals the received power from a transmitter located at a distance of 5 m away. As expected, the analytical results of $p_{UIC}^{\text{cell}}$ are in quite good agreement with corresponding simulation results, but there exist gaps between the analytical results of $p_{UC}^{\text{cell}}$ and corresponding simulation results. Meanwhile, the gaps between the analytical and simulation results of $p_{UC}^{\text{D2D}}$ are very small, which implies that the stochastic equivalence models can provide a good approximation for the interference in UIC networks. An interesting observation from the figure is that the increasing rate of $p_{UIC}^{\text{cell}}$ is larger than that of $p_{UC}^{\text{D2D}}$ (as $\chi$ decreases), which means UIC is more effective to improving
the performance of D2D links than that of cellular links. This is because, the mean interferer-receiver distance of D2D links is shorter than that of cellular links (as is analyzed in subsection III.D) and thus UIC can cancel more interference for D2D links.

To measure the effect of UIC on the successful transmission probabilities, we define a new metric called UIC coefficients as follows: \( \rho_{d}^{UIC} (l) = L_{I_{d}^{eq}}(P_{d}^{-1}T^{\alpha}) / L_{I_{d}}(P_{d}^{-1}T^{\alpha}) \) and \( \rho_{c}^{UIC} (r) = L_{I_{c}^{eq}}(P_{c}^{-1}T^{\alpha}) / L_{I_{c}}(P_{c}^{-1}T^{\alpha}) \). The UIC coefficients \( \rho_{d}^{UIC} (l) \) quantify how much \( \rho_{d} \) is improved by UIC with given \( r \) and \( l \). According to the definitions, \( \rho_{d}^{UIC} \) and \( \rho_{c}^{UIC} \) can be obtained as \( \rho_{d}^{UIC} = \int_{0}^{\infty} \rho_{d}^{UIC} (r) \cdot L_{I_{d}}(P_{d}^{-1}T^{\alpha}) \cdot f_{r} (r) \cdot dr \) and \( \rho_{c}^{UIC} (l) = \int_{0}^{\infty} \rho_{c}^{UIC} (l) \cdot L_{I_{c}}(P_{c}^{-1}T^{\alpha}) \cdot f_{l} (l) \cdot dl \), and when \( \rho_{d}^{UIC} (r) = \rho_{d} \) and \( \rho_{c}^{UIC} (l) = 1 \), \( \rho_{d}^{UIC} \) reduce to \( \rho_{d} \) and \( \rho_{c}^{UIC} \) reduce to \( \rho_{c} \) (see (9) and (18) respectively. Fig.5 shows numerical results of the UIC coefficients. As expected, \( \rho_{d}^{UIC} \) is larger than \( \rho_{d} \) when \( l = r \), which is consistent with the foregoing analysis on Fig.4.

V. NETWORK PERFORMANCE WITH SIC

In this section, we study how SIC affects the successful transmission probabilities in D2D-enabled cellular networks. The analysis is based on the stochastic equivalence models proposed in subsection III.C.

A. Successful Transmission Probability of Cellular Links

We conduct the analysis on a typical cellular link that comprises a typical SIC-capable BS located at the origin and its associated cellular user located at \( x_{0} \), where \( \| x_{0} \| = r \). According to Lemma 1, the interferers for the typical cellular link constitute an equivalent PPP \( \Phi^{eq}_{c-intf} \{ x_{0} \} \) with density \( \lambda^{eq}_{c-intf} \) and transmission power \( P_{c} \). We assume the equivalent interferers are ordered by their received power at the typical BS such that \( P_{c} g_{i} \| x_{i} \|^{-\alpha} > P_{c} g_{j} \| x_{j} \|^{-\alpha} \), \( \forall 0 < i < j \). Before deriving the successful transmission probability of the typical cellular link, we first present two useful lemmas.

First, we study the successful transmission probability of the typical cellular link after canceling \( n \) strongest equivalent interferers. Given that \( n \) strongest equivalent interferers have been canceled, the received SIR of the typical cellular link can be expressed as

\[
\text{SIR}_{c}^{(n)} = \frac{P_{c} g_{0} \| x_{i} \|^{-\alpha}}{I_{c}^{eq}(n)},
\]

where

\[
I_{c}^{eq}(n) = \sum_{j=1}^{n} P_{c} g_{j} \| x_{j} \|^{-\alpha}
\]

is the cumulative interference for the typical cellular link. Then, the successful transmission probability of the typical cellular link given that \( n \) strongest equivalent interferers have been canceled can be defined as

\[
p_{c}^{(n)} = \mathbb{P} \left[ \text{SIR}_{c}^{(n)} > T \right].
\]

The expression of \( p_{c}^{(n)} \) is given by the following lemma.

