Fundamental Lower Bound for Node Buffer Size in Intermittently Connected Wireless Networks

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Abstract—We study the fundamental lower bound for node buffer size in intermittently connected wireless networks. The intermittent connectivity is caused by the possibility of node inactivity due to some external constraints. We find even with infinite channel capacity and node processing speed, buffer occupation in each node does not approach zero in a static random network where each node keeps a constant message generation rate. Given the condition that each node has the same probability p of being inactive during each time slot, there exists a critical value $p_c(\lambda)$ for this probability from a percolation-based perspective. When $p < p_c(\lambda)$, the network is in the supercritical case, and there is an achievable lower bound for the occupied buffer size of each node, which is asymptotically independent of the size of the network. If $p > p_c(\lambda)$, the network is in the subcritical case, and there is a tight lower bound $\Theta(\sqrt{n})$ for buffer occupation, where n is the number of nodes in the network.

1 INTRODUCTION

Scaling properties of capacity, connectivity and delay of large-scale wireless networks has received considerable attention in the past several years, since the seminal work on capacity of wireless networks by Gupta and Kumar [1]. Traditionally study on these topics focuses on the assumption of maintaining always full connectivity. However, there is the case where only intermittent connectivity between source and destination is guaranteed, thus a complete path from the source to the destination does not exist all the time. This type of networks are sometimes referred to as Delay/Disruption Tolerant Networks (DTNs) [2]. Properties including capacity, delay and storage of DTNs, routing schemes and other related network design strategies have been studied in [3], [4], [5], [6], [7].

We consider wireless networks where intermittent connectivity is caused by the possibility of *node inactivity* or *link inactivity* due to some practical constraints, and such constraints may vary in different types of networks. In [8], O. Dousse *et al.* studied the latency of wireless sensor networks with uncoordinated power saving mechanisms, where constraint on the network is limited node energy and nodes switch between active (on) mode and inactive (off) mode. In [9], W. Ren and Q. Zhao considered a cognitive radio network where secondary users should keep inactive until the availability of wireless channel, and constraint for the secondary networks is the existence of primary users. In [10], Z. Kong and E. M. Yeh studied a mobile wireless network where the link between two nodes might break (turn inactive) when distance between them is out of the transmission range. One common feature of the above three papers is that they are all based on the theory of *percolation* (see [13], [14], [15]). These constraints above are external in some sense, which implies that inactivity and waiting caused by them cannot be eliminated via improving processing speed of each node or physical conditions of wireless channels. In intermittently connected networks, adequate buffer is required for each node to temporarily store the packets not ready to be sent out. With external constraints, the minimum buffer size requirements for each node do not approach zero as the network capacity and node processing speed approach infinity. Hence, there exists a lower bound on node buffer size.

As we know, in an always full-connected wireless network, buffer is also required for queuing, and queuing delay is the waiting time between the point of entry of a packet in the transmit queue to the actual point of transmission. Throughput capacity of mobile wireless networks with limited node buffer has been investigated by J. D. Herdtner and E. Chong in [11]. In [12], S. Bodas *et al.* have studied scheduling methods in multi-channel wireless networks in the small-buffer regime. A fundamental difference of always full-connected networks from intermittently connected networks is that if the actual workload of the network keeps constant, queuing delay and minimum required buffer size decrease as the capacity and node processing speed increase. Further, as the capacity and node processing speed of the network go to infinity, the minimum required buffer size and queuing delay both go to zero.

In this paper we focus on node buffer occupation in static random wireless networks with intermittent con-

nectivity where node inactivity is possible[1]. To investigate the fundamental requirements on node buffer size posed by the possibility of node inactivity, we assume the capacity and node processing speed can be regarded as infinity, compared with the actual utilization of the network capacity. Such assumption makes sense in networks where message generation rate is slow and capacity and processing speed are adequate. Even if the network capacity and processing speed are limited, results in this paper present a lower bound on node buffer occupation. As in [8], [9] and [10], we take advantage of percolation theory to study this problem. To simplify the analysis, we assume each node has the same probability p of being inactive, and states of nodes (active or inactive) change over time. We find a critical value for p, $p_c(\lambda)$, where λ is the constant node density. If $p < p_c(\lambda)$, the network is in the *supercritical case*, where there exists a unique infinite connected cluster of active nodes at any time a.s. when the network size goes to infinity. In contrast, if $p > p_c(\lambda)$, the network is in the *subcritical case*, where no such infinite cluster exists a.s. when the network size goes to infinity. Minimum buffer occupation is quite different in the two cases. In the supercritical case, there is an achievable lower bound on occupied buffer size which is asymptotically independent of the size of the network; while in the subcritical case, buffer occupation increases as size of the network grows, and there is a tight lower bound $\Theta(\sqrt{n})$ for the achievable minimum node buffer size, where n is the number of nodes.

Node buffer requirement is a crucial consideration in large scale networks where resource of a single node is limited, and a typical type of such network is wireless sensor networks (WSNs). In [20] and [21], Y. Liu *et al.* have studied diagnosis for failures (inactivity) and scalability in large WSNs. Since converge-cast is common in WSNs, we will extend the results to converge-cast in our future work, which can be applied to such WSNs. Further study can be extended to multicast [18] and mobile networks [19].

This paper is organized as follows. In Section 2, we present the network model and some basic assumptions. In Section 3 and 4, we analyze the node buffer occupation in the supercritical case and subcritical case, respectively. In Section 5, we discuss the effects of state-changing frequency on buffer occupation, and the results for networks with finite capacity. Finally, we conclude this paper in Section 7.

2 NETWORK MODEL AND ASSUMPTIONS

2.1 Node Locations and Direct Links

We consider a Poisson point process on $\mathbb{R}^2$ with constant point density λ. Locations of nodes in the network are

1. For networks with link inactivity, we can get similar results as in networks with node inactivity, but the analysis is more complicated.

the points within the square region $\mathcal{B} = \left[-\frac{L}{2}, \frac{L}{2}\right]^2$. Let n denote the number of nodes in the network. According to the property of Poisson point process, $\frac{n}{\lambda L^2} \to 1$ as $L \to \infty$.

Each node covers a disk shaped area with radius r. To simplify the analysis, r is the same for all nodes. Let $X_i (1 \leq i \leq n)$ denote the random position of node v_i. Two nodes v_i and v_j are directly connected via a *direct link* if and only if $||X_{v_i} - X_{v_j}|| \leq 2r$, where $||X_{v_i} - X_{v_j}||$ is the Euclidean distance between v_i and v_j. Without loss of generality, we assume $2r = 1$.

The set of all nodes in the network is denoted by $\mathcal{N}(\lambda, L)$. When $L \to \infty$, $\mathcal{B} \to \mathbb{R}^2$, the corresponding set of nodes is defined as $\mathcal{N}_\infty(\lambda) = \lim_{L \to \infty} \mathcal{N}(\lambda, L)$.

According to continuum percolation theory, there is a critical value for λ, λ_c, and there exists a unique infinite connected cluster in $\mathcal{N}_\infty(\lambda)$ (*giant cluster*, denoted by $C(\mathcal{N}_\infty(\lambda))$) if and only if $\lambda > \lambda_c$. To assure the majority of the network is connected, we make the following assumption.

Assumption 1 (On Node Density): Node density in the network is large enough to guarantee percolation, i.e. $\lambda > \lambda_c$.

In this paper, we mainly analyze communications of nodes within the giant cluster. We denote the nodes belonging to the giant cluster by the term *connected nodes*.

For a finite network, we define the giant cluster as the largest connected cluster in the network. The number of connected nodes n_c approaches to a constant proportion of n, i.e., $\frac{n_c}{n} \to c_\lambda$ as $n \to \infty$, where c_λ is determined by λ.

2.2 External Constraints and Node Inactivity

In this paper, instead of restricting our study on a specific type of external constraints, we consider the general effects of external constraints on the network: they make each nodes switching between active state and inactive state. During active state, a node can transmit or receive messages, while during inactive state it can neither transmit nor receive messages. Transmission between two nodes is possible only if both the transmitter and the receiver are active.

We assume the external constraints in the network are in a synchronized time-slotted manner with a slot length T_{EC}, which implies that the state of each node changes only at the beginning of a time slot. Further, the effects of external constraints satisfy the following assumptions:

1) States of each active nodes vary from one time slot to another, and are i.i.d. over different time slots.
2) The probability of being inactive is a constant p for all nodes in the network.
3) States of different nodes are i.i.d.

The network with external constraints is denoted by $\mathcal{CN}(\lambda, L, p)$ (or $\mathcal{CN}_\infty(\lambda, p)$ if $L = \infty$). Because of the possibility of node inactivity, we cannot guarantee a

complete path connecting a pair of nodes all the time. Hence, the network is *intermittently connected*.

2.3 Traffic Pattern and Buffering

We only consider the traffic of nodes within $C(\mathcal{N}(\lambda, L))$, i.e., the connected nodes.

