An Infection Approach for Resource Allocation in Cognitive Radio Networks

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Abstract—This paper presents an evolution framework of resource allocation by infection among secondary users (SUs) in an OFDMA-based cognitive radio cellular networks. Each primary user (PU) sells his extra sub-channels to SUs in his sensing range to achieve the highest payoff and each SU may come across another in his sensing range to make infection. We first analyze the sensing range to guarantee the connectivity of all SUs and then two different infection processes among SUs are considered. For the infection with local knowledge, each SU learns from the SU he meets with in his sensing range, who has larger bandwidth than that of himself, to achieve higher payoff. While in the case without local knowledge, each SU learns from another by probability which may not always help to achieve higher payoff each time. However, we prove the existence and convergence of evolutionary equilibrium (EE) for both cases, and show some interesting properties such as the impact of cheating of SUs in the above infection processes. Besides, we proposed two algorithms for the two infection process to converge to EE in a distributed manner. Simulation results show that the algorithms with local knowledge can equally share the extra resources among SUs efficiently, which is actually the overall optimal solution (OOS). While for the algorithm without local knowledge, we find that though EE exists, OOS cannot always be achieved. Furthermore, we optimize the second algorithm to make EE approximate to OOS.

I. INTRODUCTION

Recently, the increasing application of wireless devices has stimulated dramatic demand for spectrum. Due to the limitation of spectrum, cognitive radio [1][2][3] acts as a promising technology that provides service to both licensed uses (i.e. primary users) and unlicensed users (i.e. secondary users), which greatly improves the utility of the scarce spectrum. Since secondary users (SUs) can only utilized the idle spectrum when primary users (PUs) are not active temporally to avoid collisions [4], resource allocation becomes a key issue in provision of efficient spectrum utilization as well as guaranteed QoS.

Auction is an efficient mechanism to allocate resources in market [5][6][7][8]. In [6], two pricing schemes for auction-based power allocation to achieve efficiency and fairness are proposed, respectively. In [7], a general spectrum auction problem is formulated as an optimization problem and emphasized on the choice of market clearing price. A general framework for truthful double spectrum auctions called TRUST is proposed in [8], wherein an external auctioneer with complete information operates the auction to enable the POs and SUs to trade spectrum dynamically.

However, in this mechanism, each SU has to acknowledge the information of quantities of the channels, based on which he can compare and choose the best to bid for. In fact, each SU’s acquiring of channel information before accessing results in large cost on the whole system. Due to the limitation of this kind of models, much effort on channel information sharing has been made to reduce the cost mentioned above. In [9], the algorithm of merging and splitting is proposed in the formation of coalition to maximize the overall utility of the system. In [10], the authors analyze two energy-based cooperative detection methods using weighted combining. Collaborative spectrum sensing to combat shadowing or fading effects is proposed in [11]. Evolutionary game is also introduced as the model to describe and analyze the collaboration of the secondary users. In [12], a distributed learning algorithm is raised to help the secondary users to adopt the best strategy under replicator dynamics. In [13], the behavior and evolution of the secondary users are analyzed under the deterministic and stochastic models of dynamic evolutionary game, where SUs are divided into different groups and share the information of everyone’s utility within the group. These works reduce the cost on sensing by forming a cooperative group and sharing the channel information with each group member. However, most of them assume channel information can be shared among all the SUs in the group. In fact, this assumption cannot always stand because each SU’s sensing range is limited and one cannot share his information with every SU in the group.

To overcome this shortage, we introduce the mechanism of infection, in which each SU only shares his information within his sensing range.

The main contribution of this work is to propose a new resource allocation model within evolutionary framework. An infection approach is adopted to allocated the extra spectrum of primary users in a distributed manner. Every user in the system including PUs and SUs has a geographical sensing range within which he acknowledges the existence of other users by receiving “Hello” messages of them. Each PU sells his extra spectrum to SUs within his sensing range to achieve higher payoff. While, SUs share the spectrum resources though infection, that is, each interacts locally with other SUs within his sensing range, according to which they may change their strategies.

Then we derive the proper sensing range to guarantee the connectivity of the whole system, under which we propose
two infection processes with and without local knowledge respectively, to allocate spectrum resource efficiently and fairly in a distributed manner. Then we analyze the properties of existence and convergence with respect to evolution equilibrium (EE) and overall optimal solution (OOS). We find that EE cannot always be OOS in the second infection process, so we improve it to make it approximate to OOS. At last, we study the properties of cheating problem and analyze the impact of cheating on both infection processes.

The rest of this paper is organized as follows. Section II presents our system model and formulate the problem. Section III estimates the sensing range to guarantee the system’s connectivity, derives the optimal power for SU to transmit and analyzes the system final state. In Section IV and V, two infection algorithms are proposed and their properties are analyzed, including the evolutionary equilibrium, overall optimal solution and the cheating problem. Simulation results are presented in Section VI, and conclusions are drawn in Section VII.

II. SYSTEM MODEL AND PROBLEM FORMULATION

A. System Model

We consider a single cell in a cellular system $G = (M, N)$ which can be based upon WiMAX, LTE or other OFDMA-based systems, consisting of 1 base station (BS), $M$ primary users (PUs) and $N$ secondary users (SUs). We denote the set of PUs and SUs as $P = \{1, \ldots, M\}$ and $S = \{1, \ldots, N\}$ respectively.

Each PU $i$ can get sufficient sub-channels from BS and may have extra sub-channels due to his varying traffic load and the number of extra sub-channels of PU $i$ is denoted by $F_i$ ($1 \leq i \leq M$ and $F_i \geq 0$). We assume each sub-channel has the same bandwidth, denoted by $W$ and each PU has a sensing range $r_p$. Whenever PU $i$ has $F_i$ extra available sub-channels, he first collects the number of SUs $K_i$ in his sensing range who need spectrum based on the received "Hello" messages sent by SUs and then assigns the available sub-channels to them. In this procedure, PU $i$ assigns his extra sub-channels to $F_i$ SUs to achieve highest payoff if $F_i < K_i$, while assigns only $K_i$ sub-channels to SUs otherwise with the rest sub-channels reserved for the next sub-channel allocation after an exponential distributed time $t_e$ until all his extra sub-channels have been assigned.

In this model, each SU requires only one sub-channel to transmit and has a sensing area with radius $r_s$ ($r_s$ is limited due to the limitation of power), in which he can broadcast or receive "Hello" messages to or from other SUs. It is worth noting that this "Hello" message may contain the information of the spectrum that the corresponding SU is using, based on which two SUs can exchange their utilities when meeting each other and change their strategies to achieve better utilities, and we call this process "infection". Here, we further assume each SU is "honest", that is to say, once the SU is using a sub-channel, he must broadcast the "Hello" message. It is reasonable that if SU $j_1$ is out of the sensing range of all SUs except SU $j_2$, at the same time SU $j_2$ is using a sub-channel in good condition, SU $j_1$ cannot access any sub-channel without SU $j_2$’s "Hello" message. In order to avoid the impact of this negative case happening to SU $j_2$ himself as well, any SU should honestly broadcast the "Hello" message.

