Multicast Capacity in Mobile Wireless Ad Hoc Network with Infrastructure Support

Xi Chen, Wentao Huang and Xinbing Wang
Department of Electronic Engineering
Shanghai Jiao Tong University, China
Email: {qfbzcx,yelohuang,xwang8}@sjtu.edu.cn

Xiaojun Lin
Department of Electrical and Computer Engineering
Purdue University, USA
Email: linx@ecn.purdue.edu

***Abstract*—We study the multicast capacity under a network model featuring both node's mobility and infrastructure support. Combinations between mobility and infrastructure, as well as multicast transmission and infrastructure, have already been shown effective ways to increase capacity. In this work, we jointly consider the impact of the above three factors on network capacity. We assume that m static base stations and n mobile users are placed in an ad hoc network, of which the area scales with n as $f^2(n)$. A general mobility model is adopted, such that each user moves within a bounded distance from its home-point with an arbitrary pattern. In addition, each mobile node serves as the source of a multicast transmission, which results in a total number of n multicast transmissions. We focus on the situations that base stations actually benefit the capacity, and prove that multicast capacity of mobile hybrid network falls into three regimes. For each regime, matching upper and lower bounds are derived.**

***Index Terms*—Multicast capacity; mobility; hybrid networks; MANETs;**

I. Introduction

In recent years, we have witnessed a rapid development in wireless ad hoc network, in both academic and industrial fields. Gupta and Kumar have showed in their ground breaking work [1] that, even with the optimal scheduling, routing and relaying of packets, the per-node capacity of a wireless network with n nodes still decreases as $\Theta(1/\sqrt{n})$ when n tends to infinity. Many studies try to improve this poor throughput scaling by introducing different characteristics into ad hoc network, such as mobility of nodes, infrastructure support and multicast transmission schemes.

Mobility in ad hoc network was first considered by D.Tse et al. in [2]. A store-carry-forward relaying scheme was proposed and proven to sustain a $\Theta(1)$ per-node capacity, if each node can visit the whole network area with an uniform and ergodic mobility process. M. Garetto et al. study a mobility model with the restriction that each moving node is located within a circle of radius $1/f(n)$ [3]. By mapping the network to a generalized random geometric graph, the authors have proven that a $\Theta(1/f(n))$ per-node capacity is achievable. While the above results are mainly for the throughput, there also exists a significant body of works that try to find an optimal tradeoff between delay and capacity in mobile ad hoc networks (e.g., [4], [5], [6], [7], [8], [9], [10], [11]).

Infrastructure in ad hoc network can also significantly increase the network capacity. In [12], B. Liu et al. show that infrastructure can offer a linear capacity increase in hybrid network, when the number of base stations increases asymptotically faster than $\sqrt{n}$. In addition, Kozat and Tassiulas [13] prove that if the number of users served by each BS is bounded above, a per-node capacity of $\Theta(1/\log n)$ can be achieved. In [14], Agarwal and Kumar further refine this result and show $\Theta(1)$ capacity. P. Li et al. consider the impact of traffic pattern in [15], and derive the capacity results of a 3D network model in [16].

Multicast transmission refers to the transmission from single node to other k nodes, which generalizes both unicast and broadcast transmissions. In [17], X.-Y. Li proves that multicast transmission can obtain a per-flow capacity of $\Theta\left(\sqrt{\frac{1}{n\log n}}\cdot\frac{1}{\sqrt{k}}\right)$, which is larger than that of k unicast transmissions. The gain of multicast transmission results from a merge of relay paths within a minimum spanning tree.

Many following works focus on the combinations of the above characteristics, so as to further increase the network performance. In [18], X-Y. Li et al. explore the multicast capacity in a static hybrid network with infrastructure support. Establishing the multicast tree with the help of infrastructure and employing a hybrid routing scheme, the authors have showed that the achievable multicast capacity in hybrid network with m BSs is $\Theta\left(\max\left[\min\left(\frac{W_B\sqrt{m}}{n_s\sqrt{k}}, \frac{W_c\sqrt{m}}{n_s k}, \frac{W_a\sqrt{m}}{n_s k}\right), \frac{W_a}{n_s\sqrt{k}}\cdot\frac{a}{r}\right]\right)$. On the other hand, W. Huang et al. study the unicast capacity of mobile hybrid network in [19], which jointly consider the influences of node's mobility and infrastructure. The per-node capacity is $\Theta\left(\frac{1}{f(n)}\right)+\Theta\left(\min\left[\frac{m^2c}{n}, \frac{m}{n}\right]\right)$ for strong mobility, and $\Theta\left(\min\left[\frac{m^2c}{n}, \frac{m}{n}\right]\right)$ for weak and trivial mobility.

In this paper, we further study the multicast capacity scaling laws of a mobile hybrid network characterizing both mobility and infrastructure. On one hand, the combination of the above three factors can increase the network capacity significantly. On the other hand, network model featuring these three factors provides a more realistic setting than those in previous works.

In our model, n mobile users and m wire-connected base stations are placed in a wireless ad hoc network, of which the area scales with n as $f^2(n)$. Each of the n users moves around a home-point within a bounded radius. The distribution of home-points follows a cluster model (detailed in Section II).

The number of clusters is n_c, and each cluster has a radius of r. A multicast path can use either infrastructure routing, pure ad hoc routing, or both. We assume that the bandwidth for wireless channel is W. In cellular transmissions, we further divide W into uplink bandwidth W_A and downlink bandwidth W_C. The bandwidth between each pair of BSs is $W_B(n)$ [1].

Our main contributions: We divide mobility into three regimes, and present matching upper bounds and lower bounds for each regime.

- For the first regime (strong mobility regime) where $f(n)\sqrt{\gamma(n)} = o(1), \gamma(n) = \frac{\log n_c}{n_c}$, the per-node capacity of hybrid routing is:

$$\Theta\left\{\max\left[\frac{W}{\sqrt{k}f(n)}, \min\left(\frac{mW_A}{n}, \frac{m^2W_B}{kn}, \frac{mW_C}{kn}\right)\right]\right\}$$

- The second regime (weak mobility regime) stands for the situation that $f(n)\sqrt{\gamma(n)} = \omega(1)$ and $f(n)\sqrt{\tilde{\gamma}(n)} = o(1)$, where $\sqrt{\tilde{\gamma}(n)} = r\sqrt{\frac{\log(n/n_c)}{n/n_c}}$. The per-node capacity by hybrid routing is:

$$\Theta\left[\min\left(\frac{W\sqrt{n_c}}{n^R\sqrt{k}}, \frac{mW_A}{n}, \frac{n_c m^2 W_B}{(n_c-1)nk}, \frac{mn_cW_C}{nk}\right)\right]$$

- The third regime (trivial mobility regime) corresponds to $f(n)\sqrt{\tilde{\gamma}(n)} = \omega(1)$. In this regime, we prove that mobility is trivial and the network acts as a static one.

Different from previous works on hybrid network [12], [13], [14], [18], our work takes node's mobility into account. Besides, our work is the first one to consider the effect of a general mobility model (detailed in Section II) on multicast transmission. As a result, our work not only generalizes capacity results on MANETs and hybrid networks, but also provides a larger capacity under a more realistic network model featuring both mobility and infrastructure. Furthermore, we present new routing schemes to achieve such capacity.

This paper is organized as follows. We first introduce our network models and assumptions in Section II. Section III and Section IV define the uniformly dense networks, and derive multicast capacity in such networks under ad hoc routing and cellular routing, respectively. In Section V, we combine the results of previous two sections and derive multicast capacity under hybrid routing. Capacity analysis of non-uniformly dense networks is carried out in Section VI. At last, we conclude our work in Section VII.

