# Impact of Mobility and Heterogeneity on Coverage and Energy Consumption in Wireless Sensor Networks

Xiao Wang, Xinbing Wang and Jun Zhao
*Department of Electronic Engineering*
*Shanghai Jiao Tong University*
*Shanghai, China*
{*xiaojiben, xwang8, knight4088*}@*sjtu.edu.cn*

***Abstract*—In this paper, we investigate the coverage of mobile heterogeneous wireless sensor networks (WSNs). By the term heterogeneous, we mean that sensors in the network have various sensing radii, which is an inherent property of many applied WSNs. Two sensor deployment schemes are considered –uniform and Poisson schemes. We study the asymptotic coverage under uniform deployment scheme with i.i.d. and $1$-dimensional random walk mobility model, respectively. We propose the equivalent sensing radius (ESR) for both cases and derive the critical ESR correspondingly. From the perspective of critical ESR, we show that $1$-dimensional random walk mobility can increase coverage under certain delay tolerance, and thus decreases sensing energy consumption. Also, we characterize the role of heterogeneity in coverage and energy performance of WSNs with these two mobility models, and present the discrepancy of the impact of heterogeneity under different models. Under the Poisson deployment scheme, we investigate dynamic $k$-coverage of WSNs with $2$-dimensional random walk mobility model. There are many reasons for designers to require $k$-coverage rather than $1$-coverage. Both $k$-coverage at an instant and over a time interval are explored and we derive the expectation of fraction of the whole operational region that is $k$-covered, which also identifies the coverage improvement brought by mobility.**

***Keywords*-sensor networks; coverage; energy; mobility; heterogeneity**

## I. INTRODUCTION

Wireless Sensor Networks have inspired a wide range of research among which investigation on coverage is a fundamental one. The coverage of WSNs is of significance in many applications such as the security surveillance in estates, intrusion detection in battle-field or military restricted zone, etc.

In this paper, we focus on the *blanket coverage* or area coverage, which concentrates on the maximization of detection rate of targets in the sensing field. The operational region is said to be fully blanket covered if every single point in the region is sensed. Initially, stationary and flat WSNs received most attention in the coverage study. By the term flat, we mean that sensors in the network have identical sensing radius. In [15], Clouqueur studied the sensor deployment strategy to improve the coverage performance of sensor network and proposed path exposure to measure the performance. Shakkottai [16] took into account the failure probability of sensors and obtained the necessary and sufficient conditions for a sensor grid to achieve asymptotic full coverage. In [17], Liu defined three coverage performance measures and characterize asymptotic behavior of these measures. The discrepancy in behaviors among WSNs of different configuration and parameters is then demonstrated.

The $1$-coverage studied in literature mentioned above is not satisfactory in many applications and high degree of coverage is consequently demanded (cf. [6] for the reasons to require $k$-coverage rather than just $1$-coverage). Kumar [6] studied asymptotic $k$-coverage in a mostly sleeping stationary sensor network and found the coverage highly depend on the value of the function $\frac{np\pi r^2}{\log(np)}$ ($n$ is the number of sensors and $p$ is the probability that a sensor is active). The sufficient value of $\frac{np\pi r^2}{\log(np)}$ to ensure full coverage was derived under three different deployment model. Another related topic is the approach to guarantee both coverage and connectivity of WSNs. In [7], Bai and Xuan proposed deployment schemes to achieve full coverage and $k$-connectivity. And a new deployment-polygon based methodology was introduced to prove the optimality of proposed deployment patterns.

One common feature of the above papers is that they all study static WSNs. Sensor mobility is actually a concern in coverage study. Plenty of related work concentrate on refining algorithm to reposition sensors to achieve a new deployment that improves coverage. In this sense, mobility in these studies is exploited to reconfigure the topology of the network. On the other hand, in [2], Liu studied dynamic coverage which considered the coverage of sensors during their movement. Coverage over a time interval was explored, which distinguishes the work.

These previous papers study the coverage performance in flat WSNs. In this paper, we investigate the coverage as well as the sensing energy consumption property in WSNs that are both mobile and heterogeneous. Although the sensing energy consumption is much less than the energy consumption due to communication among sensors, the former should be a concern in energy-efficient WSN design since continuous sensing is usually the case in applications while the connection between sensors need not to be maintained all the time. In literature, mobility has been proved to enhance various aspects of network performance [10], [11], [12]. And many applied WSNs are actually inherently mobile [4]. On the other hand, it has been found that WSNs achieve better balance between performance and cost of sensors if opportune degree of heterogeneity is incorporated into the network by employing high-end and low-end sensors with different sensing capabilities [13]. Actually, due to various underlying reasons, sensors in WSNs are more likely to have different sensing radii. One scenario is that sensors in WSNs are products coming from different manufacturers and there is not uniform standards on the sensing range. Plus, sensors deployed in different time may also lead to heterogeneity of the network. And as the service lifetime passes by, degradation of sensing capability may be inevitable. Hence, heterogeneity is an inherent property of many WSNs.

We partition sensors into groups based on their sensing radius. The *equivalent sensing radius* (ESR) of the mobile heterogeneous WSNs will be defined to assist the analysis. The results demonstrate that the full coverage of the operational region largely depends on the value of ESR, and so does the energy consumption. In the study, we concentrate on how mobility and heterogeneity together can impact the WSN performance. We derive the critical (necessary and sufficient) ESR in stationary flat WSNs and mobile heterogeneous WSNs, respectively. The advantages and drawbacks brought by mobility and heterogeneity are analyzed based on the results. The trade-offs between coverage and delay, sensing energy consumption and cost of sensors will be presented to provide insights on WSN design.

Our main contributions are presented as below:

- Under the uniform deployment scheme, we study asymptotic coverage of heterogeneous WSNs with the i.i.d mobility model and 1-dimensional random walk mobility model. We define ESR of WSNs and obtain the critical value of ESR for the WSNs to achieve asymptotic full coverage of the operational region. We demonstrate that 1-dimensional random walk mobility reduces the energy consumption by the order $\Theta\left(\frac{\log n+\log\log n}{n}\right)$ [1] at the expense of $\Theta(1)$ delay;
- Under the uniform deployment scheme, heterogeneity is shown to impose no impact on energy consumption in stationary WSNs or WSNs with i.i.d. mobility model but to *'slightly'* increase sensing energy consumption in WSNs with the 1-dimensional random walk model;
- We study the WSNs under Poisson deployment strategy with the 2-dimensional random walk mobility model and derive the expectation of the fraction of operational region that is $k$-covered by the heterogeneous WSNs at an instant as well as over a time interval. The results hold for a general number of sensors except for the asymptotic case, and can provide guidelines on designing WSNs of high degree coverage. Plus, the results also identifies the improvement of coverage brought by mobility.

