Toward a Quality-Aware Online Pricing Mechanism for Crowdsensed Wireless Fingerprints

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***Abstract*—Fingerprinting localization systems are outstanding for its convenient deployment, where a major challenge is the high cost for collecting a huge number of received signal strength fingerprints. Mobile crowdsensing (MCS) paradigm is cost-effective for large-scale data collection; however, a quality-aware data pricing mechanism dedicated to MCSed fingerprints accommodating practical application situations including budget constraints and online data submission is still unavailable. In this paper, we present a data pricing scheme dedicated to MCSed fingerprints by enhancing the online learning technique. We first reveal the principle of fingerprints quality assessment for accurate localization. Based on the principle, we design corresponding loss and regret function, which is able to reflect values of the fingerprints with respect to localization accuracy. We then present an online pricing scheme for MCSed data, which results in that the worker's payoff is a random variable following an optimal probability density function leading to the minimum expected regret. Furthermore, we extend our scheme to application scenarios with different budget settings, where the pricing strategies for the scenarios of regret minimization with fixed budget and budget minimization for certain fingerprints quality level are investigated. Experimental results are presented to verify our theoretical analysis.**

***Index Terms*—Data acquisition, reliability estimation.**

I. Introduction

THE past decade has witnessed flourishing advances of indoor localization systems based on wireless techniques [1], where the received signal strength (RSS) fingerprinting based methodology has been widely adopted due to the convenient deployability [1], [3], [4], [12], [13], [21]. The fingerprinting based indoor localization system has two phases: In the offline phase, the site surveyor observes the RSS of Wi-Fi access points (APs) termed as RSS fingerprints at pre-determined reference points, and submits the fingerprints and the location information of these reference points to the localization server's database; in the online phase, a user in need of localization service could submit the observed fingerprints to the server, which then returns the location with the stored fingerprints match the submitted fingerprints best as the estimated location of the user.

The site survey in the offline phase requires substantial efforts, which is hardly accomplished by any single entity. The recent advances of fingerprinting localization systems utilize mobile crowdsensing (MCS) paradigm to collect fingerprints [4], [7], [10]–[14], [29]. With mobile crowdsensing, individuals with commodity mobile devices collectively contribute sensed fingerprints, where the location of the MCS workers during the offline phase is estimated by the inertial measurement unit (IMU) of the mobile device [13], [14], [29]. As the location of the worker estimated by IMU data is inaccurate, the MCSed fingerprints are usually not from the location as the worker claims, which leads to localization errors in the online phase of localization.

Efforts have been made to evaluate crowdsensed data quality [18], allocate crowdsensing tasks to appropriate workers [19], [20] and design incentive mechanisms considering the estimated data quality [17], [21], [23]–[25]; however, the proposed methods for general application scenarios are unable to distinguish high-quality fingerprints for accurate indoor localization. Moreover, how fingerprints should be priced according to the estimated quality in an online manner is still unknown to the best of our knowledge.

In this paper, we present a quality-aware online pricing scheme for MCSed fingerprints, which not only leverages the essence of fingerprinting localization to evaluate quality of fingerprints, but also enhances the online learning technique to accommodate different budget settings in practice. Our contributions are as following.

First, we propose an effective quality-assessment mechanism dedicated to quality evaluation of MCSed RSS fingerprints for accurate indoor localization. With fingerprinting based localization approach, the distribution of the RSS at each location $\vec{r}$ is used to construct an one-to-one mapping from the RSS sample space to the physical space. A point on the mean surface of the RSS sample space $\vec{\mu}(\vec{r})$ corresponds to a location in the physical space $\vec{r}$. The data noise occurs in the process of MCSing fingerprints: if the worker associates the observed fingerprints with inaccurate location information, then the $\vec{\mu}(\vec{r})$ deviates from the true value and becomes $\vec{\mu}(\vec{r'})$. Our strategy for RSS data quality assessment is to measure the deviation of

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the MCSed data from $\vec{\mu}(\vec{r})$ if the worker claims the data are observed at $\vec{r}$. In particular, we design a loss function based on the essence of the indoor localization, which reflects the difference between the MCSed RSS and a predicted true value of $\vec{\mu}(\vec{r})$. Although the real value of $\vec{\mu}(\vec{r})$ is unknown, we can still theoretically prove that the function achieves the minimum when the predicted value equals $\vec{\mu}(\vec{r})$.

Second, we design an online pricing scheme for MCSed fingerprints by enhancing the online learning technique. We formulate the pricing problem into an optimization problem, where the objective is to find the optimal probability distribution function (PDF) for pricing. In contrast to most of work about incentive mechanism design resulting in a constant price for the MCSed data, the price of the data under our scheme is a random variable following the optimal PDF. We theoretically prove that the data obtained in this way present least deviation from the true value $\vec{\mu}(\vec{r})$ with respect to each Wi-Fi access point (AP), which presents the most accurate localization performance.

Third, we extend our data pricing strategy so that it can be applied to scenarios with different budget settings. The first scenario assumes that the buyer wants to purchase the high-quality data as many as possible with constrained budget; the second scenario assumes to purchase the amount of data that can achieve a certain quality level with as least cost as possible. A straightforward problem formulation for the second scenario requires optimizing a cumulative distribution function (CDF), which is extremely difficult. Our strategy to resolve the problem is to first simplify the problem into a simple version and substitute the result into the original version, then we optimize the substituted parameters to find the final solution.

The remainder of the paper are organized as follows: Section II presents related work. Section III describes the system model based on which our work is developed. Section IV shows how to evaluate quality of wireless fingerprints. Section V presents the quality-aware online pricing mechanism based on the quality of wireless fingerprints. Section VI presents the enhanced data pricing mechanism with budget constraints. Section VII shows the experimental results, and Section VIII presents conclusion remarks.