II. Models and Assumptions

We consider a wireless network that consists of n mobile users (also referred as mobile stations, MSs) moving over a bidimensional surface. Communications are carried out in a wireless channel with bandwidth W by ad hoc routing, or by cellular routing with the help of $m = \Theta(n^M)$ base stations (BSs). BSs are connected to each other by optical fibers with bandwidth W_B.

We use $X_i(t)$ to denote the position of the $i-$th MS at any given time t, and $Y_i(t)$ for the $i-$th BS. Since BSs are statically placed in the network, we have $\forall t,\ Y_i(t) \equiv Y_i(0) \triangleq Y_i$. When referring to both MSs and BSs, we adopt $Z_i(t), 1 \le i \le n+m$ to denote their locations. Z_i^h is used to denote the location of home-point for the $i-$th node. We further define operator $\|\cdot\|$ as the Euclidian distance between two points: $d_{ij} = \|Z_i - Z_j\|$.

A. Mobility Model

Definition 1: *(Network Extension)* We assume that the network area $\mathcal{O}$ is a Torus, or a square with wrap-around conditions. The side length of the network scales with n according to a non-decreasing function $f(n) = n^\alpha$, where $\alpha \in [0, 1/2]$. We normalize the whole network to a unit Torus for convenience. Correspondingly, any value representing a distance in the network should be scaled down by $1/f(n)$.

Remark 1: $\alpha = 0$ corresponds to the *dense network* [1], of which the size remains constant while node density increases linearly with n. And $\alpha = 1/2$ represents the *extended network* [4], in which network area increase linearly with n while node density remains constant.

Remark 2: Our cluster and mobility models are similar to that of [3] and [19]. In order to characterize the non-uniform distribution and limited motion observed in the real mobility (see [20] and [21], etc), home-points are introduced. Such a model can provide a better demonstration of the preferential attachment phenomena in real networks. Furthermore, by setting $n_c = n$ and $f(n) = \Theta(1)$, our mobility and cluster model can generalize other classical mobility models, such as i.i.d. mobility model [4], hybrid random walk model [22] and Brownian motion model [5].

Definition 2: *(Home-Point Cluster Model)* We assume that there are $n_c = \Theta(n^C)$ clusters with radius $r = \Theta(n^{-R})$ (before network extension). All the clusters are independently and uniformly distributed in network $\mathcal{O}$. Then each of the n home-points is randomly assigned to a cluster and placed in it uniformly and independently.

Definition 3: *(MS Mobility)* We assume that $\{X_i(t)\}$ are independent, stationary, ergodic and rotation-invariant processes with stationary distribution $\phi_i(X)$:

$$\phi_i(X) = \phi_i(X - X_i^h) = \frac{s(\|X - X_i^h\|)}{\int_{\mathcal{O}} s(\|X - X_i^h\|)dX} \quad (1)$$

where $s(d)$ is an arbitrary non-decreasing function with finite range.

Definition 4: *(Mobility Regimes)* Let $\gamma(n) = \frac{\log n_c}{n_c}$ and $\tilde{\gamma}(n) = r^2\frac{\log(n/n_c)}{n/n_c}$. We say that MS's mobility is strong if $f(n)\sqrt{\gamma(n)} = o(1)$. Weak mobility corresponds to $f(n)\sqrt{\gamma(n)} = \omega(1)$ and $f(n)\sqrt{\tilde{\gamma}(n)} = o(1)$, while trivial mobility corresponds to $f(n)\sqrt{\tilde{\gamma}(n)} = \omega(1)$

Instead of placing BS in a regular pattern as [12], [18], in this paper the distribution of the locations of BSs matches that

[1]In real systems, BSs may not connect to each other. However, by setting W_B as a function of n, we can map them into a network where BSs can communicate with each other utilizing frequency channels with bandwidth at least $W_B(n)$. In fact, such a mapping will not change the main results in this paper. In the rest of this paper, we use W_B for short.

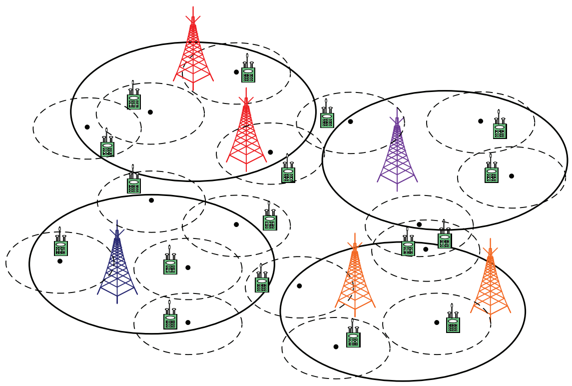

Fig. 1. An example network under cluster and mobility models. We use solid lines for cluster boundaries, dash lines for mobility boundaries, and solid points for home-points.

of MSs, so as to achieve better utilization of BSs. For the $j-$th BS, we randomly select a point Q_j according to the cluster model, then uniformly and independently place the BS in this cluster with distribution $\phi(Y-Q_j)$.

In this paper, we focus on the case that clusters do not overlap with each other w.h.p., which is guaranteed by $C < 2R$. Also, we assume $0 \leq R \leq \alpha$, so that clusters will not shrink with increasing n. In order to ensure that every cluster is served by BSs w.h.p., i.e. $m = \omega(n_c)$. An example network under cluster and mobility models is showed in Fig. 1.

B. Communication and Interference Models

Base stations communicate with each other through optical fibers with bandwidth W_B. Available bandwidth in all the wireless channels is W. In ad hoc routing, transmissions fully occupy the wireless bandwidth W. In cellular routing, wireless bandwidth is further divided into uplink bandwidth W_A and downlink bandwidth W_C. All the communications in wireless channels are characterized by Protocol Model, which is defined as followed.

***Definition** 5: (Protocol Model)* Both BSs and MSs adopt the same transmission range R_T (correspondingly, same transmission power). At each time slot t, a wireless transmission from node i to node j is successful only if: 1)

$$\|Z_i(t) - Z_j(t)\| \leq R_T$$

and 2)for any other node l that transmits at t,

$$\|Z_l(t) - Z_j(t)\| \geq (1+\Delta)R_T$$

where Δ is a constant defining a protection zone around the receiver.

We assume all of the n MSs are source nodes, each of which randomly selects other k MSs as destinations and transmits the same data to them. The definitions of *Feasible Throughput* and *Asymptotic Multicast Per-node Capacity* λ are similar to those in previous works, such as [17], [19], etc.

III. Multicast Capacity in Uniformly Dense Network by Ad Hoc Routing

In this section, we first provide the definition and characteristics of uniformly dense networks. It is showed that when a network falls into the strong mobility regime, it is equivalent to classify it as an uniformly dense network. Then matching upper and lower bounds are presented in both pure ad hoc routing and cellular routing. For pure ad hoc routing, we derive the bounds by mapping the mobile network into a random geometric graph. It is worth to note that our work is the first one to analysis the multicast capacity under such a general mobility model. For cellular routing, we divide the routing scheme into three phases and achieve matching upper and lower bounds in each phase, as well.

A. Preliminary for Uniformly Dense Network

***Definition** 6: (Local Density)* The local density of nodes at any given point X_0 is defined as:

$$\rho^n(X_0) = \sum_{i=1}^{n} E[\mathbf{1}_{X_i^{(n)} \in B(X_0, 1/\sqrt{n})} | \mathbf{H}_{n+m}]$$

where $B(X_0, 1/\sqrt{n})$ is the disk centering at X_0 with radius $1/\sqrt{n}$. $\mathbf{H}_{n+m} = \{Z_i^h\}_{i=1}^{n+m}$ defines the Borel-field of home-points. $E[\cdot]$ stands for expectation, and $\mathbf{1}_{[\cdot]}$ represents the indicator function.