[1]The following asymptotic notations are used throughout this paper. Given non-negative functions $f(n)$ and $g(n)$:

1) $f(n) = \Theta(g(n))$ means that for two constants $0 < c_1 < c_2$, $c_1 g(n) \le f(n) \le c_2 g(n)$ for sufficiently large $n$.
2) $f(n) \sim g(n)$ means that $\lim_{n\to+\infty} \frac{f(n)}{g(n)} = 1$.

The rest of the paper is organized as follows. In Section II, the system model and several performance measures are defined. In Section III, we present our main results. The asymptotic coverage problem in mobile heterogeneous WSNs is studied in Section IV and we discuss the impact of mobility and heterogeneity based on the results. In Section V, we study $k$-coverage under Poisson deployment model. Finally, we conclude our work in Section VI.

## II. System Model and Performance Measures

In this section, we describe the system model regarding sensing, deployment and mobility pattern, respectively and present several measures to assess the coverage performance of mobile heterogeneous WSNs.

### A. Deployment Scheme

Let the operational region of the sensor network $\mathcal{A}$ be an unit square and this square is assumed to be a torus. We consider two models according to which the sensors are deployed.

- Uniform Deployment Model — $n$ sensors are randomly and uniformly deployed in the operational region, independent of each other.
- Poisson Deployment Model — Sensors are deployed according to a 2-dimensional Poisson point process with density parameter $n$.

These random deployment strategies are favored in the situation that the geographical region to be sensed is hostile and inimical. Under such circumstance, wireless sensors might be sprinkled from aircrafts, delivered by artillery shell, rocket, missile or thrown from a ship, instead of being placed by human or programmed robots.

### B. Sensing Strategy

Basically, we employ the *binary disc sensing model* in this study, where we assume that a sensor is capable to sense perfectly within the disc of radius $r$ centered at the sensor. Beyond this sensing area, the sensor cannot sense. Here, $r$ denotes the *sensing radius* of a sensor.

Further, our study takes into account the general case that sensors in the network have different sensing radii. We assume that there are $u$ groups $G_1, G_2, \cdots, G_u$ in this heterogeneous network, where $u$ is a positive constant. For $y = 1, 2, \cdots, u$, group $G_y$ consists of $n_y = c_y n$ sensors, where $n$ is the total number of sensors in the network and $c_y (y = 1, 2, \cdots, u)$ is called the *grouping index* which is a positive constant invariant of $n$ (i.e., $c_y = \Theta(1)$) and $\sum_{y=1}^{u} c_y = 1$. For a given group $G_y$, sensors in this group possess identical sensing radius $r_y$.

### C. Mobility Pattern

Sensors move according to certain mobility patterns.

- I.I.D. MOBILITY MODEL — The sensing process is partitioned into time slots with unit length. At the beginning of each time slot, each sensor will randomly and uniformly choose a position within the operational region and remain stationary in the rest of the time slot.
- 1-DIMENSIONAL RANDOM WALK MOBILITY MODEL — Sensors in each group are classified into two types of equal quantity, H-nodes and V-nodes. And sensors of each type move horizontally and vertically, respectively. The sensing process is also divided into time slots with unit length. At the very beginning of each time slot, each sensor will randomly and uniformly choose a direction along its moving dimension and travel in the selected direction a certain distance $D$ which is a random variable uniformly distributed from 0 to 1. [2] We do not set requirements on the velocity of sensor during its movement, but sensors must reach destination within the time slot.
- 2-DIMENSIONAL RANDOM WALK MOBILITY MODEL — Each sensor in group $G_y (y = 1, 2, \cdots, u)$ randomly and independently chooses a direction $\theta \in [0, 2\pi)$ according to probability distribution function (p.d.f.) $f_{\Theta}^{(y)}(\theta)$, and selects a velocity $v \in [0, v_{\max})$ according to p.d.f. $f_V^{(y)}(v)$.

The i.i.d. mobility pattern is widely used since it can provide some intuitions and characterize the upper or lower bound. We will present the main approach to asymptotic coverage problem under this model. The 1-dimensional mobility model is motivated by certain networks that nodes move along determined tracks such as the networks employed in streets, systems consisted of satellites moving in fixed orbits and so forth. The 2-dimensional random walk model can highly exploit the randomness of the motion of nodes and is suitable to depict realistic situation that the statistics about the habit of moving platforms is unknown.

### D. Performance Measures

To assess the coverage performance of the wireless sensor networks, we propose four measures.

- ASYMPTOTIC COVERAGE — The ESR of the mobile heterogeneous WSNs with i.i.d. mobility model is $r_\star = \sqrt{\sum_{y=1}^{u} c_y r_y^2}$ and the ESR for 1-dimensional random walk mobility model is $r_\diamond = \sum_{y=1}^{u} c_y r_y$. Let $\mathcal{C}$ denote the event that the operational region is fully covered. Then if
$$\lim_{n\to\infty} \mathbb{P}(\mathcal{C}) = 1, \text{if } r_\star \geq cR_\star(n) \text{ for any } c > 1;$$
$$\lim_{n\to\infty} \mathbb{P}(\mathcal{C}) < 1, \text{if } r_\star \leq \hat{c}R_\star(n) \text{ for any } 0 < \hat{c} < 1,$$
$R_\star(n)$ is the critical ESR under the i.i.d. model. Similarly, we can define the critical ESR $R_\diamond(n)$ for the 1-dimensional random walk model.
- K-COVERAGE AT AN INSTANT — A point in the operational region is said to be $k$-covered at an instant $t (t \geq 0)$ if it is sensed by at least $k$ different sensors (they may come from various groups), where $k$ is a positive integer. Let $\eta_{(t)}$ be the fraction of the whole operational region that is $k$-covered at instant $t$.
- K-COVERAGE OVER A TIME INTERVAL — A point in the operational region is said to be $k$-coverage during a time interval $\mathcal{T} = [0, t)$ if it has been sensed by at least $k$ different sensors (they may come from various groups) at the end of the interval. Let $\eta_{(\mathcal{T})}$ denote the fraction of the whole area that is $k$-covered within $\mathcal{T}$.

[2] Long distance travel is energy-consuming. And if the sensor can travel beyond the dimension of the operational region (i.e., $D > 1$), it can always cover the area along its moving dimension.

## III. MAIN RESULTS

We summarize our main results in this paper as follows:

1. Under the uniform deployment scheme:

(1-a) With i.i.d. mobility model, the critical ESR is $R_\star(n) = \sqrt{\frac{\log n + \log\log n}{\pi n}}$.

(1-b) With 1-dimensional random walk mobility model, the critical ESR is $R_\diamond(n) = \frac{3(\log n + \log\log n)}{4n}$.

2. Under the Poisson deployment scheme with the 2-dimensional random walk mobility model:

(2-a) $\mathbb{E}\{\eta_{(t)}\} = 1 - \frac{\Gamma\left(k, \pi n \sum_{y=1}^{u} c_y r_y^2\right)}{(k-1)!}$, where $\pi r_y^2 < |\mathcal{A}|$ holds for arbitrary $y$, and $\Gamma(a, b)$ is the upper incomplete gamma function, defined as $\Gamma(a, b) = \int_b^{+\infty} t^{a-1} e^{-t} \mathrm{d}t$.