We assume that each available sub-channel is shared among the SUs who select this sub-channel. It is notable that this does not mean each SU occupies a part of the bandwidth of the sub-channel. In fact, it can be achieved by separating the SUs in the time-dimension benefitting from the property of OFDMA technique, that is, the total available time slots on a sub-channel is assigned to the SUs selecting this sub-channel but the equivalent bandwidth occupied by the responding SU is $\frac{W}{N}$, where $n$ is an integer and may be different for different SUs using the same sub-channel, which will be discussed later.

According to above statement, each sub-channel forms an infection tree, to record which SUs belongs to it which is acknowledged by BS and the corresponding PU, which is shown in Figure 1. If SU $j_2$ comes across SU $j_1$ and finally accesses the sub-channels of SU $j_1$, that is, SU $j_2$ is infected by SU $j_1$, SU $j_2$ is added to the infection tree as a leaf node.

Consider the infection tree in Figure 1. It is obvious that if now he infects SU $j_6$, he will share half of his equivalent bandwidth with SU $j_6$, both using $\frac{W}{2}$. Still, if SU $j_1$ infects SU $j_6$, he does not share his own equivalent bandwidth. Instead, SU $j_6$ informs the BS and then the BS tells SU $j_3$ to share half of his equivalent bandwidth with SU $j_6$ to guarantee the fairness among all the SUs in the same tree.

B. Problem Formulation

Based on our system model, each SU can access the sub-channels either assigned by a certain PU, or infected by other SUs within his sensing range.

In the former case, each PU assigns his extra sub-channels to SUs in his sensing range to achieve highest payoff. We denote the set of SUs to whom PU $i$ assigns his extra sub-channels as $SU_i = \{SU_{i1}, SU_{i2}, \ldots, SU_{iF_i}\}$. The payoff of the PU $i$’s assignment is based on the utilities of SUs in $SU_i$. If PU can achieve payoff $p$ for unit SU’s utility, the total payoff for PU $i$ can be written as:

$$U_{P_i} = p \sum_{k=1}^{F_i} U_{SU_{ik}} - C_p F_i W \quad (1)$$
where $U_{SU,i,k}$ is the utility of $SU_{i,k}$, and $C_p$ is the cost of unit bandwidth charged by BS. We assume that $U_P > 0$, so that each PU is willing to help BS assign extra spectrum, which can avoid the collision between PUs and SUs on spectrum.

In the latter case, sub-channels are shared by infection among SUs, which can be formulated as an evolutionary game. We denote the total number of extra sub-channels that can be utilized by SUs as $N_c = \sum_{i=1}^{M} F_i$ and the sub-channels owned by PU $i$ is $c_{i,k}$ (1 $\leq k \leq F_i$).

If SU $j$ is using sub-channel $c_{i,k}$, the transmitting rate of SU $j$ is

$$R_S_j(c_{i,k}) = \frac{W}{2^n} \log_2(1 + \frac{P_j|h_{j,c_{i,k}}|^2}{\sigma^2})$$

(2)

where $P_j$ is the transmitting power, $h_{j,c_{i,k}}$ is the channel and antenna gain, $\sigma^2$ is the variance of the noise and $n$ represents the location of a certain SU in the infection tree of the sub-channel, that is, SU is on the $n$th level from the root of this tree (i.e. $n=3$ for SU $j_1$ in Figure 1).

Then the utility function of SU $j$ in sub-channel $c_{i,k}$ can be derived as:

$$U_{S_j}(c_{i,k}) = R_{S_j}(c_{i,k}) - C_r \frac{W}{2^n} - \frac{\pi q_j}{2^n} P_j$$

(3)

where $\pi > 0$ is the unit price of power in the cell, $q_j > 0$ is an SU-specific parameter denoting the relation between unit currency and unit transmitting rate, and $C_r$ represents the unit cost of the equivalent bandwidth such as the price of the sub-channel, etc., which is eventually paid to BS.

It is worth noting that $p \sum_{k=1}^{F_i} U_{SU,i,k}$ in (1) is paid to PU by BS instead of SUs as a kind of compensation for assigning extra sub-channels to SUs. On the other hand, the cost term $C_r \frac{W}{2^n}$ in (3) is paid to the BS directly, so that for one sub-channel, BS can have a payoff of $C_r \frac{W}{2^n}$ from SUs. In our model, we assume that $F_i C_r W > p \sum_{k=1}^{F_i} U_{SU,i,k}$, that is, the payoff of BS from SUs is larger than the compensation to PU $i$, so that BS can benefit from PUs' assignment of extra sub-channels instead of his.

We analysis our model theoretically in an static scenario where PUs and SUs are all static or have little mobility, which has no influence on the connectivity of the system. And the properties of a dynamic scenario will be analyzed in the simulation part.

During the infection process in the static scenario, an SU selects another SU in his sensing range after an exponential distributed time $t_x$, and decides how to make infection with each other. The infection process is decided by a third side (e.g. BS, PU or SU nearby) using the infection factors of the two SUs. Note that the infection factor, denoted by $\eta_j$, represents how much equivalent bandwidth of a sub-channel the SU to be infected will utilize if a infection is successful, which can be defined as:

$$\eta_j = \begin{cases} \frac{1}{2^n} & \text{if } 2^k < \xi_{c_{i,k}} < 2^{k+1} \\ \frac{1}{2^{n+1}} & \text{if } \xi_{c_{i,k}} = 2^k \end{cases}$$

(4)

where $\xi_{c_{i,k}}$ denotes the number of SUs using sub-channel $c_{i,k}$.

Actually, infection between SUs is not always successful, which leads to another definition of infection strength for SU $j_1$ when he comes across SU $j_2$:

$$\gamma_{j_1,j_2} = \frac{\eta_{j_1}}{\eta_{j_1} + \eta_{j_2}}$$

(5)

$\gamma_{j_1,j_2}$ is the probability that SU $j_1$ will infect SU $j_2$ successfully. According to the infection strength, the third side makes a decision and then informs both of the SUs whether the infection is successful.

It is worth noting that the infection process can happen at any time, but the access time for SUs to use the new sub-channels is discrete and can only start at the beginning of each time slot, which is shown in Figure 2.

In order to describe states of infection process, we introduce two concepts of evolutionary entropy, namely, evolutionary entropy of an infection tree and the whole system respectively.