II. Related Work

Indoor Localization with Mobile Crowdsensing (MCS): MCS paradigm has been applied to fingerprinting based indoor localization systems in recent years. Wu *et al.* propose LiFS system, which leverages crowdsourcing to avoid the conventional site survey process [27], [28]. Shen *et al.* present a crowdsourcing based system *Walkie-Markie* [29] to generate indoor pathway maps from the user's contributed data. Luo *et al.* propose a self-calibrating participatory indoor localization system [13], which requires no prior knowledge about the building and user intervention including the floor planning. A fundamental study on fingerprinting localization with crowdsourcing approach is presented in [4], which provides a probabilistic model to examine the reliability of the estimated locations. The study sheds some light on the essence of the localization approach, which provides some inspiration to our work in this paper.

The work mentioned above presents novel ideas about how to utilize MCSed data in localization systems for specific purposes; however, details about how the MCSed data can be obtained are not mentioned. Moreover, designing appropriate mechanisms to obtain the MCSed is non-trivial, where many practical issues need to be considered such as the limited budget for data purchase and the method to pick out high-quality data from the entire data set.

Quality-Aware Incentive Mechanism Design for MCS: Many incentive mechanisms are proposed to motivate individuals to participate in the MCS activities, where an important issue is how to evaluate the MCSed data that are usually with considerable noise. Zhang *et al.* propose an incentive mechanism for labeling systems with the workers' expertise and the system's budget constraints considered [30]. Jin *et al.* introduce the quality of information (QoI) as the metric to evaluate the quality of sensory data [23]. Tham *et al.* take timeliness of data into consideration of quality [31], where it is assumed that the quality of contributed data will go downhill with time. Kawajiri *et al.* provide a framework aiming to level up the quality rather than the size of data directly considering the monetary consumption [18].

The data quality evaluation methods used in the work mentioned above are for general purpose, where the essence of indoor localization is not incorporated into the mechanism design. Wen *et al.* propose a quality-driven auction scheme for MCS system and tailor the mechanism for indoor localization application [21]; however, the tailored scheme relies on experiments on human's sense of locations, which introduces unpredictable factors. Zhao *et al.* design an online incentive mechanism to deal with the practical issue that the worker submits their data one-by-one in a random order, instead of in batch as assumed in other work [32]. While efforts have been made to deal with issues of MCS systems in an ad hoc manner, a systematical study of the MCS considering both the data quality evaluation and the practical application scenario is still unavailable.

Online Learning Mechanisms: The pricing mechanism proposed in this paper utilizes the idea of online learning [34], which is a natural approach for the practical online data submission scenario. Zinkevich proposes an effective algorithm Generalized Infinitesimal Gradient Ascent (GIGA), which presents a general form of online optimization algorithm [35]. The framework is then extended for a general data acquiring problem [26]. In particular, it models a general situation that a learner with a limited budget purchases data from agents coming in an online manner, and the learner needs to propose a hypothesis and a price in each round to obtain the data. The main contribution in [26] is its introduction of budget constraint into the online learning framework; however, the good in the purchase issue is not specified and the quality of the good is assumed to be known. This generalization of the problem brings much convenience in designing key components of the approach such as loss and regret function, which actually is the major challenge for application of online learning technique to practical application scenario. Our work in this paper proposes a design of online learning scheme for MCSed data pricing for indoor localization, which sheds light on practical use of online learning techniques.

III. System Model

This section presents the system model in our work. We summarize main notations in the paper in Table I for the convenience of readers.

MCS System Model: Consider a MCS system for fingerprints collection with three kinds of entities, MCS workers, purchase platform and buyers. The buyer is purchasing fingerprints for a classic fingerprinting based localization system, where the knowledge of the indoor space is perfectly known, including the map, the locations of reference points, and the Service Set

TABLE I
NOTATION TABLE

Section	Symbol	Definition
IV	$\vec{r}$	Location in the physical space
	$\vec{\mu}(\vec{r})$	Mean value of RSS fingerprints at $\vec{r}$
	S	Two dimensional Cartesian space
	$\mathcal{Y}$	Normalized Gaussian additive noise
	σ	Amplitude of noise
	$\mathcal{P}(\vec{r})$	Observed value of RSS fingerprints
	$\vec{\delta}$	Maximum estimation error of IMU
	$P_{\text{low}}, P_{\text{high}}$	Boundary of E in 1-D sample space
	$F_{\text{error}_{\vec{r}}}$	Probability of localization error
	N	Size of data sequence
V	P_t	Value of submitted RSS fingerprint at t
	$\mathcal{Z}$	Sampled data space
	h_t	Hypothesis of RSS value at t
	$\mathcal{H}$	Hypothesis space
	$\ell(h_t, P_t), \ell(h_t)$	Loss function
	η	Learning rate
	c_t	Worker's cost
	π_t	Buyer's price
	M	Upper bound of data price
	B	Total budget
	T	Size of online data sequence
	R	Regret function
	$\mathcal{L}$	Risk function
VI	β, λ, μ	Parameters in optimization
	$G_t(c)$	Cumulative distribution of cost
	q_t	Probability of purchasing data at t
	L	Lagrangian function

Identifiers (SSIDs) of Wi-Fi APs available. The buyer issues the request of RSS fingerprints at the target indoor space to the platform, which then assigns the task to registered MCS workers that are roughly around the area. The chosen workers continuously submitting the current location and corresponding RSS fingerprints observed to the platform in an opportunistic sensing manner [12]–[14], [29], where the location of the worker is estimated with dead reckoning by utilizing the inertial measurement unit (IMU) of the worker's mobile device such as accelerometer, magnetometer and gyroscope [13], [29].

The worker's mobile device installs an agent programm such as an APP that is dedicated to perform the opportunistic sensing and interact with the purchasing platform. The agent checks the percentage of resource in the device the MCS task occupies, and calculates the cost of the job. The agent representing the worker also decides if the buyer's offered price is worth of the efforts, where the worker's expectation of taking MCS jobs also can be set to the agent as configuration parameters. If the agent decides not to make the deal, the data submitted to the platform will not be authorized to be exposed to the buyer. We acknowledge the challenge for appropriately designing the agent program, but our work in this paper focuses on how the quality-aware pricing mechanism should be designed for the platform.