(2-b) $\mathbb{E}\{\eta_{(\mathcal{T})}\} = 1 - \frac{\Gamma\left(k, n \sum_{y=1}^{u} c_y \mathbb{E}\{S_{(\mathcal{T},y)}\}\right)}{(k-1)!}$, where $\mathbb{E}\{S_{(\mathcal{T},y)}\}$ denotes the expected area covered by a sensor in group $G_y$ during the time interval $\mathcal{T}$ and $\mathbb{E}\{S_{(\mathcal{T},y)}\} < |\mathcal{A}|$ holds for arbitrary $y$ and $\mathcal{T}$, $\Gamma(a, b)$ is the upper incomplete gamma function.

## IV. ASYMPTOTIC COVERAGE UNDER UNIFORM DEPLOYMENT SCHEME

In this section, we analyze the asymptotic coverage under uniform employment scheme with the i.i.d. and 1-dimensional random walk mobility model, respectively.

### A. Overview of Dense Grid

It is relatively difficult to analyze coverage by check whether all points in the operational region are covered. To approach to this problem, certain idea has been presented in [5] and [6], that is to transform the coverage of all points

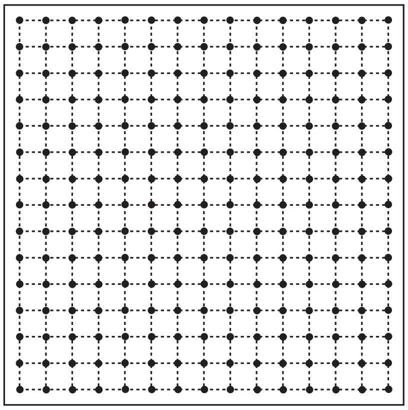

Figure 1: Dense Grid Within the Operational Region

within the operational region to the coverage of certain set of points.

The set of points, denoted by $\mathbb{M}$, is all the grid points of a $\sqrt{m}\times\sqrt{m}$ grid on the operational region. From LEMMA 3.1 and THEOREM 4.1 in [6], we know that if $m$ is large enough (i.e., the grid is dense enough), the sensing radius that sufficient to guarantee the asymptotic coverage of points in $\mathbb{M}$ will be sufficient to ensure the asymptotic coverage of the entire operational region as well. And the necessary sensing radius for $\mathbb{M}$ is also necessary for the whole operational region to achieve full coverage. Then we can focus on the coverage of the dense grid. And we will derive the critical (both necessary and sufficient) ESR for the sensor network to achieve full coverage of the dense grid.

### B. Critical ESR Under I.I.D. Mobility Model

Let $\mathcal{G}$ denote the event that the dense grid $\mathbb{M}$ is covered. And we derive the critical ESR to guarantee asymptotic full coverage of $\mathbb{M}$.

**Definition IV.1.** *For mobile heterogeneous WSNs with i.i.d. mobility model, $r_\star(n)$ is the critical sensing radius for $\mathbb{M}$ if*

$$\lim_{n\to\infty}\mathbb{P}(\mathcal{G})=1, \textit{if } r_\star\geq cr_\star(n) \textit{ for any } c>1;$$
$$\lim_{n\to\infty}\mathbb{P}(\mathcal{G})<1, \textit{if } r_\star\leq \hat{c}r_\star(n) \textit{ for any } 0<\hat{c}<1.$$

We have the following useful lemmas.

**Lemma IV.1.** *Given $x=x(n)$ and $y=y(n)$ both of which are positive functions of $n$, then $(1-x)^y\sim e^{-xy}$ if $x$ and $x^2y$ approach $0$ as $n\to+\infty$.*

*Proof:* Refer to Appendix. ■

**Lemma IV.2.** *If $r_\star(n)=\sqrt{\frac{\log n+\log\log n+\xi}{\pi n}}$ and $m=m(n)=n\log n$, for any fixed $\theta<1$,*

$$m\prod_{y=1}^{u}\left(1-\pi r_y^2(n)\right)^{c_y n}\geq\theta e^{-\xi}, \tag{1}$$

*holds for all sufficient large $n$.*

*Proof:* See Appendix. ■

*1) Necessary ESR for Full Coverage of Dense Grid:* Let $\overline{\mathcal{G}}$ denote the event that the dense grid $\mathbb{M}$ is not fully covered and we have the following proposition.

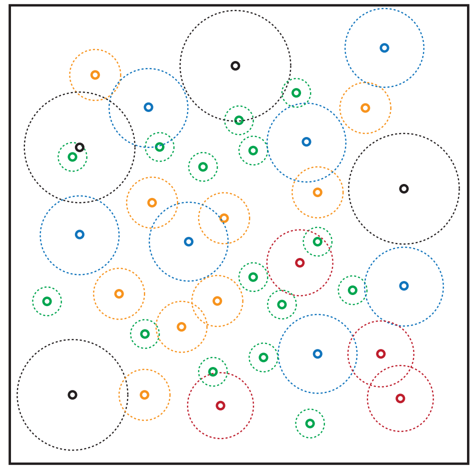

Figure 2: Coverage Under I.I.D Model

**Proposition IV.1.** *In the mobile heterogeneous WSNs with i.i.d. mobility model, if $r_\star(n)=\sqrt{\frac{\log n+\log\log n+\xi(n)}{\pi n}}$ and the density of the dense grid $\mathbb{M}$ is $m=n\log n$, then*

$$\liminf_{n\to+\infty}\mathbb{P}(\overline{\mathcal{G}})\geq e^{-\xi}-e^{-2\xi},$$

*where $\xi=\lim_{n\to+\infty}\xi(n)$.*

*Proof:* To begin with, we study the case where $r_\star(n)=\sqrt{\frac{\log n+\log\log n+\xi}{\pi n}}$ for a fixed $\xi$. Apply the Bonferroni inequality, we have that

$$\begin{aligned}\mathbb{P}(\overline{\mathcal{G}}) &\geq \sum_{P_i\in\mathbb{M}}\mathbb{P}(\{\text{some point } P_i \text{ is not covered}\})\\ &\geq \sum_{P_i\in\mathbb{M}}\mathbb{P}(\{P_i \text{ is the only uncovered point}\})\\ &\geq \sum_{P_i\in\mathbb{M}}\mathbb{P}(\{P_i \text{ is not covered}\})\\ &\quad -\sum_{P_i,P_j\in\mathbb{M}}^{P_i\neq P_j}\mathbb{P}(\{P_i \text{ and } P_j \text{ are not covered}\})\end{aligned}\tag{2}$$

Respectively, we can evaluate the two terms on the right hand side of (2). As for the first term, we have

$$\begin{aligned}
&\mathbb{P}(\{P_i \text{ is not covered}\})\\
&= \prod_{y=1}^{u} \mathbb{P}(\{P_i \text{ is not covered by sensors in } G_y\})\\
&= \prod_{y=1}^{u} \left(1-\pi r_y^2(n)\right)^{c_y n}. \qquad (3)
\end{aligned}$$

Using Lemma IV.2, we bound the first term that for any $\theta < 1$,

$$\sum_{P_i \in \mathbb{M}} \mathbb{P}(\{P_i \text{ is not covered}\}) \geq \theta e^{-\xi}, \qquad (4)$$

for all $n > N_\xi$.