Traffic Pattern of Connected Nodes: For each connected node in the network, as a source, it randomly chooses a permanent destination among other connected nodes (uniformly), and this source-destination relationship does not change over time. Each connected node generates messages to its corresponding destination node in a multihop fashion at a constant rate, r_g, which does not vary over different nodes.

Buffering: In each hop, if the transmitter or the receiver is inactive, the message should be kept in the buffer of the transmitter until both nodes are active. As we define before, if a node (as a source) or its first intermediate node toward destination is inactive, it cannot send any message. Yet we can still assume the source node "sends" messages at rate r_g but temporarily stored in the buffer of itself.

We define the *per-node throughput capacity* as the maximum bits per second each connected node can send to its chosen destination node. Now we give a basic assumption on channel capacity and per-node throughput capacity in this paper.

Assumption 2 (On Capacity and Processing Speed): First, the channel capacity for every directly connected nodes is large enough to be viewed as infinity, compared to the actual transmission rate of each node. As a result, per-node throughput capacity can also be viewed as infinity compared to r_g. Second, node processing speed is also infinitely large, compared to the state-switching frequency $\frac{1}{T_{EC}}$.

As Assumption 2 states, the capacity of the network and processing speed is infinity, which implies that once a node and its next hop turn active, they can transmit and receive message without delay[2]. If all nodes in one path are active, the message can be transmitted from one end to the other without delay. This helps us focus on the limits posed by intermittent connectivity on node buffer size in the network, and other factors are considered ideal.

Maximum Buffer Occupation in Each Time Slot: Since the capacity is infinity, buffered messages in each node are transmitted only at the beginning of each time slot within a very small time interval. On the other hand, the message generation rate r_g is finite and constant, and the buffer occupation of one node will not decrease

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2. The propagation delay is omitted in the network. Since the channel capacity and processing speed is infinity, the queuing delay in each node is zero.

but only possibly grow during the time interval except the beginning. Therefore, in each time slot, the size of occupied buffer in each node is maximum at the end of the time slot. For a connected node w, we use $S_w^{(L)}(t)$ (or $S_w^{(\infty)}(t)$) to denote the occupied buffer size of w at the end of time slot t in $\mathcal{CN}(\lambda, L, p)$ (or $\mathcal{CN}_\infty(\lambda, p)$).

Message Slot: We assume the transmission path of each message does not change if the states of all the nodes in the network do not change. Therefore, it is obvious that the messages generated by one node during one time slot must exist at the same node at the end of a time slot. We call the messages generated by u during time slot t whose destination is v a *message slot*, denoted by $m_{u \to v}^{(t)}$. If only the source or destination is specified, the notation is simplified as $m_{u \to}^{(t)}$ or $m_{\to v}^{(t)}$. If the generating time slot is not specified, the notations can be further simplified as $m_{u \to v}$, $m_{u \to}$ and $m_{\to v}$. The size of one message slot is $r_g T_{EC}$.

2.4 Percolation of Active Nodes

According to Assumption 1, a giant cluster always exists a.s. as the size of the network goes to infinity. With external constraints, for each time slot, we consider the percolation phenomenon among active nodes.

Since the states of nodes are i.i.d., the distribution of active nodes in $\mathcal{CN}_\infty(\lambda, p)$ is according to a Poisson Point Process with constant point density $(1 - p)\lambda$. Therefore, there exists a critical value for $p_c(\lambda) = 1 - \frac{\lambda_c}{\lambda}$ such that:

1) If $p < p_c(\lambda)$, $\mathcal{CN}_\infty(\lambda, p)$ is in the supercritical case, where there exists a unique infinite connected cluster of active nodes a.s. during each time slot. Let $C(\mathcal{CN}_\infty(\lambda, p), t)$ denote the infinite connected cluster of active nodes (active giant cluster) during time slot t.

2) If $p > p_c(\lambda)$, $\mathcal{CN}_\infty(\lambda, p)$ is in the subcritical case, where there does not exist a unique connected cluster of active nodes a.s. during each time slot.

In the supercritical case, $C(\mathcal{CN}_\infty(\lambda, p), t) \subseteq C(\mathcal{N}_\infty(\lambda))$, i.e. the active giant cluster is part of the giant cluster.

To illustrate this point, we first suppose there is a node $v \in C(\mathcal{CN}_\infty(\lambda, p), t)$ but $v \notin C(\mathcal{N}_\infty(\lambda))$. Then $C(\mathcal{N}_\infty(\lambda)) \cap C(\mathcal{CN}_\infty(\lambda, p), t) = \emptyset$, because otherwise v can connect to $C(\mathcal{N}_\infty(\lambda))$ through the nodes belonging to both sets. Therefore, there exist two disjoint infinite connected clusters in $\mathcal{N}_\infty(\lambda)$, which contradicts the fact that $C(\mathcal{N}_\infty(\lambda))$ is unique.

Let $\theta(\lambda, p|active)$ denote the probability that an active node belongs to the active giant cluster in an arbitrary time slot, then we have

$$\theta(\lambda, p|active) \begin{cases} > 0, & p < p_c(\lambda) \\ = 0, & p > p_c(\lambda) \end{cases}.$$

The definition of supercritical and subcritical cases above is in terms of infinite-sized network. However,

when the network size is finite, we can also use the value of p to determine which case it belongs to. Buffer occupations in the two cases behave quite differently when we let the network size approach infinity. Our work will focus on the order of the buffer occupation as a function of the network size in the two cases respectively.

3 LOWER BOUND FOR BUFFER SIZE IN THE SUPERCRITICAL CASE

In this section, we study the achievable lower bound for buffer size of connected nodes in $\mathcal{CN}(\lambda, L, p)$ when $p < p_c(\lambda)$. The main result is that the expected value of minimum buffer occupation of each connected node is asymptotically independent of the size of the network, as stated in Theorem 1.

Theorem 1: For a uniformly randomly selected connected node w in $\mathcal{CN}_\infty(\lambda, p)$ with $p < p_c(\lambda)$, at the end of an time slot t,

$$\mathbb{E}\left(S_w^{(\infty)}(t)\right) \geq b_1 r_g T_{EC}. \tag{1}$$

And it is achievable with some routing scheme that

$$\mathbb{E}\left(S_w^{(\infty)}(t)\right) \leq c_1 r_g T_{EC} < \infty. \tag{2}$$

b_1 and c_1 are positive constants, and $r_g T_{EC}$ is the length of each message slot. The lower bound $b_1 r_g T_{EC}$ holds for all possible schemes, while the upper bound $c_1 r_g T_{EC}$ holds in the optimal case. Therefore, $\Theta(1)$ is the fundamental achievable lower bound for buffer size in the supercritical case.

3.1 Proof of Inequality (1)

For the lower bound in Inequality (1), each connected node should at least buffer the messages generated by itself before it and its neighbors turn active. Since at least the source should be active when it sends out a message, the average waiting time before a message is sent out of its source is larger than $\frac{T_{EC}}{1-p}$. Applying Little's Law,

$$\mathbb{E}\left(S_w^{(\infty)}(t)\right) \geq r_g T_{EC} \frac{T_{EC}}{T_{EC}(1-p)} = b_1 r_g T_{EC},$$

where b_1 is determined by p, which does not depend on the size of the network.

In the following parts of this section, we first present a routing scheme, and then prove that under this scheme, the buffer size requirement specified in Inequality (2) is achieved.

3.2 Optimal Routing Scheme

Assumption 3 (On ORS): In Optimal Routing Scheme (ORS), we assume that each connected node knows whether it belongs to the active giant.

Assumption 3 is important in the Optimal Routing Scheme which relies on the active giant. Before describing the ORS, we first introduce the *source extending path*.

3.2.1 Source Extending Path (SEP)

Starting from each connected node, we draw a ray with a permanent random direction, and let R_u denote the ray starting from u. Then divide the ray into a string of segments of constant length l_s, as illustrated in Fig. 1. The nearest connected node to each segment endpoints is a *flag node*, and we connect every two neighboring flag nodes by the shortest path. After that, if any node in the path overlaps with more than two other nodes in the path, remove the redundant nodes, as shown in Fig. 1. The infinite path starting from u is termed as its *source extending path of u (SEP_u)*. In Optimal Routing Scheme, each message slot is first transmitted along the source extending path.
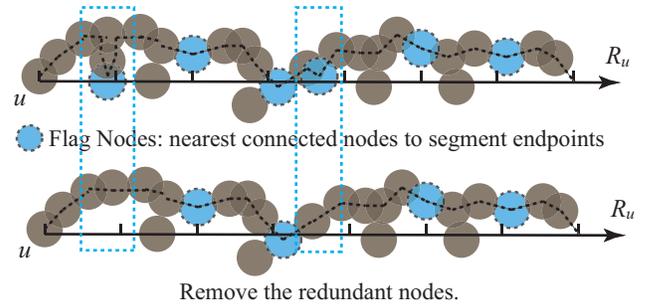

Fig. 1. Source Extending Path.

3.2.2 Optimal Routing Scheme

There are 3 stages of message forwarding in the optimal routing scheme. Consider the transmission of message slot $m_{u \to v}$.