***Definition** 7: (Uniformly Dense Network)* We say a network is uniformly dense if for any $X \in \mathcal{O}$, there exist two positive constants q and Q, such that

$$q < \rho^n(X_0) < Q$$

We introduce two lemmas from [19] and [23].

***Lemma** 1:* Suppose that Z_i^h are placed on $\mathcal{O}$ according to the cluster model. Let $\mathcal{A}$ be any given regular tessellation of $\mathcal{O}$, with area $|\mathcal{A}| \geq (16+\delta)\gamma(n)$, for some small $\delta > 0$. Let $N(\mathcal{A})$ be the number of home-points of both BSs and MSs inside $\mathcal{A}$. Then it holds w.h.p. :

$$\frac{n|\mathcal{A}|}{2} \leq \inf N(\mathcal{A}) \leq \sup N(\mathcal{A}) \leq 2n|\mathcal{A}|. \qquad (2)$$

***Lemma** 2:* If $f(n)\sqrt{\gamma(n)} = o(1)$and $m = O(n)$, where $\gamma(n) = \frac{\log(n_c)}{n_c}$, then the network is uniformly dense.

***Definition** 8: (Link Capacity)* Link capacity between node i and j is defined by the maximal long term data flow between them:

$$\mu^S(i,j) = E[\mathbf{1}_{(i,j)\in\pi^S(t)} | \mathbf{H}_{n+m}]$$

where S is any given scheduling under protocol model, and $\pi^S(t)$ denotes the set of transmission pairs scheduled by S.

B. Upper Bound in Uniformly Dense Network by Ad Hoc Routing

By ad hoc routing, we means that MSs only exchange information in wireless channel with a bandwidth W, ignoring the effect of BSs.

***Definition** 9: (Scheduling Scheme S^*)* S^* schedules the transmission from node i to node j at time slot t only if the following two condition are both satisfied: 1) $\|Z_i(t) - Z_j(t)\| < R_T = \frac{c_T}{\sqrt{n}}$, and 2) for any other node l, whether active or silent, $\|Z_l(t) - Z_j(t)\| > (1+\Delta)R_T$.

It can be proven that scheduling S^* is optimal in order sense. On one hand, $\forall R_T = o(1/\sqrt{n})$ can not fully guarantee connectivity and results in a performance decline. On the other, $\forall R_T = \omega(1/\sqrt{n})$ will cause too much interference and decreases the link capacity, as well.[2]

The next lemma is provided by [3]. It establishes a relationship between link capacity and encounter probability in mobile ad hoc networks.

***Lemma** 3:* In uniformly dense network with bandwidth W, under scheduling S^*, the link capacity between two nodes (at least one of them is MS) is:

$$\mu^{S^*}(i,j) = \Theta\left(W \cdot Pr\left\{d_{ij} \le \frac{c_T}{\sqrt{n}} | \mathbf{H}_{n+m}\right\}\right) \quad (3)$$

***Corollary** 1:* The link capacities between MSs i,j and BS l are:

$$\mu(X_i^h, X_j^h) = \Theta(W \cdot \frac{f^2(n)}{n}\eta(f(n)\|X_i^h - X_j^h\|)) \quad (4)$$

$$\mu(X_i^h, Y_l^h) = \Theta(W \cdot \frac{f^2(n)}{n}s(f(n)\|Y_l^h - X_i^h\|)) \quad (5)$$

By mapping the network into a Generalized Random Geometric Graph $G(n, n_c, r, \mu)$, we can calculate the asymptotic capacity more conveniently. In $G(n, n_c, r, \mu)$, we use n vertices to represent the home-points. For each pair of vertices (i,j), we connect them with an edge of capacity $\mu(X_i^h, X_j^h)$. The following proposition provides an upper bound of multicast capacity in the network.

***Proposition** 1:* Define d_{iD}^h as the length of minimum spanning tree that covers source node i and the corresponding home-points of its k destinations. If throughput λ is sustainable, we have:

$$\lambda \sum_s d_{sD}^h \le \sum_{ij} \mu_{ij}^h d_{ij}^h \quad (6)$$

Also, we have a classic result on d_{iD}^h [17].

***Lemma** 4:* In a d-dimensional cube of side length a, Euclidian minimum spanning tree of n randomly and uniformly distributed nodes has an asymptotic length of $\tau(d)n^{\frac{d-1}{d}}$, where $\tau(d)$ only depends on d.

***Corollary** 2:* $\forall i,\ 1 \le i \le n, d_{iD}^h = \Theta(\sqrt{k})$.

***Theorem** 1:* The upper bound of per-node multicast capacity in uniformly dense network by ad hoc routing is:

$$\lambda = O\left(\frac{W}{\sqrt{k}f(n)}\right) \quad (7)$$

Proof: In the first step, we try to simplify equation (6) and derive a preliminary expression of upper bound using Proposition 1. We introduce a sum: $T^n(d) = -\sum_i \sum_j \mu_{ij}^n \mathbf{1}_{d_{ij}^h > d}$, which represents the opposite value of the summation of link capacities provided by links that satisfy $d_{ij}^h > d$. By definition, it is obvious that $T(d+\Delta) - T(d) = \sum_i \sum_j \mu_{ij}^n \mathbf{1}_{d < d_{ij}^h < d+\Delta}$. So we have : $\sum_i \sum_j \mu_{ij}^n d_{ij}^h = \int x dT^n(x)$. Combining this with Proposition 1 and Corollary 2, it follows:

$$\lambda = O\left(\frac{1}{n\sqrt{k}} \int x dT^n(x)\right) \quad (8)$$

In the second step, we calculate $\int x dT^n(x)$ to finish the proof. Let $D = \Theta(1/f(n))$, we can divide $\int x dT^n(x)$ into $\int_{x \le D} x dT^n(x)$ and $\int_{x > D} x dT^n(x)$.

On one hand, $\int_{x \le D} x dT^n(x)$ is the summation of link capacities whose home-points are within distance $\Theta(1/f(n))$. Assume that there is a simple curve $\mathcal{L}$ dividing $\mathcal{O}$ into A_0 and B_0. The information flow crossing $\mathcal{L}$ is $\sum_{i \in A_0} \sum_{j \in B_0} \mu_{ij}^n$. Define C_0 as the set of nodes, of which the distances from $\mathcal{L}$ are not larger than $\frac{y_0}{f(n)}$. Combining with premise that $x \le D$, we have:

$$\sum_{i \in A_0} \sum_{j \in B_0} \mu_{ij}^n = \sum_{i \in C_0} \sum_{j \in B_0} \mu_{ij}^n$$
$$\le \sum_{i \in C_0} \mu_i^n \le W \cdot N(C_0) \le \Theta\left(\frac{W \cdot n}{f(n)}\right)$$

Substituting this into (8), it follows:

$$\lambda = O\left(\frac{W}{\sqrt{k}f(n)}\right) + O\left(\frac{1}{n\sqrt{k}} \int_{x > \frac{1}{f(n)}} x dT_n(x)\right)$$