Let $s_y(n) = \pi r_y^2(n)$ and $s_\star(n) = \pi r_\star^2(n)$, then $s_\star(n) = \sum_{y=1}^{u} c_y s_y(n) = \frac{\log n + \log\log n + \xi}{n}$. Hence for all $y = 1, 2, \cdots, u$, $s_y(n) = \Theta\left(\frac{\log n + \log\log n + \xi}{n}\right)$ which indicates that $s_y(n)$ and $s_y^2(n)(c_y n)$ approach 0 as $n \to +\infty$. From Lemma IV.1, we obtain that for arbitrary positive constant $\alpha$,

$$(1 - \alpha s_y(n))^{c_y n} \sim e^{-\alpha n (c_y s_y(n))}. \qquad (5)$$

Thus, for two points $P_i$ and $P_j$ in $\mathbb{M}$, we obtain that

$$\begin{aligned}
&\mathbb{P}(\{P_i \text{ and } P_j \text{ are not covered}\})\\
&\geq \prod_{y=1}^{u} \left(1-4\pi r_y^2(n)\right)\left(1-2\pi r_y^2(n)\right)^{c_y n}\\
&\sim e^{-2n\sum_{y=1}^{u}(c_y s_y(n))}. \qquad (6)
\end{aligned}$$

and on the other hand, we have the upper bound

$$\begin{aligned}
&\mathbb{P}(\{P_i \text{ and } P_j \text{ are not covered}\})\\
&\leq \prod_{y=1}^{u} \pi r_y^2(n)\left(1-\pi r_y^2(n)\right)^{c_y n} + \prod_{y=1}^{u}\left(1-2\pi r_y^2(n)\right)^{c_y n}\\
&\sim e^{-2n\sum_{y=1}^{u}(c_y s_y(n))}. \qquad (7)
\end{aligned}$$

From (6) and (7), we know that

$$\mathbb{P}(\{P_i \text{ and } P_j \text{ are not covered}\}) \sim e^{-2n\sum_{y=1}^{u}(c_y s_y(n))}. \qquad (8)$$

Then the following result holds

$$\begin{aligned}
&\sum_{P_i, P_j \in \mathbb{M}}^{P_i \neq P_j} \mathbb{P}(\{P_i \text{ and } P_j \text{ are not covered}\})\\
&\sim m^2 e^{-2n\sum_{y=1}^{u}(c_y s_y(n))}\\
&= (n\log n)^2 e^{-2ns_\star(n)}\\
&= (n\log n)^2 e^{-2(\log n + \log\log n + \xi)}\\
&= e^{-2\xi}. \qquad (9)
\end{aligned}$$

Using (4) and (9) into (2), we have

$$\mathbb{P}(\overline{\mathcal{G}}) \geq \theta e^{-\xi} - e^{-2\xi}. \qquad (10)$$

for any $\theta < 1$.

As for the case that $\xi$ is a function of $n$ with $\xi = \lim_{n\to+\infty} \xi(n)$, we know $\xi(n) \leq \xi + \delta$ for any $\delta > 0$ all $n > N_\delta$. Since $\mathbb{P}(\overline{\mathcal{G}})$ is monotonously decreasing in $r_\star$ and thus in $\xi$, we have

$$\mathbb{P}(\overline{\mathcal{G}}) \geq \theta e^{-(\xi+\delta)} - e^{-2(\xi+\delta)}, \qquad (11)$$

for all $n > N_{\theta,\delta}$. Then the result follows. ■

From Proposition IV.1, $\mathbb{P}(\overline{\mathcal{G}})$ is bounded away from zero with positive probability if $\lim_{n\to+\infty}\xi(n) < +\infty$. Combined with Definition IV.1, we know that $r_\star \geq \sqrt{\frac{\log n + \log\log n}{\pi n}}$ is necessary to achieve the full coverage of the dense grid $\mathbb{M}$.

*2) Sufficient ESR for Full Coverage of Dense Grid:* Let $\mathcal{F}_i$ denote the event that point $P_i$ in $\mathbb{M}$ is not covered. If $r_\star = cr_\star(n)$ where $c > 1$, then

$$\begin{aligned}
\mathbb{P}\left(\bigcup_{i=1}^{m}\mathcal{F}_i\right) &\leq \sum_{i=1}^{m}\mathbb{P}(\mathcal{F}_i)\\
&\leq (n\log n)\prod_{y=1}^{u}\left(1-\pi r_y^2(n)\right)^{c_y n}\\
&\sim (n\log n)e^{-n\pi(r_\star)^2}\\
&= \frac{1}{(n\log n)^{c^2-1}}.
\end{aligned}$$

For any $c > 1$, we then have the following result

$$\lim_{n\to+\infty}\mathbb{P}\left(\bigcup_{i=1}^{m}\mathcal{F}_i\right) = 0.$$

Therefore, $r_\star \geq \sqrt{\frac{\log n + \log\log n}{\pi n}}$ is sufficient to guarantee the full coverage of the dense grid.

*3) Critical ESR for Full Coverage of Operational Region:* The density of the dense grid $m = n\log n$ is sufficiently large to evaluate the coverage problem of the whole operational region. Refer to LEMMA 3.1 in [6] and using similar approach in THEOREM 4.1 in [6], we can demonstrate that $r_\star \geq \sqrt{\frac{\log n + \log\log n}{\pi n}}$ is sufficient to achieve the full coverage of the whole operational region. On the other hand, points in $\mathbb{M}$ constitute a subregion of the operational region, which indicates that the necessary condition for the dense grid is surely the necessary condition for the whole operational region.

Hence, we have the following theorem

**Theorem IV.1.** *Under the uniform deployment scheme with i.i.d. mobility model, the critical ESR for mobile heterogeneous WSNs to achieve asymptotic full coverage is* $R_\star(n) = \sqrt{\frac{\log n + \log\log n}{\pi n}}$.

*C. Critical ESR Under 1-Dimensional Random Walk Mobility Model*

Under the 1-dimensional random walk mobility model, the sensing process is slotted and sensors make use of each time slot to move and sense. We use $\mathcal{G}^\tau$ to denote the event that $\mathbb{M}$ is covered in a given time slot $\tau$, and $\mathbb{P}_\tau(\mathcal{G}^\tau)$ to denote the corresponding probability. Similarly, we define the critical ESR for 1-dimensional random walk model.