It is worth mentioning that the crowd workers are only able to roughly estimate their own locations with the IMU data, and such estimations are very limited not only in accuracy but also in availability. The IMU data based location estimation schemes will incur considerable cumulative errors; some scheme relies on special features in the environment such as the dramatic geomagnetic change near to the server room to calibrate the cumulative errors [15], but such features are not always available anywhere. The existing Wi-Fi signals could also help estimate the user's location information as in [13], [29]; however, such schemes could possibly work well only in certain indoor environments such as hallways, because it requires long-enough space to observe the signal change as the landmark. The work mentioned above partially utilizes the data from IMUs to improve the performance of the indoor localization, or to provide a coarse-grained indoor map. Although not able to provide normal indoor localization services, the roughly estimated location information along with the corresponding fingerprints observed can be a useful source of crowdsensed RSS fingerprints.

Fingerprinting Localization Model: The target indoor space S covered by the localization system can be regarded as a two-dimensional Cartesian space with $S \subset \mathbb{R}^2$. We use $\vec{r} = (x, y)$ to denote a location in S. We adopt a general model for radio propagation as in [4], where the RSS can be observed at $\vec{r}$ with respect to a given AP is

$$\mathcal{P}(\vec{r}) = \mu(\vec{r}) + \sigma\mathcal{Y}, \tag{1}$$

where $\mu(\vec{r})$ represents how the mean of RSS readings vary with respect to locations, $\mathcal{Y}$ is the normalized Gaussian additive noise with $\mathcal{Y} \sim \mathcal{N}(0, 1)$ and σ is the amplitude of the noise. This means that the RSS can be observed at $\vec{r}$ has a fixed but unknown mean value $\mu(\vec{r})$.

The radio propagation model is essentially a generalized model from the LNPL model [1], [14], which itself has been widely adopted by the industrial standardization organizations [38], [39]. Such a modeling approach with the Gaussian distribution assumption is validated by a number of work on indoor localization [3], [6], [14]; moreover, our experimental results with the EVARILOS testbed data [40] also support such modeling [5].

According to the analysis in [4], a point on the mean surface of the RSS sample space $\vec{\mu}(\vec{r})$ corresponds to a location in the physical space $\vec{r}$, which is the basic principle of fingerprinting localization. However, the RSS readings observed at a location $\vec{r}$ can be impacted by the IMU error, thus the observed $\mu(\vec{r})$ could deviate from the true value of the mean of the RSS readings and actually means $\mu(\vec{r'})$, which could result in that the user supposed to be localized at $\vec{r}$ is estimated at $\vec{r'}$. This is the root cause of inaccurate location estimation of MCS based fingerprints collection approach. Another important information indicated by the fingerprinting localization model is that the closer the reported fingerprint is to the true value of $\mu(\vec{r})$, the higher quality the fingerprint has.

Online Learning Model: We formulate the quality-aware pricing mechanism design issue under the online learning framework. The reported fingerprints from MCS workers form a data space $\mathcal{Z}$. Since the unavailability of the true value of $\mu(\vec{r})$ denoted by $\mu^*(\vec{r})$, we could only make the hypothesis of $\mu^*(\vec{r})$, and all hypotheses of the value form a hypothesis space $\mathcal{H}$. The fingerprint submitted at time point t for a location is denoted by $P_t \in \mathcal{Z}$. According to P_t and all such data submitted before time point t, we have a hypothesis of $\mu(\vec{r})$ at time point t, which is denoted by $h_t \in \mathcal{H}$. In the following, we focus on the situation that the submitted data are for a fixed location, and we use μ^* to denote the true value of the mean of fingerprints for simplicity of presentation. Then the difference between P_t and h_t is termed as the loss function $\ell(h_t, P_t) : \mathcal{H} \times \mathcal{Z} \to \mathbb{R}$. Our goal is to find the final hypothesis $\overline{h}$ that is the average over all yielded h_t, so that the difference between $\overline{h}$ and μ^* is minimized.

The challenge is that we have no idea about μ^*, and the classic online learning technique as described above is only able to

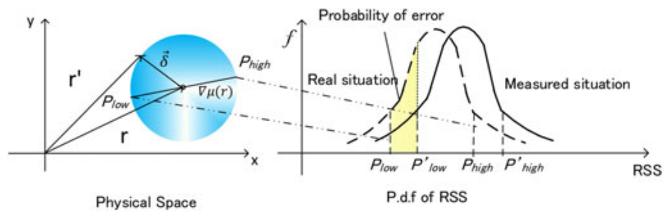


Fig. 1. Fingerprinting error.

minimize the total loss, which is the sum of $\ell(\overline{h}, P_t)$ over all t. However, even if the the total loss is minimized, it is unable to guarantee that the the gap between $\overline{h}$ and μ^* is minimized; our work in this paper provides a smart design of the loss function ℓ to this end.

Moreover, a rational buyer has concerns about data purchasing budget, and different buyers have different budget settings. Suppose the worker's cost for collecting the data is c_t at time t ($t = 1, 2, \ldots, T$), and the buyer is willing to pay π_t for the data. If $\pi_t \geq c_t$, then the deal can be made; otherwise, the buyer can not have access to the data. The buyer may want to obtain the best data possible with a fixed budget B, or obtain a number of data that make the localization system to achieve certain level of accuracy with least possible unfixed budget. The budget settings are also not considered in the classic online learning technique, and we are to deal with the issue in the latter part of the paper.