On the other hand, we try to find a simple lower bound $S^n(d)$ of $T^n(d)$ to calculate $\int_{x>D} x dT^n(x)$. $S^n(d)$ should satisfy 1) $\sup S^n(d) = 0$ and 2) $T^n(d) \ge S^n(d)$. And this results in:

$$\int_{x>D} x dT^n(x) \le \int_{x>D} x dS^n(x) \quad \forall D \ge 0$$

By definition of transmission range, we have:

$$Pr\{d_{ij}(t) \le R_T\} \le 2 \int_{\mathcal{O}} \phi^n(x) \mathbf{1}_{x > \frac{d_{ij}^h - R_T}{2}} dx$$

Considering definition of $T(d)$, we have:

$$T(d) \ge -2nW \cdot \int_{\mathcal{O}} \phi^n(x) \mathbf{1}_{x > \frac{d_{ij}^h - R_T}{2}} dx = S(d)$$

For any constant M, it follows:

$$\int_{x>D} x dT^n(x)$$
$$\le \frac{-W}{n\sqrt{k}} \int_{y > \frac{M}{f(n)}} 2ny \cdot \frac{d}{dy}\left(\int_{X \in \mathcal{O}} \phi^n(\|X\|)\xi(\|X\|) dX\right) dy$$

where $\xi(\|X\|) = \mathbf{1}_{\|X\| > \frac{d_{ij}^h - R_T}{2}}$

We take $M = 10$ (which does not affect the result in order sense), and when n is large enough, for any $y \ge \frac{10}{f(n)}$, it is

[2] A formal proof for the optimality of scheduling S^* is presented in [19].

true that $\frac{y-R_T}{2} > \frac{y}{3}$. So that:

$$\int_{x>D} x dT^n(x)$$
$$\leq \frac{-W}{n\sqrt{k}} \int_{y>\frac{10}{f(n)}} 2ny \cdot \frac{d}{dy}\left(\int_{X\in\mathcal{O}} \phi^n(\|X\|)\xi(\|X\|)dX\right) dy$$
$$< \frac{4\pi W}{\sqrt{k}} \int_y f^2(n)y^2\phi\left(f(n)(\frac{y}{3})\right)dy = \Theta\left(\frac{W}{\sqrt{k}f(n)}\right)$$

Finally, we have:

$$\lambda = O\left(\frac{W}{\sqrt{k}f(n)}\right) + O\left(\frac{W}{\sqrt{k}f(n)}\right) = O\left(\frac{W}{\sqrt{k}f(n)}\right) \quad \blacksquare$$

C. Lower Bound in Uniformly Dense Network by Ad Hoc Routing

In the following part of this section, we will derive asymptotically matching lower bound on multicast capacity in uniformly dense network by ad hoc routing. We have already mapped a mobile network into static graph, which makes the establishment of a multicast routing possible and realistic. We employ the next algorithm from [17] to set up the multicast routing in graph $G(n, n_c, r, \mu)$.

Algorithm 1 Optimal ad hoc routing for multicast transmission $U_i = \{s_i, d_{i,1}, d_{i,2}, \ldots, d_{i,k}\}$

---

1. Divide $\mathcal{O}$ into squarelets, of which the side length is $c/f(n)$. And we will have $f^2(n)/c^2$ such squarelets. We denotes the squarelet in row i column j as (i, j).
2. We use the following scheme to generate the Euclidian Spanning Tree $EST(U_i)$
   (1) Let the $k+1$ nodes form $k+1$ components;
   (2) repeat (3) and (4) for $g = 1, 2, 3, \ldots, k$;
   (3) in the gth step, divide $\mathcal{O}$ into at most $k-g$ square cells, each with side length $\frac{1}{\sqrt{k-g}}$;
   (4) choose a cell that contain two nodes from different connected components, then connect them using Manhattan routing and merge the corresponding components.
3. For each link uv in $EST(U_i)$, assume that u and v are located inside squarelet (i_u, j_u) and (i_v, j_v). In squarelet (i_u, j_v) or (i_v, j_u), choose a node w, so that uwv is the Manhattan routing connecting u and v. Repeat this for each link in $EST(U_i)$, and merge the corresponding link into Manhattan tree $MT(U_i)$.
4. For each link uw, find a node in each of the squarelets that are crossed by line uw . Obtain a path $P(u, w)$ by connecting all of this nodes. Then delete those not in U_i and obtain the multicast tree $MTR(U_i)$.
5. Return $MTR(U_i)$.

***Lemma** 5:* By Algorithm 1, the probability that a random multicast flow will be routed through a certain squarelet A is at most $c'\sqrt{k}c/f(n)$.

***Theorem** 2:* With the MTR generated by Algorithm 1, the sustainable per-node multicast capacity by ad hoc routing in dense network is $\lambda = \Theta(\frac{1}{\sqrt{k}f(n)})$.

Proof: In a random geometric graph G, number of edges from squarelet A to squarelet B is $N(A)N(B)$. If a per-node capacity λ is sustainable, each of these edges should not be overloaded. Considering the maximum of flows crossing one node defined by Lemma 5, we have:

$$\frac{\lambda\frac{c'cn\sqrt{k}}{f(n)}}{\underline{N}(A)\underline{N}(B)} = \Theta\left(\lambda \cdot \frac{\sqrt{k}f^3(n)}{n}\right)$$
$$\leq W \cdot \mu^n(\overline{d}_{A,B}) = W \cdot g(n)\eta(\sqrt{5}c)$$

where $\overline{d}_{A,B} = \frac{\sqrt{5}c}{f^3(n)\sqrt{k}/n}$.

To sum up, the sustainable per-node multicast capacity in uniformly dense network by ad hoc routing is:

$$\lambda = \Theta\left(\frac{W\cdot g(n)\eta(\sqrt{5}c)}{\frac{\sqrt{k}f^3(n)}{n}}\right) = \Theta\left(\frac{W}{\sqrt{k}f(n)}\right) \quad \blacksquare$$

IV. Multicast Capacity in Uniformly Dense Network by Cellular Routing

In this section, we consider the impact of infrastructure in multicast capacity of a mobile network. Multicast flows will be routed through BSs. We divide the bandwidth of air channels into uplink bandwidth W_A and downlink bandwidth W_C. We further assume that the bandwidth of optical fibers connecting BSs is W_B.

***Definition** 10: (Cellular Routing R^C)* Cellular routing R^C consists of three phases. In the first phase, a multicast source node routes the packets to a BS. In the second phase, the packets are routed to the cells that contain destinations. In the last phase, BSs of these cells broadcast packets to the destinations.

A. Upper Bounds in Uniformly Dense Network by Cellular Routing

We first explore the upper bound in each phase of cellular routing R^C, then combine them together to obtain the overall upper bound.

1) Upper Bound in Phase 1: In phase 1, n MSs act as sources, and try to forward their messages to BSs. Scheduling policy S^* is applied in this phase, and the uplink bandwidth for each BS is W_A.

***Lemma** 6:* The per-node capacity upper bound of uniformly dense network with m BSs and n MSs, in phase 1 of cellular routing R^C, is :

$$\lambda_{p1} = O\left(\frac{mW_A}{n}\right)$$

Proof: Because of the limited number of antennas, a BS can serve at most $\Theta(1)$ MSs in each time slot. That means one BS can at most receive $\Theta(W_A)$ traffic in each time slot. By applying a simple TDMA or FDMA scheme, the maximum of total available uplink resource is $\Theta(W_A \cdot m)$. All the n MSs share this resource equally, which results that per-node capacity of each MS can not exceed $\Theta\left(\frac{mW_A}{n}\right)$. ■

2) Upper Bound in Phase 2: In phase 2, BSs exchange messages received from MSs through optical fibers, each of which has a bandwidth of W_B. We map the network only consisting BSs into a random geometric graph $G_B(m, W_B)$. m vertices are used to represent the BSs. The capacities of edges connecting each pair of BSs (i, j) are W_B. The next proposition holds in such a graph.