  I Source u starts sending $m_{u \to v}$ along its source extending path SEP_u hop by hop, and each node on the path keeps a copy of $m_{u \to v}$. Before transmission on each hop begins, the node that newly received $m_{u \to v}$ in the source extending path must finish sending $m_{u \to v}$ to its other neighbors that do not belong to SEP_u, as shown in Fig. 2. [3] This process stops when any node that has received the messages belongs to the active giant, $C(\mathcal{CN}_\infty(\lambda, p), t)$. At the end of this stage, the set of nodes that contains a copy of $m_{u \to v}$ (including nodes on SEP_u and their neighbors) is called the *source buffering path* of $m_{u \to v}$, $SBP_{m_{u \to v}}$.

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3. This is to make sure that there are no nodes that intersect the source extending path but do not have the copy of $m_{u \to v}$, which is necessary in the proof of Lemma 1.
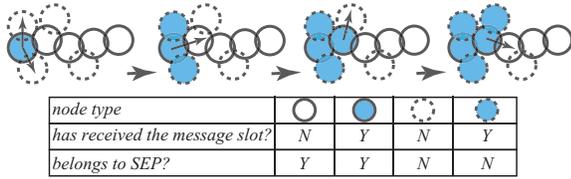
Fig. 2. Message transmission in the first stage.

II In this stage, the message is forwarded as near to the destination as possible. Assume w_1 in the source buffering path belongs to the active giant in time slot t, i.e. $w_1 \in C(\mathcal{CN}_\infty(\lambda, p), t) \cap SBP_{m_{u \to v}}$. The transmission starts from w_1 but no longer along SEP. Instead, w_1 sends the messages generated and being generated by the source to w_2 via the shortest path in $C(\mathcal{CN}_\infty(\lambda, p), t)$, where

$$w_2 = \underset{w' \in C(\mathcal{CN}_\infty(\lambda, p), t)}{\arg\min} ||X_{w'} - X_v||,$$

i.e., w_2 is the nearest node in $C(\mathcal{CN}_\infty(\lambda, p), t)$ to v. [4] When this transmission is performed, nodes in $SBP_{m_{u \to v}}$ release the copies of $m_{u \to v}$.

III In the last stage, we select the shortest path from w_2 to v as the *destination buffering path* of $m_{u \to v}$, $DBP_{m_{u \to v}}$. $m_{u \to v}$ is further transmitted hop by hop along $DBP_{m_{u \to v}}$. During each hop, if the transmitter or the receiver is inactive, the messages should be buffered in the transmitter until both are active.

We define the *source buffering radius* and *destination buffering radius* of message slot $m_{u \to v}$, denoted by $RSB_{m_{u \to v}}$ and $RDB_{m_{u \to v}}$, as

$$RSB_{m_{u \to v}} = \max_{w_i \in SBP_{m_{u \to v}}} ||X_{w_i} - X_u||,$$

$$RDB_{m_{u \to v}} = \max_{w_i \in DBP_{m_{u \to v}}} ||X_{w_i} - X_v||.$$

With ORS, we can prove that buffer occupation in each connected node is finite. Since in each time slot the active giants exists, SBR and DBR cannot be very large. Therefore, each connected node needs only to buffer the messages from near sources or to near destinations. That is the intuition of the proof of Inequality (2) with ORS.

3.3 Finite Buffering Radius

With ORS, both the source buffering path and destination buffering path are finite, as more precisely demonstrated in the following lemma.

Lemma 1: For a uniformly randomly chosen message slot $m_{u \to v}$, its corresponding buffering radii $RSB_{m_{u \to v}}$ and $RDB_{m_{u \to v}}$ satisfy

$$\mathbb{P}(RSB_{m_{u \to v}} \geq R) \leq \beta_s(R+1)e^{-\alpha_s R}, \quad \text{and}$$
$$\mathbb{P}(RDB_{m_{u \to v}} \geq R) \leq \beta_d(R+1)e^{-\alpha_d R},$$

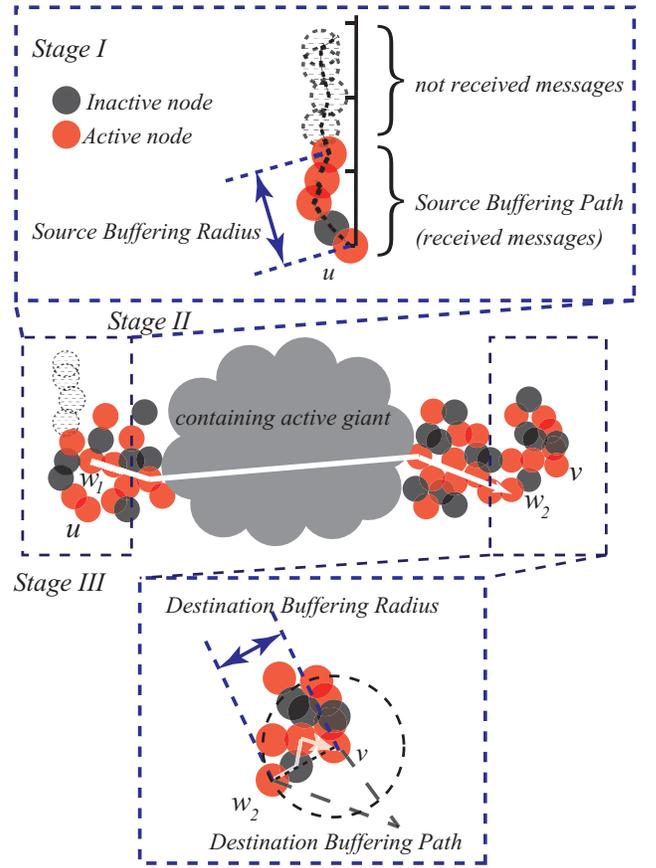4. If $v \in C(\mathcal{CN}_\infty(\lambda, p), t)$, then $w_2 = v$.

Fig. 3. Optimal Routing Scheme.

where α_s, α_d and β_s, β_d are constant with $\alpha_s, \alpha_d > 0$ and $\beta_s, \beta_d < \infty$.

Proof: The proof will use the concept of *vacant components*. A vacant component means a continuous area that is not covered by nodes, see [14]. As we only consider active nodes here, a vacant component means a continuous area that is not covered by active nodes.

We first prove the result for RDB. Consider the distance between v and the first node in $DBP_{m_{u \to v}}$, w_2. According to ORS, at the time when w_2 receives $m_{u \to v}$ (the end of Stage II), there is no node belonging to $C(\mathcal{CN}_\infty(\lambda, p), t)$ within the circle centered at v with radius $r = ||X_{w_2} - X_v|| - \frac{1}{2}$. [5] It implies that a vacant component surrounds the circular area, including every diameter of it. We only consider the horizonal diameter of this circle, which is adequate to prove the result.

Let $Sg_v(d)$ denote the horizontal line segment centered at v with length d. Utilizing Lemma 6 in Section 6, we have

5. Here $\frac{1}{2}$ is the radius of the coverage area of node w_2.
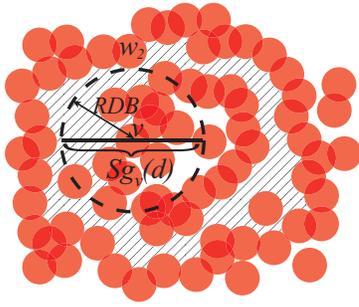
(Only active nodes are shown in this figure.)

Fig. 4. A vacant component surrounding the circular region.

Fig. 6. w_2 and v are enclosed in a circuit of connected nodes.

$$\mathbb{P}(\|X_{w_2} - X_v\| \geq R)$$
$$\leq \mathbb{P}(Sg_v(2R-1) \text{ is circulated by a vacant component})$$
$$\leq \beta e^{-\alpha(2R-1)} = \beta'_d e^{-\alpha'_d R}$$

where α'_d and β'_d are constants with $\alpha'_d > 0$ and $\beta'_d < \infty$.

Now we have proved that the distance from w_2 to v cannot be very large. However, RDB can be larger than the distance from w_2 to v (see Fig. 5). We are going to further prove the corresponding result for RDB.
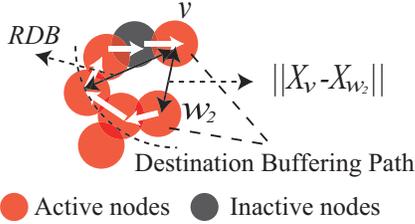

Fig. 5. An example that RDB is larger than the distance from w_2 to v.