IV. Quality Evaluation of Fingerprints

This section defines the localization error incurred by the MCSed data, which is used to evaluate the quality of the fingerprints. We are to construct loss function based on such evaluation technique, which is to be proved to have a favored property. That is, the expectation of loss $\ell(\overline{h}, P_t)$ over t achieves the minimum when $\overline{h} = \mu^*$; therefore, we could minimize $\ell(\overline{h}, P_t)$ to obtain a reasonable hypothesis of μ^*.

A. Probability Model of Data Error

Assume that the maximum location estimation error with the IMU data is $\vec{\delta}$, then the worker's actual location is at any point of the $\vec{\delta}$ neighborhood of $\vec{r}$ as shown in left part of Fig. 1, when the worker is claiming the submitted fingerprint is observed at $\vec{r}$. The boundary of the region is $\vec{r'} = \vec{r} + \vec{\delta}$. We consider a simple situation that the worker is reporting the fingerprints with respect to a single AP; although the worker may measure the RSS readings multiple times, only one value will be submitted to the platform in order to save the cost for wireless network access. Thus the submitted data follow the 1-D Gaussian distribution according to the radio propagation model in (1), which is shown in Fig. 1.

Suppose the worker is indeed standing at $\vec{r}$, the corresponding probability density function (PDF) curve is as the dashed one in the right part of Fig. 1. According to the principle of maximum likelihood estimation (MLE), the reported fingerprints in this case should fall within the range between P_{high} and P_{low}, such that $\forall\vec{\delta}$,

$$P_{\text{high}} = \sup\left\{P | p_{\vec{r}+\vec{\delta}}\left(P; \mu\left(\vec{r}+\vec{\delta}\right)\right) \geq p_{\vec{r}}(P; \mu(\vec{r}))\right\},$$

$$P_{\text{low}} = \inf\left\{P | p_{\vec{r}+\vec{\delta}}\left(P; \mu\left(\vec{r}+\vec{\delta}\right)\right) \geq p_{\vec{r}}(P; \mu(\vec{r}))\right\}, \quad (2)$$

where sup and inf represent the least upper bound (supremum) and the greatest low bound (infimum), respectively; $p_{\vec{r}}$ and $p_{\vec{r}+\vec{\delta}}$ denote the probability density function (PDF) describing fingerprints can be observed at location $\vec{r}$ and $\vec{r}+\vec{\delta}$ respectively, with $\mu(\vec{r})$ and $\mu(\vec{r}+\vec{\delta})$ denote the corresponding mean values of the PDFs.

If the worker's actual location deviates from $\vec{r}$, the PDF of corresponding reported fingerprints will deviate from the true value of μ, as the solid curve shown in the right part of Fig. 1 with corresponding P'_{high} and P'_{low}. We now examine what will occur if such inaccurate fingerprints were stored in the database and used as if they were accurate. In the localization phase, the user actually at $\vec{r}$ will report the observed fingerprint with respect to the same AP. If the reported fingerprints fall within the range between P_{high} and P'_{high}, the system will still estimate the user's location at $\vec{r}$, although the information stored in the database is inaccurate. If the reported fingerprints fall within the range between P'_{low} and P_{high}, the system will make the right choice just according to the correct information. If the reported fingerprints fall within the range between P_{low} and P'_{low}, the system will estimate the user's location at $\vec{r'}$, which is the boundary condition that the inaccurate fingerprints from the MCS incur errors in localization.

We now consider a more general case. Given a submitted fingerprint P, the probability the fingerprint can incur localization error is equal to that it falls into the range between P_{low} and P'_{low}, where P'_{low} is the supremum with respect to $p_{\vec{r''}}(\cdot;\cdot)$. Note that $\vec{r''}$ is the true location where P is observed in $p_{\vec{r''}}(P; \mu(\vec{r''}))$. Suppose that $\mu(\vec{r''}) - \mu(\vec{r}) = d$, then the probability the submitted fingerprint can incur error is that it falls into the range $[P'_{\text{low}} - d, P'_{\text{low}}]$. We can define the quality of the data that is claimed to be collected at $\vec{r}$ by how likely the data will incur localization error:

$$F_{\text{error}_{\vec{r}}}(P; \mu(\vec{r})) = \left| \int_{P'_{\text{low}}-d}^{P'_{\text{low}}} p_{\vec{r}}\left(x; \mu\left(\vec{r''}\right)\right) dx \right|. \quad (3)$$

B. Data Error Analysis

We define that $||P - \mu(\vec{r})|| = -k \cdot ln(p(P; \mu(\vec{r})))$, where k is a positive real number. This is to quantify the distance between P and $\mu(\vec{r})$, where $-k \cdot ln(p(P; \mu(\vec{r})))$ decreases as P approaches to $\mu(\vec{r})$, achieves the minimum when $P = \mu(\vec{r})$. In Gaussian Distribution, the $-k \cdot ln(p(P; \mu(\vec{r})))$ is exactly the Euclidean distance d^2. We now rewrite the probability of error as the function of $\mu(r)$

$$\text{Err}(||P - \mu)||) = F_{\text{error}_{\vec{r}}}(P, \mu). \quad (4)$$

It is straightforward that μ^* should satisfy that

$$\mu^* = \arg\min_{\mu(\vec{r})} \mathop{\mathbb{E}}_{P}(\text{Err}(||P - \mu(\vec{r}))||)). \quad (5)$$

The following theorem shows the rationale for our definition of error.

Theorem 1: The expectation of the error $\text{Err}(||P - \mu(\vec{r}))||)$ over P achieves the minimum when $\mu(\vec{r})$ equals true value of the fingerprints μ^*.