**Definition 1**: (Evolutionary Entropy of an Infection Tree): For the sub-channel $c_{i,k}$, let vector $L_{c_{i,k}} = \{l_1, l_2, \ldots, l_{\xi_{c_{i,k}}} \}$ denotes the levels of the leaf nodes of the infection tree. We have $\sum_{a=1}^{\xi_{c_{i,k}}} \frac{1}{2^a} = 1$. Then the evolutionary entropy of the tree is defined as:

$$H(c_{i,k}) = \sum_{a=1}^{\xi_{c_{i,k}}} \frac{1}{2^a} \log_2 2^a = \sum_{a=1}^{\xi_{c_{i,k}}} \frac{l_a}{2^a}$$

(6)

**Definition 2**: (Evolutionary Entropy of the Whole System): For a system with $N_c$ sub-channels (denoted by $\{c_{1}', c_{2}', \ldots, c_{N_c}' \}$), the evolutionary entropy is defined as the summation of that of all the infection trees.

$$H = \sum_{b=1}^{N_c} H(c_{b}')$$

(7)

For an infection tree with fixed number of leaf nodes, larger evolutionary entropy means a certain sub-channel is more equally shared among the SUs belonging to this tree. Similarly, for the system with a certain number of SUs, larger evolutionary entropy represents sub-channels are more fairly allocated among SUs in this system.

Note that we do not allow every SU to access this cell so that the whole cell may not be over loaded. So in this model, we aims at fairly allocating all the sub-channels among SUs to achieve highest evolutionary entropy of the whole system, at the same time guarantee that each SU at least has the minimum equivalent bandwidth required for transmission.
III. Analysis of the Infection Game

A. Connectivity of the Whole System

Since each SU has a sensing range with radius \( r_s \), the connectivity of the system is a critical issue which is essential for the system to achieve theoretical upper bound of the whole system’s evolutionary entropy.

**Definition 3: (Connectivity)** We call two SUs are connected if one of the following two conditions holds:

- Both SUs are in the sensing range of each other;
- There is a third SU who is connected with both SUs.

Furthermore, we say a system is connected if and only if every pair of SUs in the system is connected.

We consider a square cell with area \( A \) and \( N \) SUs are uniformly distributed in this area. If \( r_s \) is large enough (e.g. \( r_s = 2\sqrt{A} \)), the system is connected. Actually, from the view of the energy saving, the smaller the sensing range of the SU is, the better. Here, we propose an evaluation of \( r_s \) in Theorem 1.

**Theorem 1:** If \( r_s \) satisfies

\[
    r_s = \sqrt{\frac{\beta A \ln N}{\pi(N - 1)}} \tag{8}
\]

where \( \beta > 1 \). As \( N \to \infty \), the probability that all SUs are connected tends to 1.

**Proof:** Neglecting the boundary effect, the geometric probability of SU \( j \) located in the cell is

\[
    p = \frac{\pi r_s^2}{A}
\]

where \( r_s \) is the sensing range.

The probability that \( k \) other SUs are in the sensing range of a randomly selected SU is

\[
    p_k = \binom{N-1}{2} p^k (1 - p)^{N-1-k}, k = 0, 1, 2, \cdots, N-1
\]

So the average number of SUs located in the sensing range of SU \( j \) is

\[
    \bar{k} = \sum_{k=0}^{N-1} k p_k = \sum_{k=0}^{N-1} k \binom{N-1}{2} p^k (1 - p)^{N-1-k}
    = \frac{(N - 1) \pi r_s^2}{A} \tag{9}
\]

According to the random graph theorem [15], when \( \bar{k} = \beta \ln N \), the probability that all SUs are connected tends to 0 if \( \beta < 1 \), and to 1 if \( \beta > 1 \), as \( N \) tends to \( \infty \).

So when \( r_s \) satisfies (8), the probability that all SUs are connected tend to 1, as \( N \) tends to infinity.

In the following sections, we will only consider the case that all SUs are connected.

B. Optimal Power for Each SU

In this subsection, we derive the optimal transmitting power for each SU. We assume that the maximum power that can be utilized is \( P_{\text{max}} \). Then for SU \( j \), we can work out the optimal power \( P^*_{j,c_{i,k}} \) easily by differentiating \( U_{S_j}(c_{i,k}) \) with respect to \( P_j \).

\[
    \frac{\partial U_{S_j}(c_{i,k})}{\partial P_j} = \frac{-W|h_{j,c_{i,k}}|^2}{2n \ln(2(\sigma^2 + P_j|h_{j,c_{i,k}}|^2))^2} - \frac{\pi q_j}{2n} \tag{10}
\]

**Lemma 1:** The optimal power for SU \( j \) in sub-channel \( c_{i,k} \) is:

\[
    P^*_{j,c_{i,k}} = \begin{cases} 
    P_{\text{max}} & \text{if } q_j > \mu_1 \\
    0 & \text{if } q_j < \mu_2 \\
    \frac{W}{\pi q_j \ln 2} - \frac{\sigma^2}{|h_{j,c_{i,k}}|^2} & \text{otherwise}
\end{cases} \tag{11}
\]

where \( \mu_1 = \frac{W|h_{j,c_{i,k}}|^2}{\sigma^2 \ln 2} \) and \( \mu_2 = \frac{W|h_{j,c_{i,k}}|^2}{(\sigma^2 + P_{\text{max}}|h_{j,c_{i,k}}|^2) \ln 2} \).

**Proof:** The term on the right hand side of (10) is strictly decreasing in \( P_j \). We can see that if \( q_j > \mu_1 \), \( \frac{\partial U_{S_j}(c_{i,k})}{\partial P_j} \) is always larger than 0 and the optimal power \( P^*_{j,c_{i,k}} = P_{\text{max}} \). If \( q_j < \mu_2 \), \( \frac{\partial U_{S_j}(c_{i,k})}{\partial P_j} \) is always smaller than 0 and the optimal power \( P^*_{j,c_{i,k}} = 0 \), meaning that the SU should better not transmit. Otherwise, the optimal power can be obtained from \( \frac{\partial^2 U_{S_j}(c_{i,k})}{\partial P_j^2} = 0 \), and \( P^*_{j,c_{i,k}} = \frac{W}{\pi q_j \ln 2} - \frac{\sigma^2}{|h_{j,c_{i,k}}|^2} \). ■

C. Scheduling in an Infection Tree

To facilitate analyzing the benefit of the scheduling, we introduce the concept of balanced tree.

**Definition 4: (Balanced Tree)** A tree is balanced if and only if for every node the heights of its two subtrees differ by at most 1.

Again consider the infection tree in Figure 1, if newcomer SU \( j_0 \) is infected by SU \( j_1 \), he will obtain half of the equivalent bandwidth of SU \( j_2, j_3 \) or \( j_4 \) instead of \( j_1 \) by base station’s scheduling, which is helpful to form a balanced infection tree. This means that in the same tree, the largest equivalent bandwidth owned by an SU cannot be more than 2 times larger than the smallest owned by another. Intuitively, scheduling is of significance to equally share the sub-channels among SUs in an infection tree, and we will prove it in the next subsection.

Furthermore, we call an infection tree is a full tree if and only if all its leaf nodes are at the same level.

D. Analysis of Final System State

The efficiency of the system state can be characterized by the concept of Pareto optimality.

**Definition 5: (Pareto Optimality)** [14]: The strategy matrix \( S^p \) is Pareto optimal if \( \not\exists S' \) such that

\[
    U_{S_j}(S') \geq U_{S_j}(S^p), \forall S_j \tag{12}
\]

with strict inequality for at least one secondary user.