Lemma 3. Given that \( n \) strongest equivalent interferers have been canceled, the successful transmission probability of the typical cellular link is

\[
p_{c}^{(n)} = \int_{0}^{\infty} \int_{0}^{\infty} \frac{2 \left( \lambda^{eq}_{c-intf} \pi d_{n}^{2} \right) \right \}^{n} c^{\lambda^{eq}_{c-intf} \xi(r,d_{n})} f_{r} (r) \cdot d_{n} \cdot dr,
\]

where \( f_{r} (r) \) is given in (2) and

\[
\xi(r,d_{n}) = \frac{\delta}{1-\delta} T r^{\alpha} d_{n}^{2-\alpha} \cdot F_{1} \left( 1, 1-\delta; 2-\delta; \frac{T r^{\alpha}}{d_{n}^{\alpha}} \right) + d_{n}^{2}.
\]

Proof: Starting from the definitions of \( p_{c}^{(n)} \), we have

\[
p_{c}^{(n)} = \mathbb{P} \left[ \text{SIR}_{c}^{(n)} > T \right] = \mathbb{E}_{r} \left[ L_{c}^{eq}(n) \left( P_{c}^{-1}T^{\alpha} \right) \right]
\]

\[
= \int_{0}^{\infty} L_{c}^{eq}(n) \left( P_{c}^{-1}T^{\alpha} \right) \cdot f_{r} (r) \cdot dr.
\]
The calculation of $\mathcal{L}_{\rho_c^{(n)}}$ requires the distribution of the sum of the order statistics of interfering powers, which is difficult to obtain in the SIC scenario. However, it has been shown that the order statistics of received powers in modern networks are dominated by the distance [31]. Therefore, we can calculate $\mathcal{L}_{\rho_c^{(n)}}$ by relaxing the ordering of interfering powers to that of interfering distances. Denote the distance from $n$-th equivalent interferer to the origin by $d_n$, then we have

$$
\mathcal{L}_{\rho_c^{(n)}} (s) = \int_{[0,\infty)} \left( s \sum_{i=1}^{n} P_c g_i \right)^{-\alpha} e^{-s P_c d_n^\alpha} \frac{1}{d_n} \frac{d\Gamma}{\Gamma(n)} \, dd_n.
$$

(68)

From [32], the probability density function of $d_n$ is given by

$$
f_{d_n}(d_n) = e^{-\lambda_{n-1,int} \pi d_n^2} \frac{2}{d_n} \frac{\Gamma(n)}{\Gamma(n)} .
$$

(69)

By plugging (69) into (68) and letting $s = P_c^{-1} T \alpha$, we get

$$
\mathcal{L}_{\rho_c^{(n)}} (P_c^{-1} T \alpha) = \int_{0}^{\infty} e^{-\lambda_{n-1,int} \pi \xi(r,d_n)} \frac{2 \left( \lambda_{n-1,int} \pi d_n^2 \right)^n}{d_n \Gamma(n)} \, dd_n,
$$

(70)

where

$$
\xi(r,d_n) = \frac{\delta}{1-\delta} Tr \alpha d_n^2 - T \alpha d_n^2 d_n + d_n^2 .
$$

Then by plugging (2) (70) into (67), we complete the proof.

A possible approach for simplifying the expression of $p_c^{(n)}$ is to approximate $d_n$ by fixed value $\tilde{d}_n$, which equals the expectation of $d_n$, i.e.,

$$
\tilde{d}_n = \mathbb{E}[d_n] = \frac{\Gamma(n+\frac{1}{2})}{\sqrt{\pi} \lambda_{n-1,int} \Gamma(n)} .
$$

(71)

Then, $p_c^{(n)}$ can be approximated by

$$
p_c^{(n)} \approx \int_{0}^{\infty} e^{-\lambda_{n-1,int} \pi \xi(r,\tilde{d}_n)} \cdot f_r(r) \, dr,
$$

(72)

where $f_r(r)$ is given in (2) and

$$
\xi(r,\tilde{d}_n) = \frac{\delta}{1-\delta} Tr \alpha \tilde{d}_n^2 - T \alpha \frac{\tilde{d}_n^2}{d_n} + \frac{\tilde{d}_n^2}{d_n} .
$$

(73)

Next, we study the probability of canceling $n$-th strongest equivalent interferer. Given that all $n-1$ strongest equivalent interferers have been canceled, the received SIR of $n$-th strongest interferer at the typical BS can be expressed as

$$
\text{SIR}_{c-int f}^{(n)} = \frac{P_c g_n \| x_n \|^{-\alpha}}{\sum_{i=1}^{n-1} P_c g_i \| x_i \|^{-\alpha}},
$$

(74)

where

$$
I_{c-int f}^{(n)} = \sum_{x_i \in \Phi_{n-1,intf}} P_c g_i \| x_i \|^{-\alpha}
$$

is the cumulative interference for $n$-th strongest equivalent interferer. Then, the probability of canceling $n$-th strongest equivalent interferer given that all $n-1$ strongest equivalent interferers have been canceled can be defined as

$$
p_c^{(n)} = \frac{1}{(\mu + 1) \pi},
$$

(77)

where $\mu$ is given in (7).