Proof: We use $\mu = \mu(\vec{r})$ in short. Our target is to prove that

$$\mathop{\mathbb{E}}_{P}[\text{Err}(-k \cdot ln(p(P; \mu)))] - \mathop{\mathbb{E}}_{P}[\text{Err}(-k \cdot ln(p(P; \mu^*)))] \geq 0.$$

According to the Lagrange interpolation formula, there exists ξ satisfying

$$\mathop{\mathbb{E}}_{P}[\mathrm{Err}(-k\cdot ln(p(P;\mu)))] - \mathop{\mathbb{E}}_{P}[\mathrm{Err}(-k\cdot ln(p(P;\mu^*)))] = \mathop{\mathbb{E}}_{P}\left[-\mathrm{Err}'(\xi)k\cdot ln\left(\frac{p(P;\mu)}{p(P;\mu^*)}\right)\right].$$

It is obvious that $Err'(\xi)>0$ for the monotonicity of Err; considering the continuity of Err, $|Err'|$ is finite and there exists a $m>0$ satisfying that

$$\mathop{\mathbb{E}}_{P}\left[-\mathrm{Err}'(\xi)k\cdot ln\frac{p(P;\mu)}{p(P;\mu^*)}\right] \geq \mathop{\mathbb{E}}_{P}\left[-mk\cdot ln\frac{p(P;\mu)}{p(P;\mu^*)}\right].$$

According to Jensen's inequality.

$$\begin{aligned}-mk\mathop{\mathbb{E}}_{P}\left[ln\frac{p(P;\mu)}{p(P;\mu^*)}\right] &\geq -mk\cdot ln\left(\mathop{\mathbb{E}}_{P}\frac{p(P;\mu)}{p(P;\mu^*)}\right)\\ &= -mk\cdot ln\left(\int_P \frac{p(P;\mu)}{p(P;\mu^*)}p(P;\mu^*)dP\right)\\ &= -mk\cdot ln1 = 0.\end{aligned}$$

■

Assume that we have collected N data $P_1,\ldots,P_N$ from the worker, and we let $\hat{\mu}$ be the value that minimizes the $\frac{1}{N}\sum_{i=1}^N \mathrm{Err}(P_i;\mu)$. Theorem 1 above shows that when the number of data we have collected is large enough, the $\hat{\mu}$ we obtain from the data will approximate to the real mean value μ^*. We will use Hoeffding inequality to show the performance of the approximation.

Theorem 2: With probability $1-2e^{-\frac{2\epsilon^2}{N}}$, the difference between the expectation of the error of $\hat{\mu}$ obtained from collected data and the expectation of the error of μ^* is less than ϵ:

$$Pr\left(\frac{1}{N}\sum_{i=1}^N \mathrm{Err}(P_i;\hat{\mu}) - \mathop{\mathbb{E}}_{P}[\mathrm{Err}(P_i;\mu^*)] \leq \epsilon\right) \geq 1-2e^{-\frac{2\epsilon^2}{N}}.$$

Proof: We use $\mathrm{Err}(P;\mu)$ to represent $\mathrm{Err}(k\cdot ln(p(P;\mu)))$. Assume that $\mathrm{Err}(P;\mu)\in[m,M]$, with $0\leq m\leq M\leq 1$, according to Hoeffding inequality, we have

$$Pr\left(\left|\frac{1}{N}\sum_{i=1}^N \mathrm{Err}(P_i;\mu) - \mathop{\mathbb{E}}_{P}[\mathrm{Err}(P_i;\mu)]\right| \geq \epsilon\right) \leq 2e^{-\frac{2\epsilon^2}{N}}$$

for any μ. Then

$$Pr\left(\frac{1}{N}\sum_{i=1}^N \mathrm{Err}(P_i;\hat{\mu}) \leq \mathop{\mathbb{E}}_{P}[\mathrm{Err}(P_i;\hat{\mu})]+\epsilon\right) \geq 1-e^{-\frac{2\epsilon^2}{N}},$$

$$Pr\left(\frac{1}{N}\sum_{i=1}^N \mathrm{Err}(P_i;\mu^*) \geq \mathop{\mathbb{E}}_{P}[\mathrm{Err}(P_i;\mu^*)]-\epsilon\right) \geq 1-e^{-\frac{2\epsilon^2}{N}}.$$

It is obvious that the two events are independent to each other, thus

$$\begin{aligned}&Pr\left(\frac{1}{N}\sum_{i=1}^N \mathrm{Err}(P_i;\hat{\mu}) \leq \mathop{\mathbb{E}}_{P}[\mathrm{Err}(P_i;\hat{\mu})]+\epsilon,\right.\\ &\left.\frac{1}{N}\sum_{i=1}^N \mathrm{Err}(P_i;\mu^*) \geq \mathop{\mathbb{E}}_{P}[\mathrm{Err}(P_i;\mu^*)]-\epsilon\right)\\ &\geq (1-e^{-\frac{2\epsilon^2}{N}})^2 \geq 1-2e^{-\frac{2\epsilon^2}{N}}.\end{aligned} \tag{6}$$

When

$$\frac{1}{N}\sum_{i=1}^N \mathrm{Err}(P_i;\hat{\mu}) \leq -\mathop{\mathbb{E}}_{P}[\mathrm{Err}(P_i;\hat{\mu})+\epsilon],$$

$$\frac{1}{N}\sum_{i=1}^N \mathrm{Err}(P_i;\mu^*) \leq -\mathop{\mathbb{E}}_{P}[\mathrm{Err}(P_i;\hat{\mu})-\epsilon],$$

we have

$$\begin{aligned}0 &\leq \frac{1}{N}\sum_{i=1}^N \mathrm{Err}(P_i;\hat{\mu}) \leq -\mathop{\mathbb{E}}_{P}[\mathrm{Err}(P_i;\hat{\mu})]\\ &\leq \frac{1}{N}\sum_{i=1}^N \mathrm{Err}(P_i;\hat{\mu}) - \mathrm{Err}(P_i;\mu^*)+\epsilon\\ &\leq \epsilon\end{aligned}$$

Combining the (6), the theorem is proved. ■

The result shows that when N is big enough, the $\hat{\mu}$ converges to the real mean value μ^* in probability. Given that δ is far smaller than $|\vec{r}|$ and that $\mu(\vec{r})$ is continuous over $\vec{r}$, we may make an approximation that for any position $\vec{r'}$ on the circle centered at $\vec{r}$ with an arbitrary radius δ,

$$\mu(\vec{r'}) = \mu(r) + \nabla\mu(\vec{r})\delta\cos(\phi), \tag{7}$$

where ϕ is the angle between $\vec{r}$ and $\nabla\mu(\vec{r})$ in Fig. 1.