**Definition IV.2.** *For mobile heterogeneous WSNs with 1-dimensional random walk mobility model, $r_\diamond(n)$ is the critical sensing radius for the dense grid if*

$$\lim_{n\to\infty} \mathbb{P}_\tau(\mathcal{G}^\tau) = 1, \textit{if } r_\diamond \geq c r_\diamond(n) \textit{ for any } c > 1;$$
$$\lim_{n\to\infty} \mathbb{P}_\tau(\mathcal{G}^\tau) < 1, \textit{if } r_\diamond \leq \hat{c} r_\diamond(n) \textit{ for any } 0 < \hat{c} < 1.$$

*1) Failure Probability of a Point in $\mathbb{M}$:* We use $\mathcal{F}_i$ to denote the event that point $P_i$ in $\mathbb{M}$ is not covered by any sensor and refer to $\mathbb{P}(\mathcal{F}_i)$ as the failure probability of point $P_i$ in $\mathbb{M}$.

We first consider the probability that an arbitrary point $P_i$ in $\mathbb{M}$ can be successfully covered by a sensor $X_y$ in group $G_y$, and denoted probability as $\mathbb{P}_{(i,y)}$. Because of the symmetry of the topology, we only need to take into account the case that $X_y$ moves horizontally.

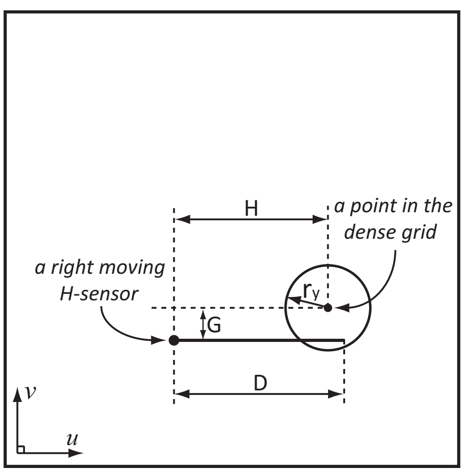


Figure 3: Coverage of a Single Point

Since $X_y$ chooses to move left or right with equal probability, we suppose $X_y$ moves right. Initially, sensors are uniformly deployed and according to the 1-dimensional random walk mobility model, sensors are always uniformly distribution at each time slot $\tau$ in the operational region seen by the points in $\mathbb{M}$. On the other hand, the dense grid is randomly and uniformly located in the operational region.

We build a Cartesian coordinate system in the operational region and denote the position of $X_y$ and $P_i$ with $(u_1, v_1)$ and $(u_2, v_2)$, respectively. Then we know that $u_1, v_1, u_2, v_1$ are uniformly distributed from 0 to 1.

Since $X_y$ does not move vertically, when considering the vertical distance between $X_y$ and $P_i$ which is denoted as $G$, we should recognize the fact that the upper boundary of the operational region is adjacent to the lower boundary. Hence, we have $G = \min\{|v_1 - v_2|, 1 - |v_1 - v_2|\}$. However, on the horizontal dimension, $X_y$ moves in certain direction and might sense $P_i$ on its way, which leads the horizontal distance to be $H = |u_1 - u_2|$. Then, we can have the p.d.f. of $G$ and $H$

$$f_G(g) = \begin{cases} 2, & 0 \leq g \leq \frac{1}{2}; \\ 0, & \text{otherwise.} \end{cases}$$

$$f_H(h) = \begin{cases} 2(1-h), & 0 \leq h \leq 1; \\ 0, & \text{otherwise.} \end{cases}$$

Point $P_i$ can be successfully covered by $X_y$ if and only if $X_y$ can enter the circle with radius $r_y$ centered at $P_i$. Hence, we have the failure probability

$$\begin{aligned} \mathbb{P}_{(i,y)} &= \mathbb{P}(G \leq r_y) \cdot \mathbb{P}(H \leq D) \\ &= (2r_y) \iint_{h \leq d} 2(1-h)\mathrm{d}h\mathrm{d}d = \frac{4}{3} r_y. \end{aligned} \quad (12)$$

Then $\mathbb{P}(\mathcal{F}_i)$ can be easily calculated.

*2) Necessary ESR for Full Coverage of Dense Grid:* Let $\overline{\mathcal{G}^\tau}$ denote the event that the dense grid $\mathbb{M}$ is not fully covered in the given time slot $\tau$. We have the following technical lemma.

**Lemma IV.3.** *If $m = m(n) = n \log n$ and $r_\diamond(n) = \frac{3(\log n + \log\log n + \xi)}{4n}$, then for any fixed $\theta < 1$,*

$$m \prod_{y=1}^{u} \left(1 - \frac{4}{3} r_y(n)\right)^{c_y n} \geq \theta e^{-\xi}, \quad (13)$$

*holds for all sufficient large $n$.*

*Proof:* Using the same technique used in the proof of Lemma IV.2, the result follows. ■

Then we present the following proposition regarding the necessary condition on ESR.

**Proposition IV.2.** *In the mobile heterogeneous WSNs with 1-dimensional random walk mobility model, if $r_\diamond = \frac{3(\log n + \log\log n + \xi(n))}{4n}$ and the density of the dense grid $\mathbb{M}$ is $m = n \log n$, then*

$$\liminf_{n\to+\infty} \mathbb{P}_\tau(\overline{\mathcal{G}^\tau}) \geq e^{-\xi} - e^{-2\xi}.$$

*where $\xi = \lim_{n\to+\infty} \xi(n)$.*

*Proof:* The technique used in this proof is similar to that used in the proof of Proposition IV.1 and we present

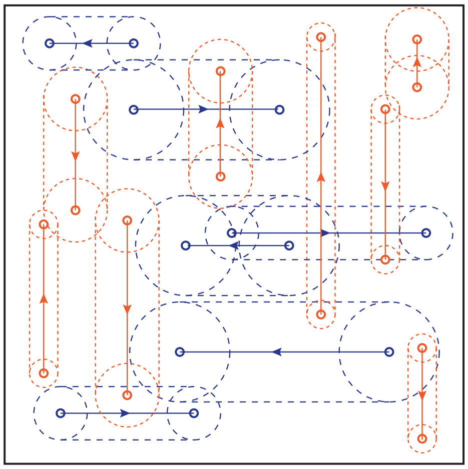

Figure 4: Coverage Under 1-Dimensional Random Walk Mobility Model

the main steps here. We begin with the case that $r_\diamond = \frac{3(\log n + \log\log n + \xi)}{4n}$ for a fixed $\xi$.

$$\mathbb{P}_\tau(\overline{\mathcal{G}^\tau}) \geq \sum_{P_i \in \mathbb{M}} \mathbb{P}_\tau(\{P_i \text{ is not covered}\}) - \sum_{P_i, P_j \in \mathbb{M}}^{P_i \neq P_j} \mathbb{P}_\tau(\{P_i, P_j \text{ are not covered}\}) \quad (14)$$

Based on (12), we can evaluate the first term of on the right hand of side of (14) and have

$$\mathbb{P}_\tau(\{P_i \text{ is not covered}\}) = \prod_{y=1}^{u} \left(1 - \frac{4}{3} r_y(n)\right)^{c_y n}. \quad (15)$$

Using Lemma IV.3, we have

$$\sum_{P_i \in \mathbb{M}} \mathbb{P}_\tau(\{P_i \text{ is not covered}\}) \geq \theta e^{-\xi}, \quad (16)$$

for any $\theta < 1$ and all $n > N_\xi$.