In Fig. 6, the shaded region contains a close circuit of connected nodes (the event that the shaded region contains a close circuit is denoted by event E_d, where d is the side length of the square). Lemma 2 in [8] shows that $\mathbb{P}(E_d) \geq 1 - \beta'_c d e^{-\alpha'_c d}$ where $\alpha'_c > 0$ and $\beta'_c < \infty$. If E_d happens, $DBP_{m_{u \to v}}$ is within the square, and thus $RDB_{m_{u \to v}} \leq \sqrt{2}d$.

$$\mathbb{P}(RDB_{m_{u \to v}} \geq R)$$
$$\leq \mathbb{P}\left(\left\{\|X_v - X_{w_2}\| \geq \frac{R}{2}\right\} \cup \overline{E_{\frac{R}{\sqrt{2}}}}\right)$$
$$\leq \mathbb{P}\left(\|X_v - X_{w_2}\| \geq \frac{R}{2}\right) + 1 - \mathbb{P}\left(E_{\frac{R}{\sqrt{2}}}\right)$$
$$\leq \beta'_d e^{-\alpha'_d R/2} + \frac{\beta'_c}{\sqrt{2}} R e^{-\frac{\alpha'_c}{\sqrt{2}} R}$$
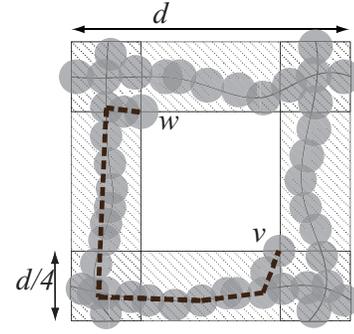$$\leq \beta_d (R+1) e^{-\alpha_d R},$$

where α_d and β_d are constants with $\alpha_d > 0$ and $\beta_d < \infty$.

The proof of RSB can utilize a similar approach as RDB. Recall the transmission of $m_{u \to v}$ in Stage I. Since the SBP contains a continuous chain of nodes on the source extending path SEP_u, and nodes directly overlap with them, then in Stage I, the active giant cannot intersect with SBP. In other words, in Stage I, SBP is circulated by a vacant component.

Here in Stage I, ORS requires transmission on each hop of SEP must guarantee all the transmitter's direct neighbors have receive copies of the messages. This is necessary for the statement: in Stage I, the active giant does not intersect with SBP. As Fig. 7 shows, it is possible in Stage I that active giant can intersect with SEP (via nodes x and y). If SBP only contains nodes of SEP, then the active giant can overlap with SBP (part of SEP), thus in this case the result of vacant component is not applicable. On the other hand, as long as the direct neighbors of
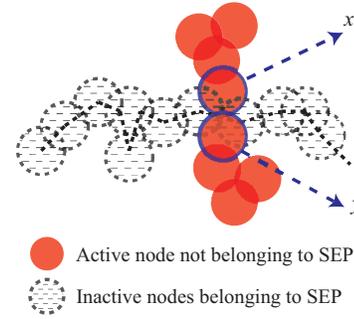

Fig. 7. Active giant intersects with SEP in Stage I.

nodes in SEP also receive and keep copies of the message, i.e. x and y also belongs to SBP, the statement above is valid

We assume $m_{u \to v}$ is generated in time slot t_0, Stage I stops in time slot $t - 1$, and Stage II starts from time slot t. Then for each time slot between t_0 and $t - 1$, there is a vacant component circulating the nodes that received $m_{u \to v}$. Specifically, we consider the vacant component in

time slot $t-1$. As in Corollary 1 in Section 6, we denote the event that u is circulated by a vacant component with diameter larger than $2R$ by $E_{V(d>2R)}(u)$, then

$$\mathbb{P}(RSB_{m_{u\to v}} \geq R) \leq \mathbb{P}(E_{V(d>2R)}(u))$$
$$\leq \beta'(2R+1)e^{-\alpha'2R} \leq \beta_s(R+1)e^{-\alpha_s R},$$

where α_s and β_s are constants with $\alpha_s > 0$ and $\beta_s < \infty$. $\square$

Lemma 1 indicates that the probability that a node belongs to the source (or destination) buffering radius of a message slot is very small when it is far from the source (or destination).

$$\mathbb{P}(w \in SBP_{m_{u\to v}}) \leq \beta_s(||X_w - X_u|| + 1)e^{-\alpha_s||X_w - X_u||},$$
$$\mathbb{P}(w \in DBP_{m_{u\to v}}) \leq \beta_d(||X_w - X_v|| + 1)e^{-\alpha_d||X_w - X_v||}.$$

3.4 Finite Hops and Latency

We have shown that the ranges of SBP and DBP tend to have relatively small values, and so do the number of hops on SBP and DBP. For SBP, "one hop" means the transmission from two neighboring nodes in the source extending path. Let $NSB_{m_{u\to v}}$ and $NDB_{m_{u\to v}}$ respectively denote the number of hops on $SBP_{m_{u\to v}}$ and $DBP_{m_{u\to v}}$.

Lemma 2: For a randomly selected $m_{u\to v}$,

$$\mathbb{P}(NSB_{m_{u\to v}} \geq N) \leq \beta_{sh}(\sqrt{N}+1)e^{-\alpha_{sh}\sqrt{N}},$$
$$\mathbb{P}(NDB_{m_{u\to v}} \geq N) \leq \beta_{dh}(\sqrt{N}+1)e^{-\alpha_{dh}\sqrt{N}},$$

where α_{sh}, α_{dh} and β_{sh}, β_{dh} are constant with $\alpha_{sh}, \alpha_{dh} > 0$ and $\beta_{sh}, \beta_{dh} < \infty$.

Proof:

First we consider $NDB_{m_{u\to v}}$. Since DBP is the shortest path connecting w_2 and v, a node cannot overlap more than two other nodes of the path. Otherwise, one node can be removed to shorten the path. Therefore, such a path with l nodes will cover at least an area with size $\pi r^2 l = \frac{l\pi}{4}$. On the other hand, this area should be within the circular region centered at v with radius $RDB_{m_{u\to v}}$.

$$\mathbb{P}(NDB_{m_{u\to v}} \geq N) \leq \mathbb{P}(\pi\frac{N}{4} \leq \pi RDB_{m_{u\to v}}^2)$$
$$\leq \mathbb{P}(RDB_{m_{u\to v}} \geq \frac{\sqrt{N}}{2}) \leq \beta_d(\frac{\sqrt{N}}{2}+1)e^{-\alpha_d\frac{\sqrt{N}}{2}}$$
$$\leq \beta_{dh}(\sqrt{N}+1)e^{-\alpha_{dh}\sqrt{N}}.$$

The proof for $RSB_{m_{u\to v}}$ is the same as $NDB_{m_{u\to v}}$, since the construction of SEP has also excluded the case that one node overlaps with three or more other nodes in the path. $\square$

In ORS, transmission latency only happens in Stage I and III. In Stage III, the single-hop-delay is (note that one node is active when it receives a message)

$$\mathbb{E}(T_1^d) = T_{EC}\sum_{k=1}^{\infty} p(1-p)^2(1-(1-p)^2)^{k-1}k = \frac{pT_{EC}}{(1-p)^2}.$$

While in Stage I, the single-hop-delay is relatively larger since in each hop the transmitter must assure the message is sent to all its direct neighbors, but it still has finite expectation $\mathbb{E}(T_1^s)$. Since the buffering hops in each stage is finite a.s., the transmission latency of each message slot is also finite. Consequently, the expected numbers of such message slots $m_{u\to}$ existing in their SBP's and of such $m_{\to v}$ existing in their DBP's are both finite, denoted by E^s and E^d (u and v are randomly selected).

3.5 Finite Buffer Occupation

In this section we finally present the proof that Inequality (2) holds when Optimal Routing Scheme is applied.

Proof of Inequality (2): In ORS, buffering can happen both in Stage I (source buffering) and Stage III (destination buffering). We first discuss source buffering. For a randomly selected connected node u, let $M_u^s(w,t)$ denote the number of such message slots $m_{u\to}$ that are buffered in w in time slot t. According to Lemma 1,

$$\mathbb{E}(M_u^s(w,t)) \leq E^s\beta_s(||X_w - X_u|| + 1)e^{-\alpha_s||X_w - X_u||}.$$

Consider the ring centered at w with radius r and width Δr ($r >> \Delta r$), and the expected number of nodes inside this ring is $2\lambda\pi r\Delta r$. We first consider how many message slots from this ring are buffered in w, then let r range from 0 to infinity and thus we can get the expected total number of message slots buffered in w, and thus get the expected value of the part of buffer occupation of w where w serves for SBP.
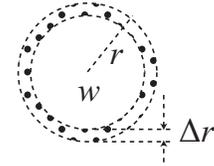

Fig. 8. The ring centered at w with radius r and width Δr. The expected number of nodes it contains is $2\lambda\pi r\Delta r$.

$$\mathbb{E}\left(S_{w,SBP}^{(\infty)}(t)\right) \leq r_g T_{EC}\int_0^{\infty} 2\lambda E^s\beta_s(r+1)e^{-\alpha_s r}\pi r \mathrm{d}r$$
$$= c_s r_g T_{EC}.$$

For destination buffering, we correspondingly have

$$\mathbb{E}\left(S_{w,DBP}^{(\infty)}(t)\right) \leq c_d r_g T_{EC}.$$

Finally, the buffer occupation of w is the sum of the above two parts,

$$\mathbb{E}\left(S_w^{(\infty)}(t)\right) \leq (c_s + c_d)r_g T_{EC} = c_1 r_g T_{EC},$$

where c_1 is determined by p and λ, which does not depend on the size of the network. $\square$

The lower bound $b_1 r_g T_{EC}$ in Inequality (1) holds for all possible schemes, while the upper bound $c_1 r_g T_{EC}$ in Inequality (2) holds with ORS. Therefore, $\Theta(1)$ is the fundamental achievable lower bound for buffer size in the supercritical case with respect to n or L.