This means that in a Pareto-optimal channel allocation \( S^p \), one cannot improve the payoff of any SU without decreasing the payoff of at least one other SU.
Actually, with the infection process going on, the system will finally reach a stable state in which each SU does not want to change his strategy any more, and we call this state evolutionary equilibrium (EE), defined as:

**Definition 6:** (Evolutionary Equilibrium - EE): The strategy matrix \( S^* = \{S_1^*, S_2^*, \ldots, S_j^*, \ldots, S_N^*\} \) defines an evolutionary equilibrium, if for every SU \( j \), we have:

\[
U_{S_j}(S^*, S_{-j}^*) \geq U_{S_j}(S^*, S_{-j}^*)
\]

(13)

for any strategy \( S' \), where \( S_{-j}^* \) is the strategy matrix excluding the strategy of SU \( j \).

Obviously, EE is Pareto optimal. However, EE may not always be the state with the largest evolutionary entropy of the whole system. Here, we introduce the concept of overall optimal solution (OOS), which is the state with largest entropy.

**Definition 7:** (Overall Optimal Solution - OOS) We call the system is in the state of OOS if and only if the following equation holds:

\[
l_{j_1,c_{i_1}} - l_{j_2,c_{i_2}} \leq 1, \forall j_1, j_2 \in S
\]

(14)

where \( l_{j_1,c_{i_1}} \) denotes the level of leaf node SU \( j_1 \) in sub-channel \( c_{i_1} \) and \( l_{j_2,c_{i_2}} \) denotes that of leaf node SU \( j_2 \) in \( c_{i_2} \).

**Theorem 2:** The evolutionary entropy of the whole system will be the largest, if and only if the OOS is reached.

**Proof:** We prove it by contradiction. Consider there exists two leaf nodes SU \( j_1 \) and \( j_2 \) with the difference of their levels larger than 2, we assume the entropy of the system is the largest. Whether \( c_{i_1} \) and \( c_{i_2} \) is the same sub-channel or not, we have \( l_{j_1,c_{i_1}} - l_{j_2,c_{i_2}} \geq 2 \). Note that, a sub-channel corresponds to an infection tree, the entropy of tree \( c_{i_1} \) and tree \( c_{i_2} \) can be represented as:

\[
H(c_{i_1}) = \sum_{a=1}^{c_{i_1}} \frac{l_{a,c_{i_1}}}{2^{a,c_{i_1}}}
\]

\[
H(c_{i_2}) = \sum_{d=1}^{c_{i_2}} \frac{l_{d,c_{i_2}}}{2^{d,c_{i_2}}}
\]

We further assume that leaf node SU \( j_3 \) has the same parent as SU \( j_1 \). If SU \( j_1 \) is infected by SU \( j_2 \), the levels of them are both updated to \( (l_{j_2,c_{i_2}} + 1) \), and the level of SU \( j_3 \) is changed to \((l_{j_1,c_{i_1}} - 1)\). So the entropies of \( c_{i_1} \) and \( c_{i_2} \) can be updated as:

\[
H'(c_{i_1}) = \sum_{a \in [1, c_{i_1}]/(j_1,j_3)} \frac{l_{a,c_{i_1}}}{2^{a,c_{i_1}}} + \frac{l_{j_1,c_{i_1}} - 1}{2^{j_1,c_{i_1} - 1}}
\]

\[
H'(c_{i_2}) = \sum_{d \in [1, c_{i_2}]/j_2} \frac{l_{d,c_{i_2}}}{2^{d,c_{i_2}}} + \frac{2l_{j_2,c_{i_2}} + 1}{2^{j_2,c_{i_2} + 1}}
\]

Then the increment of the whole system’s entropy is

\[
\Delta H = H'(c_{i_1}) + H'(c_{i_2}) - H(c_{i_1}) - H(c_{i_2})
\]

\[
= \frac{2(l_{j_2,c_{i_2}} + 1)}{2^{j_2,c_{i_2} + 1}} + \frac{l_{j_1,c_{i_1}} - 1}{2^{j_1,c_{i_1} - 1}} - \frac{2l_{j_1,c_{i_1}} - l_{j_2,c_{i_2}}}{2^{j_1,c_{i_1}}} - \frac{l_{j_2,c_{i_2}}}{2^{j_2,c_{i_2}}}
\]

\[
= \frac{1}{2^{j_2,c_{i_2}}} - \frac{1}{2^{j_1,c_{i_1}}} > 0
\]

(15)

that is, the entropy of the system increases after the infection, which obviously contradicts to the previous assumption.

Still, if the system is in the state of OOS, obviously the evolutionary entropy of the whole system is the largest, since any SU cannot increase the the whole system’s evolutionary entropy by changing his strategy.

Based on the above theorem, we can easily calculate the largest entropy which is shown in the following corollary.

**Corollary 1:** Given the total number of extra sub-channels \( N_{\epsilon} \) and the number of SUs \( N \), the largest evolutionary entropy of the whole system is

\[
H_{\text{max}} = N_{\epsilon}(\alpha - 1) + \frac{N}{2^\alpha}
\]

(16)

where \( \alpha = \lceil \log_2 \frac{N}{N_{\epsilon}} \rceil \).

**Proof:** The average number of leaf nodes of an infection tree is \( \frac{N}{N_{\epsilon}} \). So when the entropy of the system is the largest, all the leaf nodes will at least on the level \( \alpha = \lceil \log_2 \frac{N}{N_{\epsilon}} \rceil \) of an infection tree.

We let \( \eta \) denote the number of the leaf nodes which are on the level of \( \alpha \) and \( y \) the number of the leaf nodes on the level of \( \alpha + 1 \). Then we have the equations:

\[
\begin{cases}
  x + y = N \\
  x + \frac{\eta}{2} = N_{\epsilon}2^\alpha
\end{cases}
\]

Then we obtain \( x = N_{\epsilon}2^{\alpha + 1} - N \) and \( y = 2N - N_{\epsilon}2^{\alpha + 1} \), based on which the largest entropy can be calculated as

\[
H_{\text{max}} = \frac{\alpha}{2^\alpha} + \frac{\alpha + 1}{2^{\alpha + 1}} = N_{\epsilon}(\alpha - 1) + \frac{N}{2^\alpha}
\]

In the following two sections, we propose two infection algorithms with and without perfect local knowledge respectively, by which the system can converge to EE efficiently and reach OOS as well.

**IV. Infection with Local Knowledge**

In this section, we propose an infection algorithm to make the system efficiently converge to EE, which is shown in Table A and B.
In this algorithm, "Hello" message broadcasted by any SU should contain the information of infection factor $\eta$ of himself. If SU $j_1$ comes across SU $j_2$ with $\eta_{j_1} < \eta_{j_2}$, SU $j_1$ is quite willing to use the sub-channel of SU $j_2$ instead of that of his to increase his own utility, while SU $j_2$ is not that willing because the infection process may lower his utility. So the third side mentioned previously should make a decision on infection based on the infection factors of two sides. As a result, SU $j_1$ is successfully infected by SU $j_2$ with probability $\gamma_{j_2j_1}$ defined in (5).