Now we consider the thresholds of RSS value within which the estimated location will be at $\vec{r}$ rather than the boundary of the circle centered at $\vec{r}$. According to the MLE principle [36], it is clear that determining the location at $\vec{r}$ requires the RSS value fall into an interval with a higher threshold P_{high} and a lower one P_{low}, and this interval ensures that the RSS value of $\vec{r}$ in it should be higher than that at the boundary of the circle centered at $\vec{r}$. Then we can derive P_{high} and P_{low} as follows,

$$\begin{aligned}P_{\mathrm{high}}(\mu) &= \mu(\vec{r}) + \frac{\nabla\mu(\vec{r})\delta\min\{\cos\phi,\sin\phi\}}{2},\\ P_{\mathrm{low}}(\mu) &= \mu(\vec{r}) - \frac{\nabla\mu(\vec{r})\delta\min\{\cos\phi,\sin\phi\}}{2}.\end{aligned} \tag{8}$$

Thus we have the specific form of the error

$$P_{\mathrm{error}_{\vec{r}}}(P;\mu) = \int_{P_{\mathrm{high}}(\mu)}^{P_{\mathrm{high}}(P)} \frac{1}{\sqrt{2\pi}\sigma}e^{-\frac{(x-P)^2}{2\sigma^2}}dx. \tag{9}$$

V. Quality-Aware Online Pricing Mechanism Design

In this section, we present our online pricing mechanism design, based on the the standard of data quality evaluation presented in the previous section.

A. Loss and Regret Function

Theorem 1 has shown that the localization error incurred by the noisy data can be utilized to evaluate the data quality. Note that the error function is a concave function measuring the distance between P and $\mu(\vec{r})$, which is quite similar to the loss function defined in the online learning framework. In order to determine the price for fingerprints reported sequentially, we transform the error function into the loss function, without changing its monotonicity and the favored property that the function achieves the minimum in the real mean value μ^*. In particular, the concrete form of the loss function is

$$\ell_t(h_t, P) = \int_{P_{\text{low}}(h_t)}^{P_{\text{low}}(P)} \frac{dP}{\frac{\partial}{\partial h_t} F_{\text{error}_{\vec{r}}}(\frac{P-h_t}{P}; P)}. \tag{10}$$

In our model, the hypothesis h_t is the parameter of the probability distribution function of the RSS data in location $\vec{r}$, and P_t is the value of measured RSS data at time point t. Considering the convexity of the problem, we introduce online convex optimization techniques and rewrite the loss function $\ell_t(h_t, P_t) = \ell_t(h_t)$ for simplicity. The regret function can be defined as

$$R = \sum_{t=1}^{T} \ell_t(h_t) - \sum_{t=1}^{T} \ell_t(h^*), \tag{11}$$

where h^* is the optimal choice, causing the least loss in our solution space $\mathcal{H}$. The regret function reflects how the data deviate from the desired value, i.e. the real mean of RSS.

The goal of online learning is to obtain the best hypothesis when data are submitted sequentially. We here use *Online Gradient Descent (OGD)* algorithm [34] as the learning rule for h_t. The basic idea of the OGD algorithm is to minimize the loss of the current hypothesis, which is derived utilizing the data in past iterations. Compared with the traditional gradient descent method, the OGD algorithm calculates the gradient in an online manner, where the information of the complete data set is unavailable. It has been proved that the OGD has an upper bound of regret of $O(\sqrt{T})$, which ensures that the average regret tends to zero when T goes to infinite. With OGD, we obtain a h_t in each time point t according to

$$h_t = h_{t-1} - \eta \nabla \ell_t(h_{t-1}). \tag{12}$$

The traditional online learning technique assumes that the buyer can access all the data submitted online. Our data purchasing process collects submitted data that have high quality; however, with the online learning framework, even if the quality of a datum is very low, it can still provide certain information to calibrate the hypothesis. Consequently, we need to compensate the information loss incurred by giving up low-quality data, so that the influence on the loss can be neutralized. In particular, the estimation of loss is $E(\sum_{t=0}^{T} \delta_t \ell_t) = \sum_{t=0}^{T} q_t \ell_t$, where δ_t is the function showing whether the data is chosen and where q_t denotes the probability that the data submitted at t is acquired by the mechanism. However, the definition of regret in (11) still includes all the loss at each time t whether it has been purchased or not. In order to get an unbiased estimator of the regret, we define

$$\hat{\ell}_t(h) = \begin{cases} \frac{\ell_t(h_t)}{q_t} & \text{for data chosen,} \\ 0 & \text{else.} \end{cases} \tag{13}$$

Algorithm 1: RSS data Pricing Mechanism.

Require: a sequence of data $d_1, \ldots, d_T$ coming in time $1, \ldots, T$ with each data possessing a cost c_t, $c_t \in [0, M]$
Ensure: a final hypothesis $\overline{h} \in H$
1: **for** t $= 1, \ldots, T$ **do**
2: The mechanism acquires a hypothesis h_t from OGD
3: The mechanism posts a price π_t according to a distribution G_t over $[0, M]$
4: **if** $\pi_t \geq c_t$ **then**
5: The mechanism sends the loss function $\ell(h_t)/q_t$ back to the OGD and pay for the posted price π_t
6: **else**
7: The platform rejects the price and the mechanism sends 0 to the OGD
8: **end if**
9: **end for**
10: Return the final hypothesis $\overline{h} = \frac{1}{T}\sum_{t=1}^{T} h_t$

With the unbiased estimator acquired, we can consider the mechanism as an OGD that recieves $\hat{\ell}$ in each round t.