***Proposition** 2:* In a random geometric graph, traffic λ is sustainable only if, for any partition (S, D) of vertices, it holds: $\lambda \sum_{s\in S}\sum_{d\in D} \leq \sum_{i\in S}\sum_{j\in D}\mu_{ij}^n$.

***Lemma** 7:* The per-node capacity upper bound of uniformly dense network with m BSs and n MSs, in phase 2 of cellular routing R^C, is :

$$\lambda_{p2} = O\left(\frac{m^2 W_B}{kn}\right)$$

3) Upper Bound in Phase 3: In phase 3, each BS transmits messages in its own cell. Optimal scheduling policy S^* is applied in this phase again, and the downlink bandwidth for each BS is W_C. We map the network consisting of m BSs and n MSs into a random geometric graph $G_C(m+n, W_C)$. m vertices are used to represent the BSs, and the other n are for MSs. Edges only exist from BSs to MSs, with capacity $\mu(Y_i^h, X_j^h)$. There is no link among BSs, as well as MSs.

***Definition** 11: (Inner and outer tessellations)* A torus is regularly divided into multiple squarelets. Assume that, in this torus, there is a subset Ψ bounded by a closed and convex boundary $\mathcal{L}$. The inner tessellation of this torus, denoted as $\underline{\Psi}$, contains squarelets strictly within Ψ. And the outer tessellation is a union of $\underline{\Psi}$ and all the squarelets crossed by boundary $\mathcal{L}$, and is denoted as $\overline{\Psi}$.

***Proposition** 3:* Given an arbitrary non-increasing function s(x) with finite support and $X \in \mathcal{O}$, it holds:

$$\int_{Y\in\mathcal{O}} s(f(n)\|Y-X\|)dY \sim \frac{1}{f^2(n)}$$

***Lemma** 8:* The per-node capacity upper bound of uniformly dense network with m BSs and n MSs, in phase 3 of cellular routing R^C, is :

$$\lambda_{p3} = O\left(\frac{mW_C}{kn}\right)$$

Proof: By Proposition 2, we can derive an expression of upper bound in phase 3:

$$\lambda_{p3} \leq \frac{\sum_{i:Y_i^h\in U_{\mathcal{L}_\mathcal{C}}}\sum_{j:X_j^h\in V_{\mathcal{L}_\mathcal{C}}}\mu(Y_i^h, Y_j^h)}{\sum_{s:Y_s^h\in U_{\mathcal{L}_\mathcal{C}}}\sum_{d:Y_d^h\in V_{\mathcal{L}_\mathcal{C}}}\lambda_{sd}} \quad (9)$$

where $\mathcal{L}_\mathcal{C}$ is an arbitrary, simple, closed and convex curve, which divides $\mathcal{O}$ into $U_{\mathcal{L}_\mathcal{C}}$ and $V_{\mathcal{L}_\mathcal{C}}$. Define $\underline{U}_{\mathcal{L}_\mathcal{C}}$ and $\overline{U}_{\mathcal{L}_\mathcal{C}}$ as the inner and outer tessellations of $U_{\mathcal{L}_\mathcal{C}}$, respectively. Define $\overline{N}_{A_U}$ and $\underline{N}_{A_U}$ as the upper and lower bounds on the number of vertices within an arbitrary squarelet $A_U (A_U \in U_{\mathcal{L}_\mathcal{C}})$. Define $\underline{V}_{\mathcal{L}_\mathcal{C}}$, $\overline{V}_{\mathcal{L}_\mathcal{C}}$, $\overline{N}_{A_V}$ and $\underline{N}_{A_V}$ for $V_{\mathcal{L}_\mathcal{C}}$, similarly. Define $\overline{d}_{A_U,A_V}$ and $\underline{d}_{A_U,A_V}$ as the maximal and minimal distances between points of A_U and A_V.

Denote $\mu_C^{m+n} = \sum_{i:Y_i^h\in U_{\mathcal{L}_\mathcal{C}}}\sum_{j:X_j^h\in V_{\mathcal{L}_\mathcal{C}}}\mu(Y_i^h, X_j^h)$. We can express the upper and lower bounds of μ_C^{m+n} as:

$$\mu_C^{m+n} \geq \sum_{A_U\in\underline{U}_{\mathcal{L}_\mathcal{C}}}\sum_{A_V\in\underline{V}_{\mathcal{L}_\mathcal{C}}}\mu^h(\overline{d}_{A_U,A_V})\underline{N}_{A_U}\underline{N}_{A_V}$$

$$\mu_C^{m+n} \leq \sum_{A_U\in\overline{U}_{\mathcal{L}_\mathcal{C}}}\sum_{A_V\in\overline{V}_{\mathcal{L}_\mathcal{C}}}\mu^h(\underline{d}_{A_U,A_V})\overline{N}_{A_U}\overline{N}_{A_V}$$

We can select tessellations $\underline{U}_{\mathcal{L}_\mathcal{C}}$, $\overline{U}_{\mathcal{L}_\mathcal{C}}$, $\underline{V}_{\mathcal{L}_\mathcal{C}}$ and $\overline{V}_{\mathcal{L}_\mathcal{C}}$ carefully, such that their squarelets are of area $(16+\beta)\gamma(n)$. According to Lemma 1, we have:

$$\mu_C^{m+n} \geq \frac{1}{16}mn(16+\beta)^2\sum_{A_U\in\underline{U}_{\mathcal{L}_\mathcal{C}}}\sum_{A_V\in\underline{V}_{\mathcal{L}_\mathcal{C}}}\mu^h(\overline{d}_{A_U,A_V})\gamma^2(n) \quad (10)$$

$$\mu_C^{m+n} \leq 16mn(16+\beta)^2\sum_{A_U\in\overline{U}_{\mathcal{L}_\mathcal{C}}}\sum_{A_V\in\overline{V}_{\mathcal{L}_\mathcal{C}}}\mu^h(\underline{d}_{A_U,A_V})\gamma^2(n) \quad (11)$$

Since $\gamma^2(n) = o(1)$ in uniformly dense network, (10) and (11) can be seen as upper and lower Riemann sum with mesh size decreasing to 0 as $n\to\infty$. Combining with Proposition 3 and Corollary 1, we have:

$$\begin{aligned}\mu_C^{m+n} &\sim \int_{X\in V_{\mathcal{L}_\mathcal{C}}}\int_{Y\in U_{\mathcal{L}_\mathcal{C}}} mn\mu(d_{Y,X})dYdX \\ &\sim mnW_C\cdot\frac{f^2(n)}{n}\int_{X\in V_{\mathcal{L}_\mathcal{C}}}\int_{Y\in U_{\mathcal{L}_\mathcal{C}}} s(f(n)\|Y-X\|)dYdX \\ &\sim mW_C\cdot f^2(n)\cdot\frac{1}{f^2(n)}\int_{X\in V_{\mathcal{L}_\mathcal{C}}}dX \\ &\sim mW_C\end{aligned}$$

Meanwhile, applying Lemma 1, it is easy to prove that the number of source-destination pairs crossing boundary $\mathcal{L}$ in phase 3 is $\Theta(kn)$, w.h.p.. It is an interpretation of denominator in (9). Now we can conclude that a multicast capacity upper bound in phase 3 of cellular routing R^C is:

$$\lambda_{p3} = O\left(\frac{mW_C}{kn}\right)$$ ■

As previously described, cellular routing R^C is a serial connection of the above 3 phases. According to this characteristic , the overall capacity can not exceed the capacity of any element in the serial connection. With Lemmas 6-8, we have the following theorem.