Let $t_y(n) = \frac{4}{3} r_y(n)$ and $t_\diamond(n) = \frac{4}{3} r_\diamond(n)$. And we can similarly obtain the lower bound

$$\begin{aligned} &\mathbb{P}_\tau(\{P_i, P_j \text{ are not covered}\}) \\ &\geq \prod_{y=1}^{u} \left(1 - 4\pi r_y^2(n)\right) \left(1 - \frac{8}{3} r_y(n)\right)^{c_y n} \\ &\sim e^{-2n \sum_{y=1}^{u} (c_y t_y(n))}, \end{aligned} \quad (17)$$

and the upper bound

$$\begin{aligned} &\mathbb{P}_\tau(\{P_i, P_j \text{ are not covered}\}) \\ &\leq \prod_{y=1}^{u} \pi r_y^2(n) \left(1 - \frac{4}{3} r_y(n)\right)^{c_y n} + \prod_{y=1}^{u} \left(1 - \frac{8}{3} r_y(n)\right)^{c_y n} \\ &\sim e^{-2n \sum_{y=1}^{u} (c_y t_y(n))}. \end{aligned} \quad (18)$$

Combining (17) and (18), we have

$$\begin{aligned} &\sum_{P_i, P_j \in \mathbb{M}}^{P_i \neq P_j} \mathbb{P}_\tau(\{P_i, P_j \text{ are not covered}\}) \\ &\sim m^2 e^{-2n \sum_{y=1}^{u} (c_y t_y(n))} = e^{-2\xi}. \end{aligned} \quad (19)$$

Thus, using (16) and (19) into (14), we obtain

$$\mathbb{P}_\tau(\overline{\mathcal{G}^\tau}) \geq \theta e^{-\xi} - e^{-2\xi}. \quad (20)$$

Taking into account the case that $\xi$ is a function of $n$, the conclusion still holds. ∎

From Proposition IV.2, we know that $r_\diamond \geq \frac{3(\log n + \log\log n)}{4n}$ is necessary to achieve the full coverage of $\mathbb{M}$.

*3) Sufficient ESR for Full Coverage of Dense Grid:* If $r_\diamond = c r_\diamond(n)$ where $c > 1$, then

$$\begin{aligned} \mathbb{P}_\tau\left(\bigcup_{i=1}^{m} \mathcal{F}_i\right) &\leq \sum_{i=1}^{m} \mathbb{P}_\tau(\mathcal{F}_i) \\ &\leq (n \log n) \prod_{y=1}^{u} \left(1 - \frac{4}{3} r_y(n)\right)^{c_y n} \\ &\sim (n \log n) e^{-n\pi (r_\diamond)^2} \\ &= \frac{1}{(n \log n)^{c^2 - 1}} \to 0 \text{ as } n \to +\infty. \end{aligned}$$

Therefore, $r_\diamond \geq \frac{3(\log n + \log\log n)}{4n}$ is sufficient to guarantee the full coverage of the dense grid $\mathbb{M}$.

*4) Critical ESR for Full Coverage of Operational Region:* Similar to the analysis in the i.i.d. mobility model, we can reach the following theorem.

**Theorem IV.2.** *Under the uniform deployment scheme with 1-dimensional random walk mobility model, the critical ESR for mobile heterogeneous WSNs to achieve asymptotic full coverage is* $R_\diamond(n) = \frac{3(\log n + \log\log n)}{4n}$.

### *D. The Impact of Mobility and Heterogeneity on Sensing Energy Consumption*

We discuss the impact of mobility and heterogeneity on sensing energy consumption based on the results obtained in previous parts of this section.

We used the sensing energy model as $E_y \propto r_y^2$, where $E_y$ is the energy consumption of sensors with sensing radius $r_y$. Let $\overline{E}$ denote the average energy consumption of the mobile heterogenous WSNs and then $\overline{E} \propto \sum_{y=1}^{u} c_y r_y^2$.

*1) Impact of Mobility:* First, we consider the impact of mobility and sensors are assumed to have identical sensing radius, i.e., $r_y = r_\star$ or $r_y = r_\diamond (y = 1, 2, \cdots, u)$ under i.i.d and 1-dimensional random walk mobility model, respectively. We have the following results

(a) Under I.I.D. Mobility Model:

$$\overline{E}_{i.i.d.} = \Theta\left(\frac{\log n + \log\log n}{n}\right). \tag{21}$$

(b) Under 1-Dimensional Random Walk Mobility Model:

$$\overline{E}_{r.w.} = \Theta\left(\left(\frac{\log n + \log\log n}{n}\right)^2\right). \tag{22}$$

From the derivation of ESR under i.i.d. mobility model, we can realize that i.i.d. mobility is actually quasi-static since the reshuffle of sensor positions does not increase the area of sensed region in a time slot compared with the stationary case. The energy consumption $\overline{E}_{stat.}$ in static WSNs equals that in WSNs with i.i.d model. Therefore, we obtain that

$$\overline{E}_{r.w.} = \Theta\left(\frac{\log n + \log\log n}{n}\right) \cdot \overline{E}_{stat.},$$

which indicates that 1-dimensional random walk mobility model can decrease energy consumption in WSNs. However, this improvement in energy efficiency sacrifices the timeliness of detection since under 1-dimensional random walk mobility model we evaluate the coverage performance of WSNs in a time slot $\tau$ while in stationary WSNs full coverage is maintained in any time instant. The delay to achieve full coverage is upper bounded by $\Theta(1)$. This is the trade-off between energy consumption and delay of coverage in mobile WSNs.

*2) Impact of Heterogeneity:* Second, we consider the impact of heterogeneity and obtain the following results

(a) Under I.I.D. Mobility Model:

$$\overline{E}_{i.i.d.} = \Theta\left(\frac{\log n + \log\log n}{n}\right). \tag{23}$$

(b) Under 1-Dimensional Random Walk Mobility Model:

$$\overline{E}_{r.w.} > \Theta\left(\left(\frac{\log n + \log\log n}{n}\right)^2\right); \tag{24}$$

$$\overline{E}_{r.w.} < \Theta\left(\frac{(\log n + \log\log n)^{\frac{3}{2}}}{n^{\frac{3}{2}+\frac{1}{2}d_0}}\right), \tag{25}$$

where $d_0$ is any constant such that $0 < d_0 < 1$. The lower bound in (b) comes from the Cauchy Inequality that

$$\left(\sum_{y=1}^{u} c_y\right)\left(\sum_{y=1}^{u} c_y r_y^2\right) > \left(\sum_{y=1}^{u} c_y r_y\right)^2.$$

And the upper bound results from the following inequality

$$\left(\sum_{y=1}^{u} c_y r_y^2\right) < \left(\sum_{y=1}^{u} c_y r_y\right)^{\frac{3}{2}} \left(\frac{1}{n^{d_0}}\right)^{\frac{1}{2}},$$

which can be derived from the fact that $\frac{c_y}{n^{d_0}} > r_z (y, z = 1, 2, \cdots, u)$ for sufficiently large $n$.