B. Quality-Aware Online Pricing Scheme

Assume that the MCSed data come in the sequence of $d_1, \ldots, d_T$, with each contains a cost $c_1, \ldots, c_T$. The pricing mechanism can determine how much the buyer should pay for the data. However, we have no means to know either the quality of data is good enough for localization or there will be a better one coming in the future. We formally define our *RSS data Pricing Mechanism (RPM)* in Algorithm 1.

In the algorithm, we could collect the submitted data periodically. Although the algorithm is oblivious to how to configure the period, we make the system to collect the data every 10ms when we do the experiments. Note that T is the size of the collected data sequence, where we set $T = 1000$ in our experiments, meaning we let the system collect 1000 data samples. Equation (12) is a part of the OGD algorithm [34]; (13) is involved in the algorithm in rows 4-6, where the data are chosen in the first case and not chosen in the second one. Although necessary revisions have been made to accommodate the indoor localization application scenario, the algorithm in essence is in the framework of the OGD algorithm, where the convergence property has been proved [34]. Our experimental results also show that the proposed algorithm converges in practice.

The mechanism and online learning algorithm produces a sequence of hypothesis $h_1, \ldots, h_T$. The main goal of our algorithm is to get the best hypothesis $\overline{h}$, the mean value of RSS, from the sequence. One simple approach is to average every hypothesis h_t acquired at each time t

$$\overline{h} = \frac{1}{T}\sum_{t=1}^{T} h_t. \tag{14}$$

To evaluate how well our final hypothesis $\overline{h}$ approximate to the optimal hypothesis h^*, we define the risk $\mathcal{L}(h) = \frac{1}{T}\sum_t \ell_t(h)$ of a hypothesis h as the average of $\ell_t(h)$ over t.

Lemma 1: The expectation of the risk of $\overline{h}$ is less than the $\mathcal{L}(h^*)$ plus R/T [37], that is

$$\mathop{\mathbb{E}}_{\ell_t} \mathcal{L}(\overline{h}) \leq \mathcal{L}(h^*) + \frac{R}{T}.$$

This means that if $O(R(h^*)) < O(T)$, then the $\overline{h}$ will approximate to the optimal hypothesis h^* as T goes to infinite. However, the crux of the algorithm is to find the best distribution G_t used for the mechanism to post its price.

VI. Data Pricing With Budget Constraints

The price distribution G_t is closely related to the budget setting. We here consider two scenarios: First, the buyer wants to buy the best data with a fixed budget; second, the buyer wants to buy data with certain quality with as least budget as possible.

A. Regret Minimization With Fixed Budget

In this scenario, the buyer has a fixed budget, and the target of the mechanism is to minimize the total regret defined in (11). In this section, we will give the exact form of the distribution G_t and the analysis of the regret bound according to the distribution.

1) Problem Formulation: It is well-known that the regret bound of OGD is $\frac{||h^*||^2}{2\eta} + \eta \sum_{t=1}^{T} \nabla \ell_t(h_t)^2$. Substituting (14) into the expression above, we have the regret bound of RPM as following

$$R \leq \frac{||h^*||^2}{2\eta} + \mathop{\mathbb{E}}_{\ell_t, q_t} \left(\sum_{t=1}^{T} \frac{\nabla \ell_t(h_t)^2}{q_t} \right). \quad (15)$$

At each time point t, the RPM needs to post a price π_t according to G_t in order to get a minimum regret, we thus reduce the problem of designing a mechanism into an optimization problem

$$\min \sum_{i=1}^{n} \frac{\nabla {\ell_i}^2}{1 - G_i(c_i)}$$
$$\text{s.t.} \quad \sum_{i=1}^{n} \int_{c_i}^{M} x dG_i(x) \leq B, \quad (16)$$

where $\forall c_i, 0 \leq c_i \leq M$, and $G(0) = 0, G(M) = 1$. Note that the solution we need to find for the optimization problem is the CDF G_t, which is in contrast to the traditional optimization problems where the solutions are variables. We are to use calculus of variations technique to solve it.

Theorem 3: The optimal solution of the optimization problem (18) is in the form of

$$G_t(c) = \begin{cases} 1 - \frac{\nabla \ell_t}{\sqrt{\lambda c - \beta}} & c \in (\frac{\nabla \ell_t^2 + \beta}{\lambda}, M]; \\ 0 & \text{else}. \end{cases} \quad (17)$$

Proof: We first give our function space $V = \{y | y(0) = 0, y(M) = 1\}$, and we denote our cost function as

$$M(G_1, \ldots, G_T) = \sum_{t=1}^{T} \frac{\nabla {\ell_i}^2}{1 - G_t(c_t)}.$$

Then the augmented Lagrange function is derived as

$$J(G_1, \ldots, G_T, \lambda) = M(G_1, \ldots, G_T) + \lambda \left(\sum_{t=1}^{T} \int_{c_t}^{M} x dG_t(x) - B \right).$$

According to the Gateaux Derivative, we obtain that $\forall \hat{G} \in V$,

$$\delta J|_{G_t} \left(\hat{G}_t - G_t \right) = \int_{c_t}^{M} \left(-\frac{\nabla {\ell_i}^2}{(1 - G_t(c_t))^2} + \lambda x \right) \times d\left(\hat{G}_t(x) - G_t(x) \right),$$

and if $\overline{G_t}$ is the local minimum, then we have $\delta J(\hat{G}_t - \overline{G_t}) \geq 0$. Note that $\int_0^M d(\hat{G_t}(x) - G_t(x)) = 0$ and the arbitrariness of $\hat{G}_t$, we must have

$$-\frac{\nabla {\ell_i}^2}{(1 - G_t(c_t))^2} + \lambda x = \beta \geq 0$$

hold for any x on $[c_t, M]$, thus proved. ∎

The major challenge now is to determine β and λ. Notice that $G(x)$ is not continuous, using the Stieltjes Integral, we can rewrite the constraint as following

$$\sum_{t=1}^{T} \int_{c_t}^{M} x dG_t(x) = \sum_{t=1}^{T} \left(\int_{c_t}^{M} x G_t'(x) dx + (1 - G_i(M))M \right) \leq \sum_{t=1}^{T} \nabla \ell_t \left(\frac{2}{\lambda} \sqrt{\lambda M - \beta} + \frac{c_t}{\sqrt{\lambda c_t - \beta}} - \frac{2}{\lambda} \sqrt{\lambda c_t - \beta} \right) \leq B. \quad (18)$$