A. Convergence of EE

We first consider the system with only one extra sub-channel, i.e., $N_c = 1$. If every SU operates the infection process at least once, all SUs become the leaf nodes of the infection tree. Note that this tree is already a balanced tree, which means the whole system achieves EE as well as OOS.

For the system with more than one extra sub-channel, say $N_c > 1$, EE (which is actually OOS) exists and the system will converge to EE. This statement holds because if the system has not yet reached EE, at the same time every SU has already run the infection process at least once, which we call one round, there will be at least one pair of SUs whose infection factors are different and they will run the infection process again, thus the entropy of the whole system is non-decrease.

We suppose SU $j_2$ who is using sub-channel $c_{j_2}$ with infection factor $\eta_{j_2} = \frac{1}{2^j_{j_2}}$ comes across SU $j_1$ who is using $c_{i_1}$ with $\eta_{j_1} = \frac{1}{2^{j_1}}$ and $\tau = k_{j_2} - k_{j_1} > 0$. If $c_{j_2}$ is a full tree, the expected increment of the system evolutionary entropy $\mathbb{E}[\Delta H] = (\frac{1}{2^j_{j_2}} - \frac{1}{2^{j_1}})\gamma_{j_1j_2}$. Note that $\mathbb{E}[\Delta H] = 0$ when $\tau = 1$, and $\mathbb{E}[\Delta H] > 0$ when $\tau > 1$. If $c_{j_2}$ is not a full tree, $\mathbb{E}[\Delta H] = (\frac{1}{2^j_{j_2}} - \frac{1}{2^{j_1}})\gamma_{j_1j_2} > 0$. So, before the system reaches EE, $\mathbb{E}[\Delta H] > 0$ after one round, which implies the system will converge to EE sooner or later.

B. Convergence Time of EE

To facilitate analysis, we consider the scenario that for each SU, all the other SUs are in his sensing range. In this scenario, we show the theoretical upper bound of the expected convergence time.

**Theorem 3:** For each SU, if all the other SUs are in his sensing range, the theoretical upper bound of expected convergence time can be expressed as:

$$E[t_c] < ([3 \times 2^{\log_2 N}] \mathbb{H}_\text{max} + 1)(1 + \ln N) \quad (17)$$

**Proof:** The waiting time $t$ between two infection processes for a certain SU is exponential distributed, i.e., $p_t = \lambda \exp(-\lambda t)$. Then the cumulative distribution function (CDF) of $t' = \max_{j \in N}(t_{j_1}, t_{j_2}, \ldots, t_{j_N})$ is

$$Pr(t' < t) = Pr(t_{j_1} < t)Pr(t_{j_2} < t) \cdots Pr(t_{j_N} < t) = (1 - \exp(-\lambda t))^N$$

So the probability density function (PDF) of $t'$ is

$$P(t') = N\lambda \exp(-\lambda t)(1 - \exp(-\lambda t))^{N-1}$$

The expectation of $t'$ is

$$E[t'] = \int_0^\infty N\lambda t \exp(-\lambda t)(1 - \exp(-\lambda t))^{N-1} dt$$

$$= \sum_{k=1}^N \frac{(-1)^k+1}{k} \left( \frac{1}{k} \right)^N = \sum_{k=1}^N \frac{1}{k} < 1 + \ln N$$

According to the lack of memory property of exponential distribution [16], the time that a round lasts is an independent variable. If the system has not reach OOS, the expected increment of the system evolutionary entropy in one round satisfies: $\mathbb{E}[\Delta H]_{\text{round}} \geq \mathbb{E}[\Delta H] > \frac{1}{2^{j_{max}} + 1}$. To achieve the highest entropy, the expectation of the time the system should go through is less than $\left(\frac{\mathbb{H}_\text{max}}{\mathbb{H}_\text{max}} + 1\right)$ rounds. So the convergence time satisfies:

$$E[t_c] < ([3 \times 2^{\log_2 N}] \mathbb{H}_\text{max} + 1)(1 + \ln N)$$

C. The Cheating Problem

We suppose that each SU only knows the infection factor $\eta$ of the sub-channels he is using and the information contained in the “Hello” messages transmitted by other SUs in his sensing range. If every SU’s location and sensing range are fixed, our proposed algorithm can fast converge to EE. However, each SU is rational and has the motivation to achieve a higher expected utility by announcing a smaller infection factor in his “Hello” message. In this subsection, we analyze the impact of this cheating problem.

**Proposition 1:** Each SU can get a higher expected utility if he announces a smaller infection factor than that of the true value.

**Proof:** We assume that SU $j_1$ comes across SU $j_2$ and the infection factor contained in the “Hello” message of SU $j_2$ is true. As $W$ is a constant, we assume it equals to “1”. Since the utility function of SU is in linear relation with the equivalent portion of the sub-channel, in this proof we only consider the equivalent bandwidth $B$ of the sub-channel and calculate the expected value $\mathbb{E}[B]$.

**Case 1:** SU $j_1$ is on the level $(k_{j_1} - 1)$.

- SU $j_1$ announces true information in his “Hello” message. The infection factor of SU $j_1$ is $\eta_{j_1} = \frac{1}{2^{j_{j_1}}}$ and the of SU $j_2$ is $\eta_{j_2} = \frac{1}{2^{j_{j_2}}}$, and $\mathbb{E}[B]$ can be calculated as

$$\mathbb{E}[B] = \left\{ \begin{array}{ll}
\frac{1}{2^{j_{j_1}}} & \text{if } k_{j_1} \leq k_{j_2} \\
\frac{1}{2^{j_{j_2}}} \gamma_{j_2j_1} + \frac{1}{2^{j_{j_1}}}(1 - \gamma_{j_2j_1}) & \text{if } k_{j_1} > k_{j_2}
\end{array} \right.$$  

- SU $j_1$ announces untruth infection factor $\frac{1}{2^{j_{j_1}}}$, then the expectation of $B$ is

$$\mathbb{E}'[B] = \left\{ \begin{array}{ll}
\frac{1}{2^{j_{j_1}}} & \text{if } k_{j_1} \leq k_{j_2} \\
\frac{1}{2^{j_{j_2}}} \gamma_{j_2j_1} + \frac{1}{2^{j_{j_1}}}(1 - \gamma_{j_2j_1}) & \text{if } k_{j_1} > k_{j_2}
\end{array} \right.$$
where \( \eta_j' = \frac{1}{2^{k_j_1-\Delta}} \), \( \eta_j'' = \frac{1}{2^{k_j_2}} \), \( \gamma_j' = \frac{\eta_j'}{\eta_j''} = \frac{\eta_j'}{\eta_j''} \) and 
\( \gamma_{j_2j_1} = \frac{\eta_j'}{\eta_j''} \).