4 LOWER BOUND FOR BUFFER SIZE IN THE SUBCRITICAL CASE

If $p > p_c(\lambda)$, the network is in the subcritical case. In this case, the active giant cluster does not exist during each time slot a.s. Therefore, when the size of the network goes to infinity, without utilizing the active giant, transmission latency should also go to infinity. Hence, the entire path from the source to the destination should buffer the messages.

In the optimal case, the achievable lower bound for node buffer size is presented in Theorem 2.

Theorem 2: Buffer occupation of connected nodes in the subcritical case follows:

1) The average buffer occupation among all connected nodes in $\mathcal{CN}(\lambda, L, p)$, denoted by $\overline{S}^{(L)}(t)$, has a lower bound on $\Theta(L)$, or equivalently $\Theta(\sqrt{n})$, and

$$\lim_{L \to \infty} \frac{\overline{S}^{(L)}(t)}{L} \geq b_2 r_g T_{EC}.$$

2) For a randomly selected connected node w in $\mathcal{CN}(\lambda, L, p)$ with $p > p_c(\lambda)$, at the end of time slot t, it is achievable that

$$\lim_{L \to \infty} \mathbb{E}\left(\frac{S_w^{(L)}(t)}{L} \right) \leq c_2 r_g T_{EC} < \infty.$$

b_2, c_2 are finite positive constants irrelevant to L, and $r_g T_{EC}$ is the length of each message slot.

Because of the square shape of the network region, nodes in the central part would need larger buffer than nodes near the rim. Fortunately, the edge effects do not change the order of the required buffer size with respect to L or n; it is achievable that the expected value of minimum required buffer size of each connected node has a common upper bound. We will first give a theoretical lower bound on the minimum required buffer size, and then prove it can be achieved, via presenting a constructive upper bound.

4.1 The Lower bound on the Average Buffer Occupation

In this subcritical intermittently connected network, the following lemma on the minimum message existing time (delay) can be proved.

Lemma 3: If $(1 - p)\lambda < \lambda_c$, the minimum latency of message slot $m_{u \to v}$, $T_{m_{u \to v}}$, satisfy

$$\lim_{||X_u - X_v|| \to \infty} \frac{T_{m_{u \to v}}}{||X_u - X_v||} = \gamma > 0 \quad a.s.$$

Proof for this lemma is based on the Subadditive Ergodic Theorem [16], and is similar to the proof of Theorem 1 in [8], where the only difference is that state switching in [8] is asynchronous (uncoordinated). Theorem 1 in [8] shows that in subcritical case, the minimum delay from X to Y ($T(X, Y)$) satisfies

$$(1 - \epsilon)\eta \leq \frac{T(X, Y)}{|X - Y|} \leq (1 + \epsilon)\eta$$

for any $\epsilon > 0$ whenever $|X - Y|$ is large enough, where $|X - Y|$ is the distance between X and Y. Hence, $\lim_{|X-Y| \to \infty} \frac{T(X,Y)}{|X-Y|} = \gamma' > 0$, a.s. In the proof of Theorem 1 in [8], state switching only affects the average one-hop delay, and the condition that the one-hop delay is finite (t_{off}) is sufficient for the proof. In this paper, though state switching is synchronous, since the one-hop delay is also finite, the proof method in Theorem 1 in [8] is still valid for Lemma 3 in this paper and the result is the same.

Since each connected node uniformly randomly chooses a destination, the average distance of source-destination pairs is of the order $\Theta(L)$, or equivalently, $\Theta(\sqrt{n})$. Therefore, the average minimum latency for a message slot is $\Theta(\sqrt{n})$. Since messages are generated continuously, by Little's Law, the average number of message slots generated by one node existing in the network in a given time slot is $\Theta(\sqrt{n})$. There are $\Theta(n)$ connected nodes, thus the number of all message slots existing in a given time slot is $\Theta(n\sqrt{n})$. Hence, the average number of message slots one connected node should buffer is $\Theta(\sqrt{n})$.

$\Theta(\sqrt{n})$ is a lower bound on the average minimum required buffer size of each connected node. Take the length of each message slot, $r_g T_{EC}$, into consideration, we have

$$\lim_{L \to \infty} \frac{\overline{S}^{(L)}(t)}{L} > b_2 r_g T_{EC}.$$

4.2 A Constructive Upper bound on the Minimum Buffer Occupation for All Nodes

In this section, we will present a scheme in which a $\Theta(\sqrt{n})$ buffer size requirement can be achieved in the subcritical case. To achieve this, we designate a permanent path for each source-destination pair where the number of hops is asymptotically linear to the distance between them. The path is similar to the source extending path in supercritical case, but has two ends.

For a source node u, and its destination v, we draw a straight line L_u^v connecting them, and divide it into segments with constant length l_s, as shown in Fig. 9. The end points are denoted by $\{EP_i^{u \to v}\}$, $i = 0, 1, ..., \lfloor ||X_u - X_v|| / l_s \rfloor$. At each end point of the segments, we choose the nearest connected node as a *flag node* in the path from u to v. The set of flag nodes is denoted by $\{NF_i^{u \to v}\}$, $i = 0, 1, ..., \lfloor ||X_u - X_v|| / l_s \rfloor$. Connect
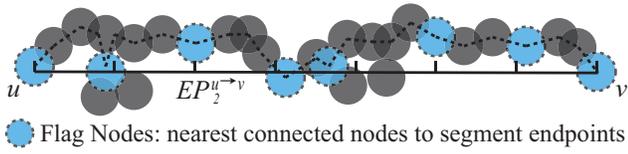
Fig. 9. The permanent path from u to v.

every two neighboring flag nodes with the shortest path between them, and thus we get the path from u to v, $P_{u \to v}$. Denote the segment of the path between $NF_{i-1}^{u \to v}$ and $NF_i^{u \to v}$ by $PS_i^{u \to v}$. All the messages generated by u is transmitted along this path.

This scheme assures that the number of hops in $P_{u \to v}$ is asymptotically linear to the $||X_u - X_v||$ (see [8]). Since the delay on each hop is finite $\left(\frac{pT_{EC}}{(1-p)^2}\right)$, the average latency is $\Theta(\sqrt{n})$. We are going to show how many paths a connected node belongs to, and then give the upper bound on the minimum required buffer size in the subcritical case.

Lemma 4: A connected node w belongs to at most $\Theta(\sqrt{n})$ paths connecting source-destination pairs.

Proof: Applying the same technique as in the proof of Lemma 1, if a square connected circuit that surrounds $EP_{i-1}^{u \to v}$ and $EP_i^{u \to v}$, then $PS_i^{u \to v}$ must be within this circuit. Then if w belongs to $PS_i^{u \to v}$, it must be inside the smallest one of such circuits, and there is no such circuits with smaller size than the distance between w and the segment. Therefore,

$$\mathbb{P}(w \in PS_i^{u \to v})$$
$$\leq \beta_p(||X_w - EP_i^{u \to v}|| + 1)e^{-\alpha_p||X_w - EP_i^{u \to v}||},$$

where β_p and α_p are constants with $\alpha_p > 0$ and $\beta_p < \infty$. Assuming the distance between w and L_u^v is d_{w,L_u^v}, we can calculate the probability that w belongs to $P_{u \to v}$. Divide $\{EP_i^{u \to v}\}$ into two subsets (as in Fig. 10): one

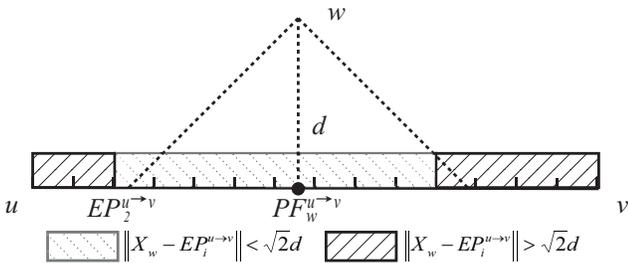Fig. 10. Calculate the probability that w belongs to $P_{u \to v}$.

includes endpoints within distance $\sqrt{2}d_{w,L_u^v}$ to w, the other includes endpoints outside this distance. For the first subset, $||X_w - EP_i^{u \to v}|| \leq d_{w,L_u^v}$; and for the second subset, $||X_w - EP_i^{u \to v}|| > ||EP_i^{u \to v} - PF_w^{u \to v}||$, where $PF_w^{u \to v}$ is the perpendicular foot from w to L_u^v.