**Proof:** Following the relaxation approach for the order statistics of interfering powers in the proof of Lemma 3, we can rewrite $\text{SIR}_{c-int f}^{(n)}$ as

$$
\text{SIR}_{c-int f}^{(n)} = \frac{P_c g_n d_n^{-\alpha}}{I_{c-int f}^{(n)}} .
$$

(78)

Then, we have

$$
p_c^{(n)} \triangleq \mathbb{P} \left[ \text{SIR}_{c-int f}^{(n)} > T \right]
$$

$$
= \mathbb{E}_{d_n, I_{c-int f}^{(n)}} \left[ \exp \left( -P_c^{-1} T d_n I_{c-int f}^{(n)} \right) \right]
$$

$$
= \mathbb{E}_{d_n} \left[ \mathcal{L}_{I_{c-int f}^{(n)}} (P_c^{-1} T d_n) \cdot f_{d_n}(d_n) \, dd_n \right].
$$

(79)

The Laplace transform of $I_{c-int f}^{(n)}$ is obtained as

$$
\mathcal{L}_{I_{c-int f}^{(n)}} (s) = e^{-\lambda_{n-1,int} \pi \xi(s,\tilde{d}_n)} \cdot f_r(r) \, dr,
$$

(80)

Hence,

$$
\mathcal{L}_{I_{c-int f}^{(n)}} (P_c^{-1} T d_n) = \exp \left( -\lambda_{n-1,int} \pi \xi(s,\tilde{d}_n) \right),
$$

(81)

where $\xi$ is given in (7). Then by plugging (69) (81) into (79), we complete the proof.

Based on Lemma 3 and 4, we can derive the successful transmission probability of the typical cellular link.

**Theorem 5.** The successful transmission probability of cellular links with infinite SIC capability is

$$
p_c^{(n)} = p_c + \sum_{n=1}^{\infty} \left( \prod_{i=1}^{n} I_{c-int f}^{(i)} \right) \left( \prod_{i=0}^{n-1} (1 - p_c^{(i)}) \right) p_c^{(n)},
$$

(82)
where \( p_c, p_c^{(n)}, p_{c-intf}^{(i)} \) are given in (6), (65), (77) respectively.

**Proof:** For the typical cellular link, we define the event of its successful transmission without SIC as

\[
E_0 : \text{SIR}_c^{(0)} > T,
\]

and the event of its successful transmission with \( n \)-level \( (n \geq 1) \) SIC as

\[
E_n : \left( \bigcap_{i=1}^{n} \text{SIR}_c^{(i)} > T \right) \cap \left( \bigcap_{i=0}^{n-1} \text{SIR}_c^{(i)} < T \right) \cap \left( \text{SIR}_c^{(n)} > T \right).
\]

Using the assumption that the interference to each user is independent, we get

\[
\mathbb{P}[E_n] = \left\{ \begin{array}{ll}
p_c^{(0)} = p_c, & n = 0, \\
\left( \prod_{i=1}^{n} p_{c-intf}^{(i)} \right) \left( \prod_{i=0}^{n-1} \left( 1 - p_c^{(i)} \right) \right) p_c^{(n)}, & n \geq 1.
\end{array} \right.
\]

Therefore, the successful transmission probability of cellular links can be obtained as

\[
p_c^{SIC} = \sum_{n=0}^{\infty} \mathbb{P}[E_n].
\]

**Corollary 1.** The successful transmission probability of cellular links with \( (\text{finite}) \) \( N \)-level \( (N \geq 1) \) SIC capability is

\[
p_c^{N-SIC} = p_c + \sum_{n=1}^{N} \left( \prod_{i=1}^{n} p_{c-intf}^{(i)} \right) \left( \prod_{i=0}^{n-1} \left( 1 - p_c^{(i)} \right) \right) p_c^{(n)},
\]

where \( p_c, p_c^{(n)}, p_{c-intf}^{(i)} \) are given in (6), (65), (77) respectively.