Then the increment of the expected equivalent bandwidth is 
\[ E'_B - E_B = \begin{cases} 0 & \text{if } k_{j_1} \leq k_{j_2} \\ \frac{2^{-k_{j_1}}(2^{k_{j_2}+1})(2^\Delta - 1)}{(2^k_{j_1} + 2^{k_{j_2}})(2^k_{j_1} + 2^{k_{j_2}})} & \text{if } k_{j_1} > k_{j_2} \end{cases} \]

So \( E'_B - E_B \geq 0 \) if \( \Delta > 0 \).

Case 2: \( SU_j_1 \) is on the level \( k_{j_1} \).

The proof is similar to Case 1. We can figure out that 
\[ E'_B - E_B = \begin{cases} 0 & \text{if } k_{j_1} \leq k_{j_2} \\ \frac{2^{-k_{j_1}}(2^{k_{j_2}})(2^\Delta - 1)}{(2^k_{j_1} + 2^{k_{j_2}})(2^k_{j_1} + 2^{k_{j_2}})} & \text{if } k_{j_1} > k_{j_2} \end{cases} \]

So \( E'_B - E_B > 0 \) if \( k_{j_1} > k_{j_2} \) and \( \Delta > 0 \).

To summarize the two cases we can conclude that \( SU_j_1 \) has the motivation to provide a smaller infection factor.

Also, if \( SU_j_1 \) provides a smaller infection factor, other SUs will come across him with smaller probability since according to the infection process 1, SUs will come across the SU with largest partition of sub-channel. Even if there is an SU coming across him, as his infection strength is smaller than that of the true infection case, the infection has a smaller probability to be successful. So, announcing the untrue information can help lower the deterioration probability of the utility. The proposition holds.

V. INFECTION WITHOUT LOCAL KNOWLEDGE

To overcome the cheating problem in the above section, we propose another infection algorithm, shown in Table C and D, by which each SU has no motivation to provide untrue infection factor. In this algorithm, "Hello" message of a SU only informs the other SUs of its existence. If \( SU_j_1 \) selects \( SU_{j_2} \), \( SU_j_2 \) and \( SU_j_1 \) will infect \( SU_j_2 \) with probability \( \gamma_{j_1j_2} \) at the same time \( SU_j_2 \) will infect \( SU_j_1 \) with probability \( \gamma_{j_2j_1} \).

As a result, there will be four cases after infection shown in Figure 3, that is, (a) both SUs remain the same, (b) \( SU_j_1 \) is infected by \( j_2 \), (c) \( SU_j_2 \) is infected by \( j_1 \), and (d) both SUs are infected by each other.

![Fig. 3. The results of infection.](image)

Suppose \( SU_j_1 \) is using sub-channel \( c_i_1 \) with infection factor \( \eta_j_1 = \frac{1}{2^{k_j_1-\Delta}} \), while \( SU_j_2 \) is using \( c_i_2 \) with \( \eta_j_2 = \frac{1}{2^{k_j_2}} \). If \( \tau = k_j_2 - k_j_1 > 0 \), the relationship between the two trees with respect to \( c_i_1 \) and \( c_i_2 \) can be categorized into following four types.

C. Infection Algorithm 2 for SUs

1: Initialize:
2: collect the "Hello" messages in his sensing range
3: randomly select one sub-channel and access
4: set an exponential distributed time \( t_e \)
5: Loop:
6: if a PU assigns a sub-channel to him
7: change his sub-channel to the new one
8: end if
9: if a SU comes across him
10: infection process 2 or 3
11: end if
12: if time \( t_e \) is up
13: randomly select an SU within his sensing range to meet
14: infection process 2 or 3
15: reset \( t_e \) to a new value
16: end if
17: repeat Step 6 to Step 16

D. Infection Process 2

1: if \( SU_j_1 \) comes across \( SU_j_2 \)
2: exchange the information of infection factors \( \eta_j_1 \) and \( \eta_j_2 \)
3: if \( \eta_j_1 = \eta_j_2 \) or \([log_2 \eta_j_1 - log_2 \eta_j_2] = 1 \) and the tree with smaller \( \eta \) is a full tree
4: SU \( j_1 \) and \( j_2 \) do not infect each other
5: else
6: \( SU_j_1 \) can infect \( SU_j_2 \) with probability \( \gamma_{j_1j_2} \)
7: \( SU_j_2 \) can infect \( SU_j_1 \) with probability \( \gamma_{j_2j_1} \)
8: end if
9: end if

Type 1: \( c_i_1 \) is a full tree while \( c_i_2 \) is not.
\[ E[\Delta H] = \frac{1}{(2^\tau + 1)^2} \left( \frac{2^{2\tau} - 2}{2^{k_i_2-1} - 2^{k_i_2}} \right) > 0 \]

Type 2: \( c_i_2 \) is a full tree while \( c_i_1 \) is not.
\[ E[\Delta H] = \frac{1}{(2^\tau + 1)^2} \left( \frac{2^{2\tau} - 1}{2^{k_i_1-1} - 2^{k_i_2}} \right) \]

Note that \( E[\Delta H] < 0 \) when \( \tau = 1 \), and \( E[\Delta H] > 0 \) when \( \tau > 1 \).

Type 3: Both \( c_i_2 \) and \( c_i_1 \) are full trees.
\[ E[\Delta H] = \frac{1}{(2^\tau + 1)^2} \left( \frac{2^{2\tau} - 2}{2^{k_i_2-1} - 2^{k_i_2}} \right) \]

Note that \( E[\Delta H] < 0 \) when \( \tau = 1 \), and \( E[\Delta H] > 0 \) when \( \tau > 1 \).

Type 4: Neither \( c_i_1 \) nor \( c_i_2 \) is a full tree.
\[ E[\Delta H] = \frac{(2^\tau - 1)^2}{2^{k_i_2-1}(2^\tau + 1)} > 0 \]

Note that we assume infection dose not occur on condition that \( |k_{j_1} - k_{j_2}| = 1 \) at the same time the tree with smaller \( \eta \) being a full tree, which can guarantee the expectation of entropy increment of any type above is larger than zero.

A. Convergence of EE

Since the expected entropy increment is always larger than zero, EE exists in the long run, but EE is not always OOS. We will show it in the simulation section.

To make the system approximate to OOS, we propose another infection process shown in Table E. Note that the system state is not always stable at OOS, so we show when the system can be stable at OOS by the following theorems.
Theorem 4: If \( N \) and \( N_c \) (\( N_c \geq 2 \)) satisfy
\[
N_c2^{\left\lfloor \log \frac{N_c}{4}\right\rfloor + 1} - N_c + 1 \leq N \leq N_c2^{\left\lfloor \log \frac{N_c}{4}\right\rfloor + 1} - 1
\]
the system is not stable at OOS all the time with a little deviation now and then.