Hence, with the sensing energy model $E_y \propto r_y^2$, heterogeneity does not make any difference to sensing energy in WSNs under i.i.d. mobility model or stationary WSNs, which can be seen from (21) and (23). However, from (22) and (24) we know that under 1-dimensional random walk mobility model, sensing energy consumption will raise due to heterogeneity. And there is a trade-off in mobile WSNs that designers must face: on one hand, heterogeneous WSNs consisted of high-end sensors with large sensing range and low-end sensors with small sensing range can reduce the cost of WSNs and guarantee a satisfactory sensing performance; on the other hand, heterogeneity will increase sensing energy consumption. From the upper bound in (25), the order of energy consumption in terms of $n$ approaches the order (i.e., $n^2$) in the case without heterogeneity. Hence, the energy efficiency will not be dramatically deteriorated by the heterogeneity.

**Remark IV.1.** *The sensing process in WSNs depends on the area covered by each sensor. And under the* 1*-dimensional random walk mobility model, the area covered by sensors is on the same order of sensing radius $r$. In stationary WSNs or WSNs with i.i.d. mobility model, however, the covered area is on the order of $r^2$. This discrepancy of dependence on sensing radius leads the impact of heterogeneity to be different under the two models.*

## V. $K$-Coverage In Mobile Heterogeneous WSNs Under Poisson Deployment Model

In this section, we study $k$-coverage at an instant and over a time interval in mobile heterogeneous WSNs. As demonstrated in [2], the sensor locations within the operational region are modeled as 2-dimensional Poisson process with density $n$. Thus, the coverage problem can be described by the frequently used Poisson-Boolean model. Besides, the 2-dimensional random walk mobility model is employed in this part.

### A. $K$-coverage at an Instant

We start with the results regarding $k$-coverage at an instant.

**Theorem V.1.** *For arbitrary $y$, $\pi r_y^2 < |\mathcal{A}|$ holds. Given an instant $t > 0$, the expectation of the fraction of operational region that is $k$-covered at instant $t$ is*

$$\mathbb{E}\{\eta_{(t)}\} = 1 - \frac{\Gamma\left(k, \pi n \sum_{y=1}^{u} c_y r_y^2\right)}{(k-1)!},$$

*where $\Gamma(a,b)$ is the upper incomplete gamma function, defined as $\Gamma(a,b)=\int_b^{+\infty} t^{a-1}e^{-t}dt$.*

*Proof:* We follow the basic idea in [2]. The condition $\pi r_y^2 < |\mathcal{A}|$ is necessary here and in the proof of [2] as well, we think. For any point $B$ in the operational region $\mathcal{A}$, define an indicator function $I_B$ on whether $B$ is $k$-covered at instant $t$ as follows,

$$I_B=\begin{cases}1, \text{ point } B \text{ is } k\text{-covered at instant } t,\\ 0, \text{ otherwise.}\end{cases}$$

Then we acquire that

$$\mathbb{E}\{I_B\}=\mathbb{P}(\{\text{point } B \text{ is } k\text{-covered at instant } t\}). \quad (26)$$

Denote the total area in $\mathcal{A}$ that is $k$-covered as $|\mathcal{V}|$. We have $|\mathcal{V}|=\int_{\mathcal{A}} I_B \mathrm{d}B$. By Fubini's theorem, exchanging the order of integral and expectation, we obtain

$$\mathbb{E}\{|\mathcal{V}|\}=\int_{\mathcal{A}}\mathbb{E}\{I_B\}\mathrm{d}B. \quad (27)$$

Let $\eta_{(t)}$ be the fraction of area that is $k$-covered at the instant $t$. Clearly, since we do not consider the boundary effects, for any point $B$ in $\mathcal{A}$, the probability that $B$ is $k$-covered is identical. Therefore, substituting (26) into (27),

$$\begin{aligned}\mathbb{E}\{\eta_{(t)}\} &= \frac{\int_{\mathcal{A}}\mathbb{P}(\{B \text{ is } k\text{-covered at instant } t\})\mathrm{d}B}{|\mathcal{A}|}\\ &= \mathbb{P}(\{B \text{ is } k\text{-covered at instant } t\}), \quad (28)\end{aligned}$$

where $B$ is any given point in $\mathcal{A}$. The equality (28) means that the expected value of $\eta_{(t)}$ is equal to the probability that any given point in $\mathcal{A}$ is $k$-covered.

Let $C(B,r)$ be the circle centered at point $B$ with radius $r$. If a sensor $X$ with sensing range $r$ is in $C(B,r)$, then sensor $X$ covers point $B$. Use $m_{(t,y,B,r_y)}$ to denote the number of sensors in group $G_y$ which have ever been in $C(B,r_y)$ at the given instant $t$. Define $M(B,t):=\sum_{y=1}^{u} m_{(t,y,B,r_y)}$. Then $M(B,t)$ is the number of sensors that cover point $B$ at the instant $t$. Thus, point $B$ is $k$-covered at instant $t$ if and only if the following inequality holds: $M(B,t)\ge k$.

Since sensors is Poisson distributed at time $t=0$ and follow the random mobility pattern, the locations of sensors at arbitrary instant $t$ can still be modeled as Poisson point process of density $n$. Further, sensors in the group $G_y(y=1,2,\cdots,u)$ can be seen to follow a Poisson point process with density $n_y=c_y n(y=1,2,\cdots,u)$. Therefore, at the given instant $t$, $m_{(t,y,B,r_y)}$ follows the Poisson distribution with density $n_y\pi r_y^2$. Namely, $m_{(t,y,B,r_z)}\sim \text{Poisson}(n_y\pi r_y^2)$.

As $M(B,t)$ is the sum of independent Poisson-distributed random variables $m_{(t,y,B,r_y)}$ $(y=1,2,\cdots,u)$, thereby $M(B,t)$ obeys a Poisson distribution with parameter $\sum_{y=1}^{u} n_y\pi r_y^2$. Then, we have

$$M(B,t)\sim \text{Poisson}\left(\sum_{y=1}^{u} n_y\pi r_y^2\right),$$

and thus we obtain the probability mass function

$$\begin{aligned}&\mathbb{P}(M(B,t)=m)\\ &= \frac{1}{m!}\left(\sum_{y=1}^{u} n_y\pi r_y^2\right)^m \exp\left(-\sum_{y=1}^{u} n_y\pi r_y^2\right). \quad (29)\end{aligned}$$

From (29), we can derive

$$\begin{aligned}&\mathbb{P}(\{B \text{ is not } k\text{-covered at instant } t\})\\ &= P(M(B,t)\le k-1)\\ &= \sum_{m=0}^{k-1}\frac{1}{m!}\left(\sum_{y=1}^{u} n_y\pi r_y^2\right)^m \exp\left(-\sum_{y=1}^{u} n_y\pi r_y^2\right)\\ &= \frac{\Gamma\left(k,\sum_{y=1}^{u} n_y\pi r_y^2\right)}{(k-1)!}\\ &= \frac{\Gamma\left(k,n\sum_{y=1}^{u} c_y\pi r_y^2\right)}{(k-1)!}. \quad (30)\end{aligned}$$

Then from (28) and (30), the result follows. ■

## B. K-coverage over a Time Interval

Similar results can be obtained for $k$-coverage over a time interval $\mathcal{T}$.