**B. Successful Transmission Probability of D2D Links**

We consider a typical D2D link with \( M \)-level SIC receivers. The assumption of finite-level SIC receiver is motivated by the limited computational capabilities of D2D users. The derivation of the successful transmission probability of D2D links is quite similar to that of cellular links, and hence we directly present the results and omit their proofs.

**Lemma 5.** Given that \( n \) strongest equivalent interferers have been canceled, the successful transmission probability of the typical D2D link is

\[
p_d^{(n)} = \frac{2 \left( \lambda d-intf \right) \pi k_n^2}{k_n \Gamma(n)} \int_0^\infty \int_0^\infty \frac{\left( \lambda d-intf \right) \pi k_n^2}{k_n \Gamma(n)} e^{-\lambda d-intf \pi (l,k_n)} f_l(l) \, dk_n \, dl,
\]

where \( f_l(l) \) is given in (1) and

\[
\lambda d-intf = \frac{\delta}{1 - \delta} T l_0^2 k_n^{2-\alpha} 2 F_1 \left( 1, 1 - \delta; 2 - \delta; \frac{T l_0^2}{k_n^{\alpha}} \right) + k_n^2.
\]

**Lemma 6.** Given that all \( n-1 \) strongest equivalent interferers have been canceled, the probability of canceling \( n \)-th strongest equivalent interferer for the typical D2D link is

\[
p_d^{(n)} = \frac{1}{(\mu + 1)\pi},
\]

where \( \mu \) is given in (7).

**Theorem 6.** The successful transmission probability of D2D links with \( (\text{finite}) \) \( M \)-level \( (M \geq 1) \) SIC capability is

\[
p_d^{M-SIC} = p_d + \sum_{n=1}^{M} \left( \prod_{i=1}^{n} p_{d-intf}^{(i)} \right) \left( \prod_{i=0}^{n-1} \left( 1 - p_d^{(i)} \right) \right) p_d^{(n)},
\]

where \( p_d, p_d^{(n)}, p_{d-intf}^{(i)} \) are given in (17), (87), (89) respectively.

**C. Discussions and Numerical Results**

Now that we have developed expressions for the successful transmission probabilities of cellular and D2D links with SIC capabilities, based on the stochastic equivalence models. It is noted that the derived analytical expressions are not the exact results of corresponding successful transmission probabilities, since approximated models are used in the derivation. We provide some numerical results to compare the analytical results with the actual (simulation) results. The system parameters are set as \( \alpha = 4, P_c = P_d = 1, \lambda_c = \lambda_d = 0.01, T = 1 \,(0 \,dB) \).

Fig.6 shows the successful transmission probabilities of cellular and D2D links with SIC. The system parameters are set as \( \alpha = 4, P_c = P_d = 1, \lambda_c = \lambda_d = 0.01, T = 1 \,(0 \,dB) \).
Successful transmission probability of cellular links with SIC vs. density of D2D links. The system parameters are set as $\alpha = 4$, $P_c = P_d = 1$, $\lambda_c = 0.01$, $T = 1$ (0 dB).

improvement for the network; however, when the SIC level is larger than 2, SIC cannot further improve the network performance. Considering the hardware complexity and cost of multi-level SIC, in practical networks, 1-level and 2-level SIC can be adopted in the receivers.

Fig.7 shows the successful transmission probability of cellular links with SIC vs. the density of D2D links. As expected, increasing the density of D2D links leads to a decrease in the successful transmission probability of cellular links. However, from this figure we can observe that SIC can compensate part of the performance loss of cellular links. For example, 1-level SIC at the cellular receiver can compensate the performance loss generated by D2D interferers of density $\lambda_d = 0.0035$ (see point A), and 2-level SIC corresponds to $\lambda_d = 0.0045$ (see point B). Moreover, the fact that point C is very close to point B also indicates that more SIC levels (> 2) cannot provide extra performance gains for the network.

VI. Conclusion

In this paper, we study the performance of IC techniques in large-scale D2D-enabled cellular networks using the tools from stochastic geometry. We first derive the successful transmission probabilities of cellular and D2D links in the network without IC capabilities as the baseline results. Then, to simplify the interference analysis, we propose the approach of stochastic equivalence of the interference, through which the two-tier interference can be represented by an equivalent single-tier interference. Based on the proposed stochastic equivalence models, we derive the successful transmission probabilities of cellular and D2D links in the networks where UIC and SIC are respectively applied. The effectiveness of these two IC techniques for improving the performance of D2D-enabled cellular networks is validated by both analytic and numerical results. Future work will extend the analysis to the maximum transmission capacity of the network.

REFERENCES