Proof: According to Theorem 2 and drawer principle, when OOS is achieved, there are at least \((N-N_c2^{\left\lfloor \log \frac{N_c}{4}\right\rfloor + 1} + N_c)\) infection trees with \( N_{\text{leaf}} = 2^{\left\lfloor \log \frac{N_c}{2}\right\rfloor + 1} \) leaf nodes and rest trees with \( N_{\text{leaf}} \) leaf nodes satisfying \( 2^{\left\lfloor \log \frac{N_c}{2}\right\rfloor} < N_{\text{leaf}} < 2^{\left\lfloor \log \frac{N_c}{4}\right\rfloor + 1} \).

Note that infection factors of SUs in different kinds of trees are \( \left\lfloor \log \frac{N_c}{4}\right\rfloor + 1 \) and \( \left\lfloor \log \frac{N_c}{2}\right\rfloor \) respectively at OOS, but Infection Process 3 in Table E is still going on. However, the entropy may decrease with a probability \( \frac{1}{2} \), with the expectation of the whole system’s entropy increment is a negative value \( \frac{3}{2}2^{\left\lfloor \log \frac{N_c}{4}\right\rfloor} \). So deviation may happen around OOS, which is shown in simulation section.

Theorem 5: If \( N \) and \( N_c \) (\( N_c \geq 2 \)) satisfy
\[
N_c2^{\left\lfloor \log \frac{N_c}{4}\right\rfloor} \leq N \leq N_c2^{\left\lfloor \log \frac{N_c}{2}\right\rfloor + 1} - N_c
\]
all SUs will be stable at OOS after a long enough time.

Under this condition, the system can reach OOS in which all SUs have the same infection factor, so that all SUs do not infect one another. Note that OOS in this situation is EE, which is shown in the simulation section as well.

B. The Cheating Problem

Proposition 2: Due to lack of local knowledge (infection factor), each SU cannot get a higher expected utility if he announces an untrue infection factor.

Proof: We assume that SU 1\(_j\) comes across SU 2\(_j\) and the information contained in the “Hello” message of SU 2\(_j\) is true. We let \( \xi_{1j} \) and \( \xi_{2j} \) denote the number of leaf nodes in the “Hello” message of SU 1\(_j\) and SU 2\(_j\). We assume the infection factors of both nodes are \( \eta_{1j} = \frac{1}{2^{\xi_{1j}}} \) and \( \eta_{2j} = \frac{1}{2^{\xi_{2j}}} \). Like the proof in the last section, we only consider the equivalent bandwidth of the sub-channel B and calculate the expected value \( E_B \).

After the infection, the strategies of both SUs can have the following 4 cases:

- SU 1\(_j\) does not learn from SU 2\(_j\) but SU 2\(_j\) learns from SU 1\(_j\): probability \( p = \gamma_{j1j2}\).
- SU 1\(_j\) learns from SU 2\(_j\) but SU 2\(_j\) does not learn from SU 1\(_j\): probability \( p = \gamma_{j2j1}\).
- SU 1\(_j\) and SU 2\(_j\) learn from each other: probability \( p = \gamma_{j1j2} \gamma_{j2j1}\).
- SU 1\(_j\) and SU 2\(_j\) does not learn from each other: probability \( p = \gamma_{j1j2} \gamma_{j2j1}\).

Case 1: SU 1\(_j\) is on the level \( k_{j1j} \), while SU 2\(_j\) is on the level \( k_{2j2} \).

- SU 1\(_j\) announces true information in his “Hello” message. To him, the expected equivalent bandwidth of the sub-channel he will use after the infection is
\[
E_B = \frac{1}{2^{k_{1j}}} \gamma_{j1j2} + \frac{1}{2^{k_{2j}}} \gamma_{j2j1}
\]
- SU 2\(_j\) announces untrue information in his “Hello” message. We assume the infection factor he announced is \( \frac{1}{2^{k_{j2}}} \) and the infection factors of both SUs are \( \eta_{j1} = \frac{1}{2^{k_{j1}}} \) and \( \eta_{j2} = \frac{1}{2^{k_{j2}}} \) and the infection strength \( \gamma_{j1j2} = \frac{\eta_{j1}}{\eta_{j2} + \eta_{j2}} \) and \( \gamma_{j2j1} = \frac{\eta_{j2}}{\eta_{j1} + \eta_{j2}} \).

So the increment of the expected equivalent bandwidth of the sub-channel is
\[
E'_B - E_B = \left( \frac{2^{k_{j2}} - 2^{k_{j1}}}{} \right) \left( 1 - \Delta \right) \left( 2^{k_{j1}} + 2^{k_{j2}} \right)
\]

We let \( E'_B - E_B > 0 \) and can obtain that \( k_{j2} < k_{j1}, \Delta < 0 \) or \( k_{j1} > k_{j2}, \Delta > 0 \). And \( E'_B - E_B < 0 \) when \( k_{j1} < k_{j2}, \Delta > 0 \) or \( k_{j1} > k_{j2}, \Delta < 0 \). Since SU 1\(_j\) does not know the value of \( k_{j2} \), he has little motivation to provide untrue information.

Case 2: SU 1\(_j\) is on the level \( k_{j1j} \), while SU 2\(_j\) is on the level \( k_{2j2} \).

- SU 1\(_j\) does not learn from SU 2\(_j\) but SU 2\(_j\) learns from SU 1\(_j\): probability \( p = \gamma_{j2j1} \gamma_{j1j2} \).
- SU 1\(_j\) learns from SU 2\(_j\) but SU 2\(_j\) does not learn from SU 1\(_j\): probability \( p = \gamma_{j1j2} \gamma_{j2j1} \).
- SU 1\(_j\) and SU 2\(_j\) learn from each other: probability \( p = \gamma_{j1j2} \gamma_{j2j1} \).
- SU 1\(_j\) and SU 2\(_j\) does not learn from each other: probability \( p = \gamma_{j1j2} \gamma_{j2j1} \).

Case 3: SU 1\(_j\) is on the level \( k_{j1j} - 1 \), while SU 1\(_j\) is on the level \( k_{2j2} \).

- SU 1\(_j\) does not learn from SU 2\(_j\) but SU 2\(_j\) learns from SU 1\(_j\): probability \( p = \gamma_{j2j1} \gamma_{j1j2} \).
- SU 1\(_j\) learns from SU 2\(_j\) but SU 2\(_j\) does not learn from SU 1\(_j\): probability \( p = \gamma_{j1j2} \gamma_{j2j1} \).
- SU 1\(_j\) and SU 2\(_j\) learn from each other: probability \( p = \gamma_{j1j2} \gamma_{j2j1} \).
- SU 1\(_j\) and SU 2\(_j\) does not learn from each other: probability \( p = \gamma_{j1j2} \gamma_{j2j1} \).
\[ E_B' - E_B = (2^{k_1} + k_{j2} + \Delta + 2^{2k_{j2}} + \Delta - 2^{2k_1} + \Delta - 2^{k_{j1}} + k_{j2}) (1 - 2^\Delta) \]

\[ (2^{k_1} + \Delta + 2^{k_{j2}})^2 (2^{k_{j1}} + 2^{k_{j2}})^2 \]

We let \( E_B' - E_B = 0 \) and can obtain that \( \Delta_1 = 0 \) or \( \Delta_2 = \frac{2^{k_1} + k_{j2} + \Delta + 2^{2k_{j2}} + \Delta - 2^{2k_1} + \Delta - 2^{k_{j1}} + k_{j2}}{2^{k_1} (1 - 2^\Delta)} \).