***Theorem** 3:* The upper bound of per-node multicast capacity in uniformly dense network by cellular routing R^C is:

$$O\left(\min\left[\frac{mW_A}{n}, \frac{m^2W_B}{kn}, \frac{mW_C}{kn}\right]\right)$$

B. Lower Bounds in Uniformly Dense Network by Cellular Routing R^C

Similar to the derivation of upper bounds by cellular routing R^C, we derive lower bounds of cellular routing R^C in 3 phases, respectively. Then a combination of the lower bounds is presented.

1) Lower Bound in Phase 1:

***Lemma** 9:* A traffic rate $\Theta\left(\frac{mW_A}{n}\right)$ is sustainable from any MS to infrastructure system in phase 1 of cellular routing R^C.

2) Lower Bound in Phase 2:

***Lemma** 10:* A traffic rate $\Theta\left(\frac{m^2W_B}{kn}\right)$ is sustainable between BSs in phase 2 of cellular routing R^C.

Proof: Consider the random geometric graph of BSs described in Section IV-A-2), the maximum traffic from source squarelet S to destination squarelet D is upper-bounded by $\lambda\cdot kn$. This traffic is sustainable only if none of the edges in the route is overload. And it holds w.h.p., if:

$$\frac{\lambda_{p2}\cdot kn}{N_b(S)N_b(D)} \sim \frac{\lambda_{p2}\cdot kn}{m^2} \leq W_B$$

Therefore, $\forall\lambda_{p2} \leq \Theta\left(\frac{m^2W_B}{kn}\right)$ is sustainable in phase 2 of cellular routing R^C. ■

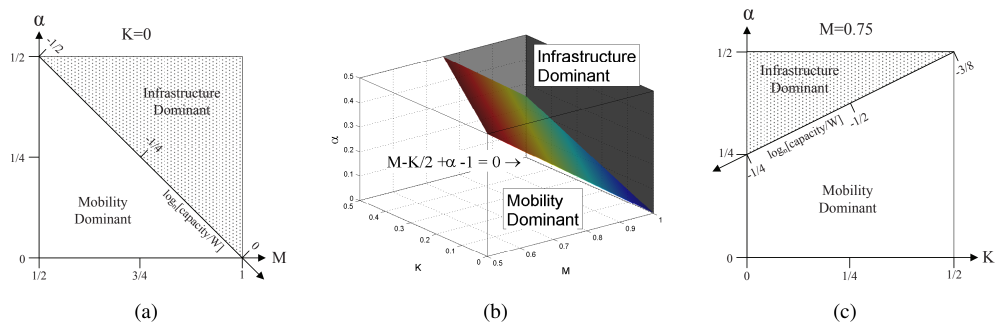

Fig. 2. Graphical illustration of capacity regimes in uniformly dense network by hybrid routing. We present the case of $\varphi = 0$, where the bandwidth between each pair of BSs is a constant. The 3D figure 2-(b) shows the partition between infrastructure dominant regime and mobility dominant regime. The leftmost figure 2-(a) presents the capacity scaling for unicast transmission. And the rightmost figure 2-(c) shows how the multicast capacity scales with K and α, when BSs number is $\Theta(n^{\frac{3}{4}})$.

3) Lower Bound in Phase 3: Let f_i denote the number of multicast flows, each of which has as least one destination inside the $i-$th BS's cell (or transmission range). We borrow the next lemma from [18] to facilitate our proof.

***Lemma** 11:* When $n_s > \max\left(\frac{32m\log m}{k}\log\frac{52m}{k}, \frac{16m}{k}\log(2n)\right)$ is satisfied, the variable $\max_j^m f_j$ is $\Theta(\frac{n_s k}{m})$ w.h.p., where n_s is the number of source nodes.

***Lemma** 12:* A traffic rate $\Theta\left(\frac{mW_C}{kn}\right)$ is sustainable from one BS to MSs in phase 3 of cellular routing R^C.

With Lemmas 9,10 and 12, we can draw the following conclusion:

***Theorem** 4:* The lower bound of per-node multicast capacity in uniformly dense network by cellular routing R^C is:

$$\Theta\left(\min\left[\frac{mW_A}{n}, \frac{m^2W_B}{kn}, \frac{mW_C}{kn}\right]\right)$$

V. Multicast Capacity in Uniformly Dense Network by Hybrid Routing

The hybrid routing utilizes both ad hoc routing and cellular routing, with the purpose of further improving the network capacity and system throughput. It is worth noticing that optimal hybrid routing allows packets to access the infrastructure system at most once. A concise explanation is that accessing infrastructure several times will directly decrease the efficiency of frequency and time resources, especially the efficiency of uplink/downlink bandwidth.

A. Upper and Lower Bounds by Hybrid Routing

***Theorem** 5:* In uniformly dense network, the upper bound of per-node multicast capacity by any hybrid routing is:

$$O\left\{\max\left[\frac{W}{\sqrt{k}f(n)}, \min\left(\frac{mW_A}{n}, \frac{m^2W_B}{kn}, \frac{mW_C}{kn}\right)\right]\right\}$$

***Definition** 12: (Hybrid Routing R^H)* Hybrid Routing R^H evaluates pure ad hoc routing and cellular routing R^C, and adaptively selects a better scheme (with larger throughput) to route the packets.

Applying Theorems 2 and 4 together, we can obtain the following theorem.

***Theorem** 6:* By hybrid routing R^H, the sustainable per-node multicast capacity in uniformly dense network is:

$$\Theta\left\{\max\left[\frac{W}{\sqrt{k}f(n)}, \min\left(\frac{mW_A}{n}, \frac{m^2W_B}{kn}, \frac{mW_C}{kn}\right)\right]\right\}$$

Theorem 6 shows that our hybrid routing R^H is optimal in order sense. Moreover, it proves that the upper bound purposed by Theorem 5 is tight.

B. Discussion on Capacity Regimes

***Proposition** 4:* The optimal frequency allocation between W_A and W_C is:

$$W_A = \frac{W}{k+1},\ W_C = \frac{kW}{k+1}$$

The multicast capacity in uniformly dense network by hybrid routing can be represented by the following parameters: $m = \Theta(n^M), k = \Theta(n^K), W_B = \Theta(W \cdot n^{\varphi})$ and $f(n) = n^{\alpha}$. If $\varphi > -M$, the bottleneck of cellular routing falls in the wireless accesses. Otherwise, the backbone transmission itself becomes the limitation. Fig. 2 provides a graphical illustration of capacity regimes.

VI. Multicast Capacity in Non-Uniformly Dense Network

In the previous sections, we discuss the multicast capacity of mobile uniformly dense network, where $f(n)\sqrt{\gamma(n)} = o(1)$. Now we will present the results in mobile non-uniformly dense network. As is described, mobility in non-uniformly dense network can be further divided into two regimes: weak and trivial. Mobility only helps in intra-cluster message delivering in weak mobility regime. As a result, we present a mixed routing scheme for both intra-cluster and inter-cluster multicast transmissions. Meanwhile, we point out that in trivial mobility regime, the impact of mobility can be ignored. Therefore, capacity results in previous literatures can be applied in this regime.