**Theorem V.2.** *Let $\mathbb{E}\{S_{(\mathcal{T},y)}\}$ denote the expected area covered by a sensor in group $G_y$ during the time interval $\mathcal{T}$. $\mathbb{E}\{S_{(\mathcal{T},y)}\}=\pi r_y^2+\mathbb{E}\{v_y\}<|\mathcal{A}|$ holds for arbitrary $y$ and $\mathcal{T}$. Given a time interval $\mathcal{T}=[0,t)$, the expectation of the fraction of the operational region that is $k$-covered during this interval is*

$$\mathbb{E}\{\eta_{(\mathcal{T})}\}=1-\frac{\Gamma\left(k,n\sum_{y=1}^{u} c_y\mathbb{E}\{S_{(\mathcal{T},y)}\}\right)}{(k-1)!},$$

*where $\Gamma(a,b)$ is the upper incomplete gamma function.*

*Proof:* As demonstrated in [2], area coverage depends on the distribution of the random shapes only through its expected area. Hence, similar to the proof of *Theorem* V.1, we can easily obtain that

$$\mathbb{E}\{\eta_{(\mathcal{T})}\}=1-\frac{\Gamma\left(k,n\sum_{y=1}^{u} c_y\mathbb{E}\{S_{(\mathcal{T},y)}\}\right)}{(k-1)!}. \quad (31)$$

Note the condition that $\mathbb{E}\{S_{(\mathcal{T},y)}\}<|\mathcal{A}|$ which means the area covered by a sensor during $\mathcal{T}$ cannot exceed the total area of the operational region otherwise the conclusion derived in [1] will not apply. ■

## VI. CONCLUDING REMARKS

In this paper, we have studied coverage in mobile and heterogeneous wireless sensor networks. Specifically, we have investigated asymptotic coverage under uniform deployment model with i.i.d. and 1-dimensional random walk mobility model, respectively. Mobility is found to decrease sensing energy consumption. On the other hand, we demonstrate that heterogeneity increases energy consumption under 1-dimensional random walk mobility model but imposes no impact under the i.i.d. model. The $k$-coverage under Poisson deployment scheme with 2-dimensional random walk mobility model has been discussed, which also identifies the coverage improvement brought by mobility.

There are several directions for future work. First, we plan to extend the results for asymptotic coverage to include the case that sensors follow Poisson deployment scheme. Second, it is interesting to consider asymptotic $k$-coverage under certain mobility models. Finally, we would like to consider coverage under uniform deployment model for a more general $n$ rather than the asymptotic case.

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## APPENDIX

*Proof of Lemma IV.1:* Taking logarithm, we have

$$\begin{aligned}\log(1-x)^y = y\log(1-x) \\ = \quad -y\sum_{i=1}^{+\infty}\frac{x^i}{i} \\ = \quad -y\left(x+\frac{x^2}{2}+\delta(x)\right),\end{aligned} \tag{32}$$

where

$$0<\delta(x)=\sum_{i=3}^{+\infty}\frac{x^i}{i}<\sum_{i=3}^{+\infty}\frac{x^i}{3}=\frac{1}{3}\frac{x^3}{1-x}<\frac{x^2}{3}. \tag{33}$$

The last step in (33) comes from the fact that $1-x>x$ when $n$ is sufficiently large.

Therefore, from (32) and (33) we have

$$e^{-xy-\frac{5}{6}x^2y}<(1-x)^y<e^{-xy-\frac{1}{2}x^2y}. \tag{34}$$

Then the result follows. ∎

*Proof of Lemma IV.2:* Let $s_y=\pi r_y^2(n)$ and taking logarithm of the left hand side of (1), we obtain

$$\begin{aligned}
&\log\left(m\prod_{y=1}^{u}\left(1-\pi r_y^2(n)\right)^{c_y n}\right)\\
&= \log(n\log n)+\sum_{y=1}^{u}\Big((c_y n)\log(1-s_y)\Big)\\
&= \log(n\log n)-\sum_{y=1}^{u}\left((c_y n)\sum_{i=1}^{+\infty}\frac{(s_y)^i}{i}\right)\\
&= \log(n\log n)-\sum_{y=1}^{u}\left(c_y n\left(\sum_{i=1}^{2}\frac{(s_y)^i}{i}+\delta_y\right)\right), 
\end{aligned}\tag{35}$$

where, similar to (33),

$$\delta_y=\delta_y(n)=\sum_{i=3}^{+\infty}\frac{(s_y)^i}{i}\le\frac{1}{3}(s_y)^2, \tag{36}$$

Substituting (36) into (35), we have

$$\begin{aligned}
&\log\left(m\prod_{y=1}^{u}\left(1-\pi r_y^2(n)\right)^{c_y n}\right)\\
&\ge \log(n\log n)-\sum_{y=1}^{u}\left((c_y n)\left((s_y)+\frac{5}{6}(s_y)^2\right)\right)\\
&\ge \log(n\log n)-n\left(\pi r_\star^2(n)\right)-\frac{5}{6}n\sum_{y=1}^{u}\left(c_y(s_y)^2\right)\\
&= -\xi-\frac{5}{6}n\sum_{y=1}^{u}\left(c_y(s_y)^2\right).
\end{aligned}\tag{37}$$

Note that $s_i<c_j$ for $i,j=1,2,\cdots,u$ when $n$ is sufficiently large. We then have

$$\sum_{y=1}^{u}\left(c_y(s_y)^2\right)\le\left(\sum_{y=1}^{u}(c_y s_y)\right)^{3/2}=\left(\pi r_\star^2(n)\right)^{\frac{3}{2}}.$$

Hence, we know that

$$\frac{5}{6}n\sum_{y=1}^{u}\left(c_y(s_y)^2\right)\le\frac{5}{6}n\left(\pi r_\star^2(n)\right)^{\frac{3}{2}}\to 0,\ \text{as } n\to+\infty.$$

Therefore, for any $\epsilon>0$, the following holds

$$\log\left(m\prod_{y=1}^{u}\left(1-\pi r_y^2(n)\right)^{c_y n}\right)\ge-\xi-\epsilon, \tag{38}$$

for all $n>N_\epsilon$. Let $\theta=e^{-\epsilon}$ and taking the exponent of both sides, the results follows. ∎