$$\mathbb{P}(w \in P_{u \to v})$$
$$\leq \sum_i \mathbb{P}(w \in PS_i^{u \to v})$$
$$\leq \sum_{||X_w - EP_i^{u \to v}|| < \sqrt{2}d_{w,L_u^v}} \mathbb{P}(w \in PS_i^{u \to v}) +$$
$$\sum_{||X_w - EP_i^{u \to v}|| > \sqrt{2}d_{w,L_u^v}} \mathbb{P}(w \in PS_i^{u \to v})$$
$$\leq \beta_1 d_{w,L_u^v}(\sqrt{2}d_{w,L_u^v} + 1)e^{-\alpha_1 d_{w,L_u^v}} +$$
$$\beta_2 d_{w,L_u^v}e^{-\alpha_2 d_{w,L_u^v}}$$
$$\leq \beta(d_{w,L_u^v}^2 + 1)e^{-\alpha d_{w,L_u^v}},$$

where β and α are constants with $\alpha > 0$ and $\beta < \infty$. Since the edge effects, nodes in the central region needs larger buffer size, and we assume w is located at the center of the network. Suppose $||X_u - X_w|| = R$, then there exists a constant c_2 that for all possible u, v and w with $R > r$,

$$\mathbb{P}(r < d_{w,L_u^v} < r + \Delta r | R > r) < c_3 \frac{\Delta r}{R}.$$

If $R < r$, $\mathbb{P}(r < d_{w,L_u^v} < r + \Delta r | R < r) = 0$. Fig. 11 illustrates the case when $R > r$. Then we calculate the expected number of lines intersecting the ring centered at w with radius r and width Δr, denoted by $NL_w(r, \Delta r)$.

$$\mathbb{E}(NL_w(r, \Delta r))$$
$$\leq \int_r^{\frac{L}{\sqrt{2}}} 2\pi R \lambda c_3 \frac{\Delta r}{R} dR$$
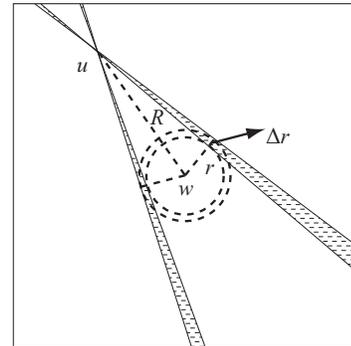$$\leq c_4 L \Delta r.$$

Fig. 11. Calculate the probability that L_u^v intersects the ring centered at w with radius r and width Δr. The shaded region is the possible area where v is located.

Let NP_w denote the total number of paths connecting

source-destination pairs that w belongs to, then we have

$$\mathbb{E}(NP_w)$$
$$\leq \int_0^{\frac{L}{\sqrt{2}}} \mathbb{E}(NL_w(r, \mathrm{d}r))\beta(r^2+1)e^{-\alpha r}$$
$$\leq \int_0^{\frac{L}{\sqrt{2}}} c_4 L\beta(r^2+1)e^{-\alpha r}\mathrm{d}r$$
$$\leq c_5 L.$$

Therefore, w belongs to at most $\Theta(\sqrt{n})$ paths connecting source-destination pairs. $\square$

If $w \in P_{u\to v}$, then the expected number of message slots generated by u that are buffered in w is of a constant order, after buffering in the network becomes stable. Therefore, the expected number of all message slots buffered in w is linear to the number of paths it belongs to. Lemma 4 gives an upper bound on this value,

$$\lim_{L\to\infty} \mathbb{E}\left(\frac{S_w^{(L)}(t)}{L}\right) < c_2 r_g T_{EC} < \infty.$$

We get this upper bound when w is at the center of the network. Taking edge effects into consideration, this upper bound still applies to all connected nodes. Hence, Theorem 2 is proved.

4.3 Variations in Time Domain

Each hop of one path has a constant expected buffer occupation (only considering the messages generated by the source of this path). Now we use simulation to investigate the variations (fluctuations) of buffer occupation over path. The path consists of 1000 nodes, and the probability of being inactive is 0.5. From Fig. 12, we can see that as the number of hops from source increases, the variation of buffer occupation in one node also increases, and buffer occupation is intensive in a small portion of time but is empty in most of time, as shown in Fig. 12.

This phenomenon can be viewed as an accumulation of messages due to the inactivity. For instance, consider the i-th and $(i+1)$-th nodes. If the $(i+1)$-th node stays inactive for several time slots, the messages are likely to accumulate at the i-th node. Although one node is not very likely to stay inactive for a long period, on a long path the accumulation can occur all the way from the source to the destination.

However, Fig. 13 illustrates the variation increases slowly as as the number of hops from source extends to infinity. We use the *Non-empty ratio* to reflect this variation, which is the proportion of time when the buffer of a node is not empty, or occupied. Small non-empty ratio means that variation is large and buffer occupation is intensive in only very few time slots. Let NE_i denote the non-empty ratio of the i-th node. We find it should at least satisfy the following equation.
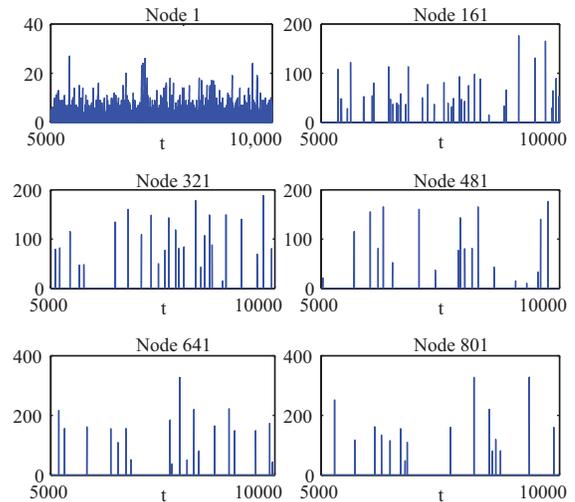
$$\lim_{i\to\infty} iNE_i = +\infty.$$

Fig. 12. Single-path buffer occupation, $p = 0.5$. (Without path flow control.)

The proof of this equation can utilize the result of the latency in subcritical case. Suppose

$$\lim_{i\to\infty} iNE_i < c < +\infty,$$

then there are at most a $\frac{c}{i}$ proportion of time slots when the buffer is occupied. Then in every of such time slots, on average there will be at least $\frac{i}{c}$ message slots in the buffer, which indicates that $\frac{i}{c}$ message slots arrive at the i-th node at almost the same time. In this case the largest difference among the latencies of these message slots is as large as $\Delta T \geq \frac{i}{c}$, then

$$\frac{\Delta T}{i} \geq \frac{1}{c} > 0,$$

contradicting the fact that latency is asymptotically linear to the path length which requires

$$\lim_{i\to\infty} \frac{T}{i} = \gamma, \quad \text{or equivalently}$$
$$\lim_{i\to\infty} \frac{\Delta T}{i} = 0.$$

Intuitively, if the non-empty ratio is already very small at the i-th node, it indicates that the interval between two time slots when the buffer is not empty is very large. Then it is difficult for the messages in the two time slots to further converge into one node in a future time slot. Therefore, the accumulation phenomenon becomes less obvious when the messages are already intensively accumulated.

In our network model, under subcritical case, it is common that each connected node serves for multiple paths. If the sources of the paths sharing one node are located distantly, the variation (fluctuation) of buffer occupation in the shared node is small, because the correlation of buffer occupations by different paths in the shared node
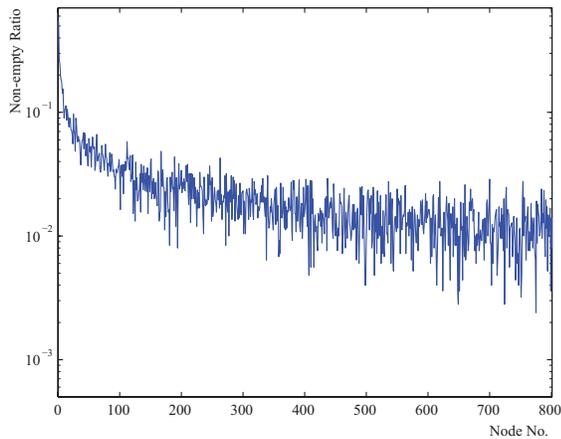
Fig. 13. Non-empty ratio of node buffer on a single path, $p = 0.5$. The ratio increases slowly as the path extends to infinity. (Without path flow control.)

is small when their sources are located distantly. Since one node belongs to $\Theta(\sqrt{n})$ paths, its variation of buffer is reduced when n increases. To verify this point, we also conduct a simulation where there are two (three) paths that shares 3 nodes, x_1, x_2 and x_3. Assume x_2 is the n-th node for both (all) paths. We expect the non-empty ratio of the n-th node will be larger than the single-path case.

The simulation result illustrated in Fig. 14 shows the ratios $\frac{NE_n^{2-path}}{NE_n^{1-path}}$ and $\frac{NE_n^{3-path}}{NE_n^{1-path}}$ respectively in the 2-path and 3-path cases. As n increases, $\frac{NE_n^{2-path}}{NE_n^{1-path}}$ becomes close to 2 and $\frac{NE_n^{3-path}}{NE_n^{1-path}}$ becomes close to 3. It shows that the different paths are likely to occupy the buffer of the shared node in different time slot; otherwise the . It indicates that when the sources of multiple paths are far away, the variation is reduced. The correlation of the buffer occupations imposed by different paths on the shared nodes is small; otherwise this variation will increase because of multiple paths.