In non-uniformly dense network without the support of BSs, a larger transmission range should be adopted to guarantee connectivity. In [3], it is proved that this transmission range should be $R_T = \Omega(1/\sqrt{n})$, which only provides a capacity

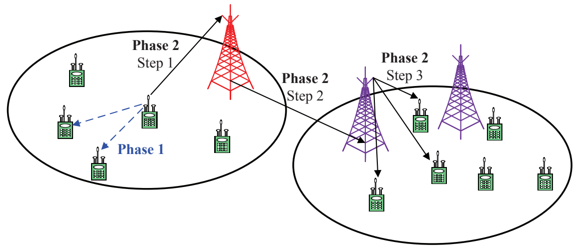

Fig. 3. An example of hybrid routing $\widetilde{R}$ in weak mobility regime

of $\Theta\left(\frac{n_c}{n^2 \log n_c}\right)$ in pure ad hoc networks. The poor capacity is a consequence of more interferences brought by larger transmission range. Considering this, we propose to decrease the transmission range of nodes, and employ BSs to guarantee connectivity. A useful lemma is cited from [19], so as to facilitate our proof.

***Lemma** 13:* Nodes with transmission range $R_T^w = r\sqrt{\frac{n_c}{n}}$ will cause no interference to those in other clusters, with high probability.

According to the new transmission range, we propose a new scheduling scheme.

***Definition** 13: (Scheduling Scheme $\widetilde{S}$)* $\widetilde{S}$ schedules the transmission from node i to node j at time slot t, only if the following two condition are both satisfied: 1) $\|Z_i(t) - Z_j(t)\| < R_T^w$, and 2) for any other node l, whether active or silent, $\|Z_l(t) - Z_j(t)\| > (1+\Delta)R_T^w$.

A. Multicast Capacity in Weak Mobility Regime

Under scheduling scheme $\widetilde{S}$, MSs in different clusters cannot communicate with each other directly in air channels. A hybrid routing is proposed to finish the transmissions.

***Definition** 14: (Hybrid Routing $\widetilde{R}$)* Hybrid Routing $\widetilde{R}$ consists of 2 phases. In phase 1, each source node transmits packets to destinations in its own cluster. In phase 2, each source node employs cellular routing (as defined in Definition 10) to transmit packets to destinations in other clusters.

Fig.3 gives an example of hybrid routing $\widetilde{S}$ in weak mobility regime.

1) Multicast Capacity in Phase 1 of $\widetilde{R}$: Since there is no inter-cluster interference in phase 1, each cluster acts like a sub-network independently.

***Theorem** 7:* In weak mobility regime, each cluster forms an independent sub-network, which is uniformly dense.

With Theorem 7, we can map our results in Section III directly into phase 1 of $\widetilde{R}$, and obtain the following theorem.

***Theorem** 8:* The per-node multicast capacity of phase 1 in $\widetilde{R}$ is

$$\widetilde{\lambda}_{p1} = \Theta\left(\frac{W\sqrt{n_c}}{n^R\sqrt{k}}\right)$$

2) Multicast Capacity in Phase 2 of $\widetilde{R}$: Since we apply cellular routing R^C in phase 2 of hybrid routing $\widetilde{R}$, we can further divide phase 2 of $\widetilde{R}$ into three serial steps, accordingly. If we can determine the parameters in each step, previous results can be mapped into phase 2 of $\widetilde{R}$.

Due to the limited space, we skip the mapping of parameters in this phase, and present corresponding lemmas directly.

***Lemma** 14:* The per-node capacity in the first step of $\widetilde{R}$'s phase 2 is:$\Theta\left(\frac{mW_A}{n}\right)$.

***Lemma** 15:* The per-node capacity in the second step of $\widetilde{R}$'s phase 2 is:$\Theta\left(\frac{n_c m^2 W_B}{(n_c-1)nk}\right)$

***Lemma** 16:* The per-node capacity in the last step of $\widetilde{R}$'s phase 2 is:$\Theta\left(\frac{mn_c W_C}{kn}\right)$

Now that we have the results in all the serial components of hybrid routing $\widetilde{R}$, the overall capacity should be restricted by the minimum throughput of all the components.

***Theorem** 9:* In weak mobility regime, the per-node multicast capacity of mobile ad hoc network with infrastructure support, by hybrid routing $\widetilde{R}$, is:

$$\Theta\left[\min\left(\frac{W\sqrt{n_c}}{n^R\sqrt{k}}, \frac{mW_A}{n}, \frac{n_c m^2 W_B}{(n_c-1)nk}, \frac{mn_c W_C}{nk}\right)\right]$$

B. Multicast Capacity in Trivial Mobility Regime

In the following, $\tilde{n} = n_c/n$ denotes the number of MSs in each cluster. Applying the connectivity criterion in [24], the critical transmission range within each cluster is $R_T^t = r\sqrt{\log(\tilde{n})/\pi\tilde{n}}$, when $f(n)\sqrt{\tilde{\gamma}(n)} = \omega(1)$. From Lemma 13, it is obvious that R_T^t will cause no inter-cluster interference. We can map each cluster into a sub-network almost the same as Theorem 7. We use $\widetilde{f}(n)$ and $\widetilde{R}_T$ respectively to denote the scaling factor and transmission range after the mapping .

***Theorem** 10:* In trivial mobility regime, nodes' mobility are negligible and the whole network acts as a static one.

Proof: Since the clusters are independent with each other in trivial mobility regime, we only need to prove that each node acts like a static one within each cluster. It is equivalent to prove that the following four conditions hold for any time slots t_0 and t ($t_0 < t$):

1) Any pair i, j that can communicate at t_0 will be able to communicate at t.
2) Any pair i, j that cannot communicates at t_0 still cannot communicate at t.
3) Any pair i, j that cause interference to each other at t_0 will still cause interference to each other at t.
4) Any pair i, j that cause no interference to each other at t_0 will not cause any interference to each other at t.

According to the mobility model, a node can at most move $D/\widetilde{f}(n)$ away from its home-point. Hence the maximal change of distance between two nodes is $4D/\widetilde{f}(n)$. Considering the definition of trivial mobility regime, it holds that $\widetilde{R}_T = \omega(\frac{1}{\widetilde{f}(n)})$. We first consider the probability of condition 1):

$$\begin{aligned} P_1 =& Pr\{d_{ij}(t) \le \widetilde{R}_T | d_{ij}(t_0) \le \widetilde{R}_T\} \\ =& Pr\left\{d_{ij}(t_0) \le \widetilde{R}_T - \frac{4D}{\widetilde{f}(n)} | d_{ij}(t_0) \le \widetilde{R}_T\right\} \\ =& \frac{\pi\left(\widetilde{R}_T - \frac{4D}{\widetilde{f}(n)}\right)^2}{\pi\widetilde{R}_T^2} \to 1 \end{aligned}$$

The probability of condition 2) follows:

$$\begin{aligned}P_2 =&1 - Pr\{d_{ij}(t) \leq \widetilde{R}_T | d_{ij}(t_0) > \widetilde{R}_T\} \\ =&1 - \frac{Pr\{\widetilde{R}_T < d_{ij}(t_0) \leq \frac{4D}{f(n)}\}}{Pr\{d_{ij}(t_0) \leq \widetilde{R}_T\}} \\ \to&1 - \frac{0}{Pr\{d_{ij}(t_0) \leq \widetilde{R}_T\}} = 1\end{aligned}$$

Let A_3 denotes the event of $\widetilde{R}_T \leq d_{ij}(t_0) \leq (1+\Delta)\widetilde{R}_T$, then the probability of condition 3) can be expressed as:

$$\begin{aligned}P_3 =&Pr\left\{\widetilde{R}_T + \frac{4D}{f(n)} \leq d_{ij}(t_0) \leq (1+\Delta)\widetilde{R}_T - \frac{4D}{f(n)} | A_3\right\} \\ =&\frac{\pi\left\{\left[(1+\Delta)\widetilde{R}_T - \frac{4D}{f(n)}\right] - \left(\widetilde{R}_T + \frac{4D}{f(n)}\right)\right\}^2}{\pi[(1+\Delta)\widetilde{R}_T - \widetilde{R}_T]^2} \\ =&\frac{\left[\Delta\widetilde{R}_T - \frac{8D}{f(n)}\right]^2}{[\Delta\widetilde{R}_T]^2} \to 1\end{aligned}$$

Derivation of condition 4)'s probability is similar to that of condition 2). Therefore, we skip the detail and present the result: $P_4 \to 1$ when $n \to \infty$.