If \( 2^{k_1} + k_{j2} + \Delta + 2^{2k_{j2}} > 2^{2k_{j1}} > 0 \), \( E_B' - E_B < 0 \) when \( \Delta > \Delta_2 \).

If \( 2^{k_1} + k_{j2} + \Delta + 2^{2k_{j2}} < 2^{2k_{j1}} < 0 \), \( E_B' - E_B < 0 \) when \( \Delta < \Delta_2 \).

Since SU \( j_1 \) does not know the value of \( k_{j2} \), still he has little motivation to provide untrue information.

Case 4: SU \( j_1 \) is on the level \( k_{j1} = 1 \), while SU \( j_1 \) is on the level \( k_{j2} = 1 \).

\[ E_B = \frac{1}{2^{k_{j1}}} \gamma_{j1,j2}^2 + \frac{1}{2^{k_{j1}}} \gamma_{j1,j2}^2 + \frac{1}{2^{k_{j2}}} \gamma_{j2,j1}^2 + \frac{1}{2^{k_{j2}}} \gamma_{j2,j1}^2 \]

\[ E_B' = \frac{1}{2^{k_{j1}}} \gamma_{j1,j2}^2 + \frac{1}{2^{k_{j1}}} \gamma_{j1,j2}^2 + \frac{1}{2^{k_{j2}}} \gamma_{j2,j1}^2 + \frac{1}{2^{k_{j2}}} \gamma_{j2,j1}^2 \]

\[ E_B' - E_B = -\frac{2^{k_{j1}} + k_{j2} (1 - 2^\Delta)^2}{(2^{k_1} + \Delta + 2^{k_{j2}})^2 (2^{k_{j1}} + 2^{k_{j2}})^2} < 0 \]

Since \( E_B' - E_B < 0 \) is always true, the SU \( j_1 \) will not provide untrue information.

VI. SIMULATION RESULTS

To evaluate the performance of our proposed algorithms, we perform simulations in a cell with multiple PUs and multiple SUs distributed in a 1000m \( \times \) 1000m square.

A. Estimation of the Sensing Range

According to Theorem 1, when \( N = 20 \), the sensing range for every SU should be in the order of \( \sqrt[20]{224.02m} \). We run the resource allocation process for 1000 times for each sensing range, count the times that all SUs are connected and calculate the probability which is showed in Figure 4. Figure 4 shows that as the sensing range increases at the region [200, 550], the probability that all SUs are connected increases.

In Figure 5, when the number of SUs \( N \) varies from 20 to 200, three full curves show the minimum value of SUs’ sensing range \( r \) if the connectivity probability of the system is larger than 95%, 90% and 85% respectively. While three dotted curves represent the theoretical value shown in equation (8) when \( \beta = 6.25, 4, 1 \) respectively. From this figure, we find that to guarantee the connectivity of the system, the sensing range of SUs can decrease as \( N \) increases. Note that the three full curves are all below the theoretical curve with \( \beta = 6.25 \).

The three full curves are nearly all below the theoretical curve with \( \beta = 4 \), at the same time above the the theoretical curve with \( \beta = 1 \). So, when \( N \) is larger than 20, the sensing range obtained from equation (8) with \( \beta = 4 \) is large enough. Later we use this sensing range to further study other properties of our model on condition that all SUs are connected.

B. Convergence of EE and OOS

In Figure 6 and Figure 7, there are 3 PUs in the cell with \{1, 2, 2\} extra sub-channels respectively. The uppermost straight line indicates the system evolutionary entropy in the state of OOS.

Figure 6 compares the convergence properties of the three infection processes with \( N = 120 \) SUs. From this figure, we can conclude that Infection Process 1 converges fastest, followed by Infection Process 3, and Infection Process 2 slowest comparatively. Also, we can see that the curve of Infection Process 1 is non-decreasing. While, the other two processes can decrease, but can still converge to OOS and EE.

Consider the system with \( N = 158 \) SUs showed in Figure 7, we can tell that in Infection Process 2, the evolutionary entropy of the system can not reach to the largest value but be stable at a smaller constant which is the state of EE. While the
The evolutionary entropy of the system in Infection Process 3 can approximate to the largest value but fluctuate below it instead of being stable, which validates Theorem 4, since $N$ satisfies Equation (19).

Figure 8 shows the distribution of convergence time when $N = 120$. We suppose the the PDF of $t_e$ is $p_t = \lambda \exp(-\lambda t_{\text{DT}})$ and run the program for 500 times and record the convergence time. From the figure, we see that the convergence time concentrates around 200 to 800 time slots.

In Figure 9, we consider 6 PUs with $\{1, 2, 3, 2, 1, 1\}$ extra sub-channels respectively. When the total number of extra sub-channels is unchanged, i.e. $N_c = 10$, the average convergence time for Infection Process 3 increases and decreases regularly as $N$ increases, which validates Theorem 4 and Theorem 5. It is worth noting that the points with average convergence time longer than 9000 time slots means that the system can not be stable at OOS. According to Theorem 4, when $150 < N < 160$ or $310 < N < 320$, the system will not be stable at OOS, which coincides with this figure. Though unstable points exist, the overall convergence trend increases with $N$.

C. The Fairness Issue (expect for a more proper results)

D. The Dynamic Property

Figure 12 shows the dynamic property of our model when the number of SUs who need spectrum varies with time going
on. When OOS is already achieved, we find that system can converge to a new OOS very fast if the number of SUs increases or decreases by not a large margin. So it is quite suitable to the realistic situations where every SU with mobility can come into or leave a cell randomly.

Figure 13 shows the dynamic property when the demands of PUs for the sub-channels vary as time goes on. Since a PU has a higher priority than any SU to access sub-channels, when he need more spectrum, the SUs who are using the corresponding sub-channels should exit at once and access other sub-channels by infection process. At the same time, if a PU has more extra sub-channels, he can assign them to the SUs nearby to achieve extra payoff. Figure 13 shows that though the demands of PUs can vary, the system can adjust to it and converge to the updated largest entropy fast which meets the requirements of the realistic system.

VII. CONCLUSION

In this paper, we put forward an evolution framework of resource allocation by means of infection and two infection processes with and without infection factor are proposed. We first derive SU’s sensing range to guarantee connectivity of the whole system. Then we analyze the existence and convergence of EE and OOS by the concept of entropy, and discuss the issue of potential cheating. Moreover, two algorithms are proposed to approximate to OOS. Numerical results show that our infection algorithm with infection factor can converge to EE and OOS efficiently and algorithm without infection can converge to EE which is not always OOS. Furthermore, we improve the latter algorithm to approximate to OOS.

REFERENCES