The time-domain variation of buffer occupation can be reduced if we apply flow control for each path, i.e. to control the amount of messages sent in one time slot of each hop. In the simulation of which the results are illustrated in Fig. 15, we restrict that in one time slot each hop can transmit at most 30 message slots. Obviously, the time-domain variation is largely reduced. In most of time slots, the maximum buffer occupation is 30, and in only very few time slots when it is up to 60. This flow control is in terms of one path, and if one node belongs to multiple paths, it applies a separate flow control for every path. Hence, such path flow control does not reduce the throughput of the network. Therefore, one node can set a queue for each path it serves for, to prevent too small non-empty ratio and too large buffer occupation in a particular time slot.

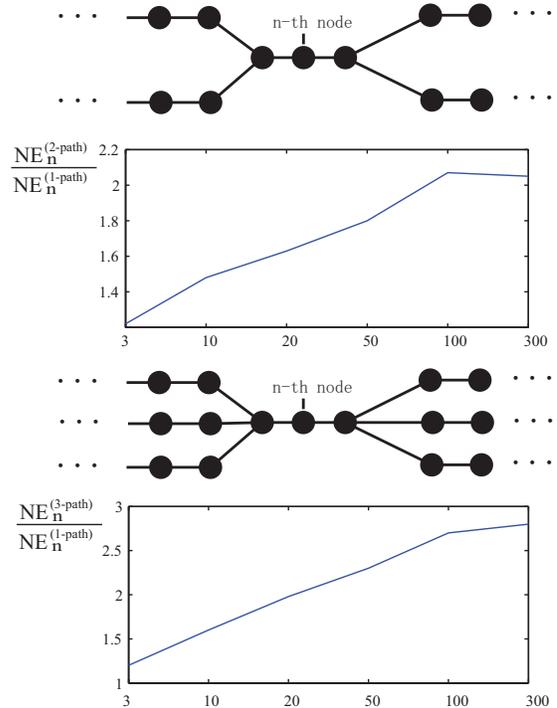The discussions above shows that the $\Theta(\sqrt{n})$ bound

Fig. 14. Multiple paths shared 3 nodes. $p = 0.5$.

can well reflect the actual buffer occupation even if we take into consideration the variations in time domain.

5 DISCUSSIONS

5.1 On the Length of Time Slot

With the infinite channel capacity assumption, results in both supercritical and subcritical case indicate that buffer size requirements for connected nodes are closely related to the length of time slot T_{EC}. T_{EC} determines the frequency that the external constraints change, and the period a node being active or inactive. Results demonstrated in Theorems 1 and 2 show that minimum buffer size requirements for connected nodes are proportional to the value of T_{EC}.

This indicates that, without decreasing the probability of inactivity p, we can still reduce the requirements on buffer size by making T_{EC} smaller, if it is controllable. For instance, in a large scale wireless sensor network where node sleeping is allowed for saving energy, the amount of messages a node buffers can be reduced by making the sleeping and active periods smaller, even if their ratio is constant. If we do not consider the power consumed by switching between sleeping and active states, the power consumption for each node is mainly determined by the ratio of the durations of sleeping and active states. Therefore, smaller periods do not impair the performance of the energy saving mechanism.

Consider a network where buffer size of each node is limited, and T_{EC} is controllable. Also assume the channel
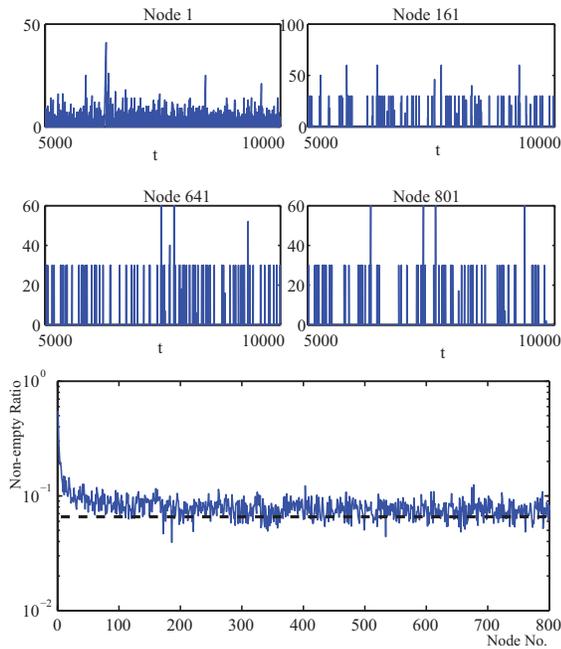
Fig. 15. Time-domain variation with path flow control, $p = 0.5$, in one time slot each hop can transmit at most 30 message slots.

capacity can be viewed as infinity due to the low message generation rate r_g, and the probability of inactivity p is relatively large that the network is in the subcritical case. If it is required to support a constant r_g as the size of the network increases, we can scale T_{EC} to $O(\frac{1}{\sqrt{n}})$ to assure limited buffer size of each node is enough.

Further consider an extreme case where $T_{EC} \rightarrow 0$, then buffer occupation also reduces to 0. This case is similar to a TDMA scheme, and is equivalent that nodes are always active but channel capacity C reduces to a constant proportion $(1-p)^2 C$. However, when T_{EC} is very small, the node processing speed can no longer be viewed as infinity, and node inactivity caused by external factors (or constraints) are no longer crucial to the network. In this case, internal factors such as capacity and processing speed are the main constraints of the network performance.

By contrast, if T_{EC} is not controllable and also node buffer size is limited, in the subcritical case, r_g cannot remain constant as the size of the network increases. In this case, r_g is at most $\Theta(\frac{1}{\sqrt{n}})$. This is the constraints on r_g even if the channel capacity is infinite.

5.2 On Finite Channel Capacity

Previous sections are based on the assumption of infinite channel capacity due to the comparatively low message generation rate, r_g. Now we are going to discuss how the results are applied to networks with finite channel

capacity. We again omit the processing delay of each node.

In both supercritical and subcritical case, each connected node on average serves as an intermediate for $\Theta(\sqrt{n})$ source-destination pairs, and thus the throughput capacity of each node is $\Theta(\frac{1}{\sqrt{n}})$. Similar and detailed analysis of throughput capacity can be found in [17]. The order of this throughput capacity is still achievable even if node inactivity exists. Therefore, r_g cannot remain constant as n increases. Let $r_g = O(\frac{1}{\sqrt{n}})$, and we show that Theorems 1 and 2 still hold.

1) In the supercritical case, a message slot with size $r_g T_{EC} = O(\frac{1}{\sqrt{n}})$ can be sent through $\Theta(\sqrt{n})$ hops within constant time. Then we can still assure the buffering path of each message consists of finite hops, and the existing time (delay) is still of constant order. Therefore, the number of message slots each node needs to buffer is still of constant order, and the size of buffered messages in one node is $O(\frac{1}{\sqrt{n}})$. Since the throughput capacity is $\Theta(\frac{1}{\sqrt{n}})$, the buffered messages does not exceed the order of the per-node throughput. Therefore, Theorem 1 still holds. Further, since $r_g = O(\frac{1}{\sqrt{n}})$, the actual amount of buffered message in each connected node is also $O(\frac{1}{\sqrt{n}})$.

2) In the subcritical case, the result in Theorem 2 still holds if only r_g does not exceed the per-node throughput capacity. With $r_g = O(\frac{1}{\sqrt{n}})$, the actual amount of buffered message in each connected node is $\Theta(\sqrt{n} r_g) = O(1)$.

The discussion above show that when the channel capacity is finite, and r_g is adjusted according to the per-node throughput capacity, minimum occupied buffer size of each connected node does not increase with the growth of the network size.

6 USEFUL RESULTS ABOUT VACANT COMPONENTS IN SUPERCRITICAL CASE

A vacant component isolates the area inside it from accessing to the nodes outside. In supercritical case, vacant components cannot be very large.

Lemma 5: Randomly select a vacant component V in the following way: first, uniformly randomly choose a point o in $\mathbb{R}^2$; second, if o is not covered by any node, then let $V = V\{o\}$ (the vacant component o belongs to), else return to the first step. (A larger vacant component are more likely to be chosen.) If $\lambda > \lambda_c$ (or $(1-p)\lambda > \lambda_c$ if only active nodes are considered) [6], then

$$\mathbb{P}\left(d(V) \geq a\right) \leq c_{v1} e^{-c_{v2} a},$$

6. If one only considers active nodes in one time slot, then in supercritical case it is $(1-p)\lambda > \lambda_c$. So does that apply to Lemma 6 and Corollary 1.

where c_{v1}, c_{v2} are constants with $c_{v1} < \infty$ and $c_{v2} > 0$.