To sum up, all of the 4 conditions hold for any time slots t_0 and t ($t_0 < t$) w.h.p.. ∎

With the help of Theorem 8, we can extend existing results of static network into trivial mobility regime of mobile network. Since pervious works, such as [12], [18], [19], etc, have already discussed capacity of static network with infrastructure support, one can refer to theses works for more details.

VII. Conclusion

This paper analyzes the multicast capacity in mobile ad hoc network with infrastructure support. Node distribution follows a cluster model, which characterizes the spatial inhomogeneity of realistic mobility. First of all, mobility is classified into three regimes: strong, weak and trivial. It is showed that, only in the first regime can mobility help to forward the inter-cluster multicast transmissions. When it is in weak regime, mobility is only helpful in delivering intra-cluster packets. Moreover, mobility's impact is negligible when it falls into trivial regime. Secondly, infrastructure support can essentially improve the network capacity. Especially in the last two regimes, infrastructure plays a major role in forwarding inter-cluster packets. Thirdly, hybrid routing schemes are proposed to achieve capacity bounds in each of the regimes. It is worth pointing out that our work not only generalizes previous results in on MANETs and hybrid networks, but also provides a new insight on a more realistic network model featuring multicast transmissions, node's mobility and infrastructure support. Our results are instructive in the design of real hybrid systems combining cellular and ad hoc transmissions.

Acknowledgment

This paper is supported by National Fundamental Research Grant (No. 2011CB302701); NSF China (No. 60832005 and No. 60934003); China Ministry of Education New Century Excellent Talent (No. NCET-10-0580); China Ministry of Education Fok Ying Tung Fund (No. 122002); Qualcomm Research Grant; Shanghai Basic Research Key Project (No. 11JC1405100); China National Science and Technology Major Project (No. 2011ZX03001-002-01 and No. 2010ZX03003-001-01); NSF Grant (CNS-0721477).

References

[1] P. Gupta and P. R. Kumar, "The capacity of wireless networks," *IEEE Trans. Inf. Theory*, vol. 46, no. 2, pp. 388–404, 2000.
[2] M. Grossglauser and D. N. C. Tse, "Mobility increases the capacity of ad hoc wireless networks," *IEEE/ACM Trans. Networking*, vol. 10, no. 4, pp. 477–486, 2002.
[3] M. Garetto, P. Giaccone, and E. Leonardi, "Capacity scaling in delay tolerant networks with heterogeneous mobile nodes," in *ACM MobiHoc 2007*, New York, USA, 2007, pp. 41–50.
[4] M. J. Neely and E. Modiano, "Capacity and delay tradeoff for ad hoc mobile networks," *IEEE Trans. Inf. Theory*, vol. 51, no. 6, pp. 1917–1937, 2005.
[5] X. Lin, G. Sharma, R. R. Mazumdar, and N. B. Shroff, "Degenerate delay-capacity tradeoffs in ad-hoc networks with brownian mobility," *IEEE/ACM Trans. Networking*, vol. 14, no. SI, pp. 2777–2784, 2006.
[6] Y. Wang, X. Chu, X. Wang, and Y. Cheng, "Optimal multicast capacity and delay tradeoffs in manets: A global perspective," in *IEEE Infocom 2011*, Shanghai, China, 2011.
[7] P. Li, Y. Fang, and J. Li, "Throughput, delay, and mobility in wireless ad hoc networks," in *IEEE Infocom 2010*, San Diego, CA, USA, 2010.
[8] L. Ying, S. Yang, and R. Srikant, "Optimal delay-throughput trade-offs in mobile ad-hoc networks," *IEEE Trans. Inf. Theory*, vol. 54, no. 9, pp. 4119 – 4143, 2008.
[9] S. Zhou and L. Ying, "On delay constrained multicast capacity of large-scale mobile ad-hoc networks," in *IEEE Infocom 2010 mini-conference*, San Diego, CA, USA, 2010.
[10] W. Huang and X. Wang, "Throughput and delay scaling of general cognitive networks," in *IEEE Infocom 2011*, Shanghai, China, 2011.
[11] H. Li, X. Liu, W. He, J. Li, and W. Dou, "End-to-end delay analysis in wireless network coding: A network calculus-based approach," in *IEEE ICDCS 2011*, Minneapolis, MN, USA, 2011.
[12] B. Liu, Z. Liu, and D. Towsley, "On the capacity of hybrid wireless networks," in *IEEE Infocom 2003*, vol. 2, San Francisco, USA, 2003, pp. 1543–1552.
[13] U. C. Kozat and L. Tassiulas, "Throughput capacity of random ad hoc networks with infrastructure support," in *ACM MobiCom 2003*, New York, USA, 2003, pp. 55–65.
[14] A. Agarwal and P. R. Kumar, "Capacity bounds for ad hoc and hybrid wireless networks," *SIGCOMM Comput. Commun. Rev.*, vol. 34, no. 3, pp. 71–81, 2004.
[15] P. Li and Y. Fang, "Impacts of topology and traffic pattern on capacity of hybrid wireless networks," *IEEE Trans. Mobile Computing*, vol. 8, no. 12, pp. 1585 – 1595, 2009.
[16] P. Li, M. Pan, and Y. Fang, "The capacity of three-dimensional wireless ad hoc networks," in *IEEE Infocom 2011*, Shanghai, China, 2011.
[17] X.-Y. Li, "Multicast capacity of wireless ad hoc networks," *IEEE/ACM Trans. Networking*, vol. 17, no. 3, pp. 950–961, 2009.
[18] X. Mao, X.-Y. Li, and S. Tang, "Multicast capacity for hybrid wireless networks," in *ACM MobiHoc 2008*, Hong Kong, China, 2008, pp. 189–198.
[19] W. Huang, X. Wang, and Q. Zhang, "Capacity scaling in mobile wireless ad hoc network with infrastructure support," in *IEEE ICDCS, 2010*, Genoa, Italy, 2010, pp. 848–857.
[20] J. H. Kang, W. Welbourne, B. Stewart, and G. Borriello, "Extracting places from traces of locations," *SIGMOBILE Mob. Comput. Commun. Rev.*, vol. 9, no. 3, pp. 58–68, 2005.
[21] Z. Yang and Y. Liu, "Understanding node localizability of wireless ad-hoc networks," in *IEEE Infocom 2010*, San Diego, CA, USA, 2010.
[22] G. Sharma, R. Mazumdar, and N. B. Shroff, "Delay and capacity trade-offs in mobile ad hoc networks: A global perspective," *IEEE/ACM Trans. Networking*, vol. 15, no. 5, pp. 981–992, 2007.
[23] M. Garetto, P. Giaccone, and E. Leonardi, "Capacity scaling in ad hoc networks with heterogeneous mobile nodes: The super-critical regime," *IEEE/ACM Trans. Networking*, vol. 17, no. 5, pp. 1522–1535, 2009.
[24] P. Gupta and P. R. Kumar, "Critical power for asymptotic connectivity," in *IEEE Conference on Decision and Control, 1998*, vol. 1, Florida, USA, 1998, pp. 1106–1110.