Proof: According to an immediate consequence of Lemma 4.4 in [14] (on Page 114, above Corollary 4.1), since in the supercritical case $\lambda > \lambda_c$, we have $\sigma^*(m, 3m, 1) \to 0$ (or $\sigma^*(3m, m, 2) \to 0$) as $m \to \infty$, where $\sigma^*(m, 3m, 1)$ (or $\sigma^*(3m, m, 2)$) is the probability for the existence of a vacant crossing from left to right (or top to bottom) in a $(m, 3m)$ (or $(3m, m)$) sized rectangular. Hence, we can find $\kappa_0 > 0$ and large enough m such that

$$\sigma^*(m, 3m, 1) < \kappa_0,$$
$$\sigma^*(3m, m, 2) < \kappa_0.$$

Consider a randomly selected point o in $\mathbb{R}^2$, according to Lemma 4.1 in [14], we have

$$\mathbb{P}\left(d(V\{o\}) \geq a\right) \leq c'_{v1}e^{-c'_{v2}a},$$

where $V\{o\}$ is the vacant component containing o ($d(V\{o\}) = 0$ if o is covered by an active node), and c'_{v1}, c'_{v2} are constants with $c'_{v1} < \infty$ and $c'_{v2} > 0$. And

$$\begin{aligned}
&\mathbb{P}(d(V\{o\}) \geq a)) \\
= \ &\mathbb{P}(d(V\{o\}) \geq a | d(V\{o\}) = 0))\mathbb{P}(d(V\{o\}) = 0) \\
&+ \mathbb{P}(d(V\{o\}) \geq a | d(V\{o\}) > 0)\mathbb{P}(d(V\{o\}) > 0) \\
= \ &0 + \mathbb{P}(d(V\{o\}) \geq a | d(V\{o\}) > 0)P_{vac},
\end{aligned}$$

where P_{vac} is the proportion of space not covered by nodes, which is a constant when the node density is given.

Recall the way that V is selected, we have

$$\begin{aligned}
&\mathbb{P}\left(d(V) \geq a\right) = \mathbb{P}(d(V\{o\} \geq a | d(V\{o\}) > 0) \\
= \ &\frac{\mathbb{P}(d(V\{o\} \geq a))}{P_{vac}} \leq c_{v1}e^{-c_{v2}a},
\end{aligned}$$

where c_{v1}, c_{v2} are constants with $c_{v1} < \infty$ and $c_{v2} > 0$. $\square$

Based on Lemma 5, we have the following result that indicates a large area is not likely to be isolated by a vacant component.

Lemma 6: Consider a randomly located line segment with length l, denoted by $Sg(l)$. If $\lambda > \lambda_c$ (or $(1-p)\lambda > \lambda_c$ if only considering active nodes), then the probability that it is circulated by a vacant component is

$$\mathbb{P}\left(Sg(l) \text{ is circulated by a vacant component}\right) \leq \beta e^{-\alpha l},$$

where α, β are constants with $\beta < \infty$ and $\alpha > 0$.

Proof: As shown in Fig. 16, we draw a string of unit squares, beginning at one endpoint of $Sg(l)$ along the extended ray, denoted by $\{Sq_i\}$. If a vacant component circulates $Sg(l)$, then it has to intersect the extended ray. Let N_i denote the number of vacant components intersecting Sq_i, and $V_{i,j}$ ($1 \leq j \leq N_i$) denote the j-th vacant component intersecting Sq_i. By Lemma 4.5 in [14], the expected number of vacant components intersecting each square is a constant, denoted by $N_{vac} = \mathbb{E}(N_i)$.
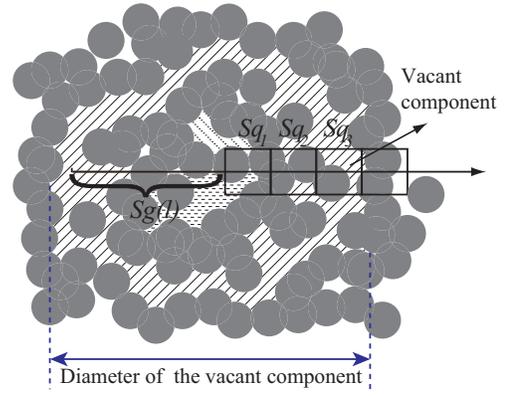

Fig. 16. A line segment surrounded by a vacant component.

Since it is supercritical case, according to Lemma 5, we have

$$\begin{aligned}
&\mathbb{P}\Big(Sq_i \text{ contains a vacant component} \\
&\qquad \text{with diameter larger than } r\Big) \\
\leq \ &\sum_{k=0}^{\infty}\mathbb{P}(N_i = k)\sum_{j=1}^{k}\mathbb{P}\left(d(V_{i,j}) \geq r\right) \\
\leq \ &\sum_{k=0}^{\infty}\mathbb{P}(N_i = k)k c_{v1}e^{-c_{v2}r} = c_{v1}N_{vac}e^{-c_{v2}r}.
\end{aligned}$$

Further,

$$\begin{aligned}
&\mathbb{P}\left(Sg(l) \text{ is circulated by a vacant component}\right) \\
\leq \ &\sum_{i=1}^{\infty}\mathbb{P}\Big(Sq_i \text{ contains a vacant component} \\
&\qquad \text{surrounds } Sg(l)\Big) \\
\leq \ &\sum_{i=1}^{\infty}\mathbb{P}\Big(Sq_i \text{ contains a vacant component} \\
&\qquad \text{with diameter larger than } l + i - 1\Big) \\
\leq \ &\sum_{i=1}^{\infty}c_{v1}N_{vac}e^{-c_{v2}(l+i-1)} \leq \beta e^{-\alpha l},
\end{aligned}$$

where α and β are constants with $\alpha > 0$ and $\beta < \infty$.

$\square$

Corollary 1: Consider a randomly located node w. If $\lambda > \lambda_c$ (or $(1 - p)\lambda > \lambda_c$ if only considering active nodes), then the probability that it is circulated by a vacant component with diameter larger than l (denoted by the event $E_{V(d>l)}(w)$) is

$$\mathbb{P}(E_{V(d>l)}(w)) \leq \beta'(l+1)e^{-\alpha' l},$$

where α', β' are constants with $\beta' < \infty$ and $\alpha' > 0$.

Proof: We draw a horizontal ray starting from w, then any vacant component that circulates w must intersect

this ray. The first segment with length l on this ray is denoted by $Sg(l)$. Similar as in the proof of Lemma 6, there are a string of unit squares starting from the endpoint of $Sg(l)$. The number of vacant components intersecting with $Sg(l)$ is denoted by N_l, and $\mathbb{E}(N_l) < N_{vac}l$. The set of such vacant components intersecting with $Sg(l)$ is Vl_j.
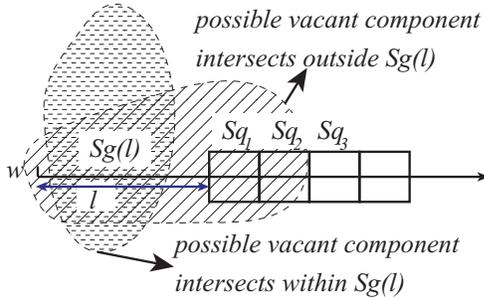

Fig. 17. Possible vacant components circulate w with diameter larger than l.

$$
\begin{aligned}
&\mathbb{P}(E_{V(d>l)}(w)) \\
\leq \quad &\sum_{k=0}^{\infty} \mathbb{P}(N_l = k) \sum_{j=1}^{k} \mathbb{P}(d(Vl_j) > l) \\
&+ \sum_{i=1}^{\infty} \mathbb{P}\Big(Sq_i \text{ contains a vacant component} \\
&\qquad \text{with diameter larger than } l+i-1\Big) \\
\leq \quad &N_{vac}l c_{v1} e^{-c_{v2}l} + \beta e^{-\alpha l} \\
\leq \quad &\beta'(l+1) e^{-\alpha' l},
\end{aligned}
$$

where α', β' are constants with $\beta' < \infty$ and $\alpha' > 0$. $\quad\square$

7 CONCLUSION

In this paper, we have studied the fundamental lower bound on node buffer size in intermittently connected wireless networks where node inactivity is possible due to external constraints. In detail, we analyzed buffer occupation when the channel capacity is infinite, and the results can be viewed as a lower bound for networks with finite channel capacity. We find when the probability of inactivity is smaller than a critical value and thus the network is in the supercritical case, the fundamental achievable lower bound on node buffer size is $\Theta(1)$, i.e. the minimum node buffer size requirements are asymptotically independent of the size of the network; when the probability of inactivity is larger than the critical value, the network is in the subcritical case, and the achievable lower bound on node buffer size increases as the network expands, with the order of $\Theta(\sqrt{n})$.

We find since node buffer size requirements are proportional to the length of time slot according to which the external constraints changes, these requirements could be reduced by decreasing the length of time slot, even if the probability of inactivity remains unchanged.

We also find when the channel capacity is finite, and message generation rate is adjusted according to the per-node throughput capacity, results with infinite channel capacity still hold.

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