The Stieltjes Integral here has its practical significance, because we assume that the cost lies between $[0, M]$, in other word, the mechanism does not accept any price higher than M, thus for any posted price c that is higher than M, the mechanism will only pay M instead of c.

Now since we get the solution of the G_t, the remaining is to determine the parameters λ and β. Recall our initial optimization problem of minimizing the regret bound. The Lagrangian is thus given as follows

$$L(\mu, \beta, \lambda) = \sum_{t} \left(\nabla \ell_t \left(\sqrt{\lambda c_t - \beta} + \mu \left(\frac{2}{\lambda} \sqrt{\lambda M - \beta} + \frac{c_t}{\sqrt{\lambda c_t - \beta}} - \frac{2}{\lambda} \sqrt{\lambda c_t - \beta} \right) \right) \right) - \mu B. \quad (19)$$

and its gradient is obtained accordingly.

$$\frac{\partial L}{\partial \lambda} = \sum_t \nabla \ell_t \left(\left(\frac{c_t}{2} - \frac{\mu}{\lambda}\right) \frac{1}{\sqrt{\lambda c_t - \beta}} + \frac{\mu}{\lambda} \frac{1}{\sqrt{\lambda M - \beta}} - \frac{1}{2} \frac{c_t^2 \mu}{\sqrt{(\lambda c_t - \beta)^3}} \right)$$

$$\frac{\partial L}{\partial \beta} = \sum_t \nabla \ell_t \left(\left(\frac{\mu}{\lambda} - \frac{1}{2}\right) \frac{1}{\sqrt{\lambda c_t - \beta}} - \frac{\mu}{\lambda} \frac{1}{\sqrt{\lambda M - \beta}} - \frac{\mu}{2} \frac{c_t}{\sqrt{(\lambda c_t - \beta)^3}} \right)$$

$$\frac{\partial L}{\partial \mu} = \sum_t \nabla \ell_t \left(\frac{2}{\lambda} \sqrt{\lambda M - \beta} + \frac{c_t}{\sqrt{\lambda c_t - \beta}} - \frac{2}{\lambda} \sqrt{(\lambda c_t - \beta)} \right) - B \tag{20}$$

According to the complementary slackness theorem, $\mu \neq 0$, which means that the constraint condition in (16) for the optimal solution is strict. An intuitive explanation of this property is that the mechanism should use up the budget to get the optimal data. It is infeasible to work out the analytic solution of the optimal value of β and λ due to the complex form of the equation itself and the fact that we do not have access to enough prior knowledge of c_t and $\nabla \ell_t$.

We here give the iterative update of these two parameters. We initially set β to a fixed value and λ to a very small value, *e.g.*, 0.01. Then at each time point t, we update the value of $\lambda^{(t)}$ iteratively according to

$$\lambda^{(t)} = \frac{T^2}{B^2 M} \left(\sum_{i=1}^{t-1} \frac{\nabla \ell_i(h_t)}{t-1} \sqrt{c_t} \right)^2 + \frac{\beta}{M} \tag{21}$$

The above iteration formula is straightforwardly derived from the constraint (18).

2) Regret Analysis: When the parameter β and λ is fixed, a higher $\nabla \ell_t$ means a higher probability that a data will be purchased, which is in accordance with the definition of the gradient, the fastest direction that the current h_t descends to the ideal one. For the parameter λ, a smaller λ means that the mechanism would tend to pay a high price for the data. The parameter β can be seen as the most dominant factor that determines the minimum price that the mechanism would pay.

Generally, a more sufficient budget B would lead to a higher β and smaller λ. When budget B goes to infinite, the λ and β goes to zero, leading to a CDF of $G(M) = 1$. That is, when the budget is unlimited, the mechanism will try to purchase the data as many as possible. For a fixed β, we give the estimation of the upper bound of the regret of RPM in theorem 4.

Theorem 4: For a fixed β, the regret of RPM is bounded with

$$R < O\left(\max\left\{ \sqrt{T}, \frac{T\theta}{\sqrt{B}} \sqrt{1 - \frac{\beta B^2}{T\theta^2 M}} \right\} \right), \tag{22}$$

where $\theta = \mathbb{E}\, \frac{1}{T} \sum_t \nabla \ell_t (2\sqrt{M} - \sqrt{c_t})$.

Proof: We prove the theorem by firstly setting the $\beta_0 = 0$, and through simple calculation, we can have an estimation of $\lambda_0 = \frac{T^2}{B^2}\theta^2$. Since that $\partial L/\partial \beta > 0$, $\partial L/\partial \lambda > 0$ hold in β_0 and λ_0, we obtain that the optimal solution (β^*, λ^*) statisfy that $\beta^* > \beta^0$, $\lambda^* > \lambda_0$. Considering the discontinuity of the G_t, we introduce $C = \{q_t | c_t < \frac{\nabla \ell_t^2}{\beta}\}$. It is obvious that all the elements in C equals to 1. We substitue λ_0 and β into the estimation of the regret bound given in (15)

$$\begin{aligned} R &\leq \frac{||h^*||^2}{2\eta} + \eta\, \mathbb{E} \left(\sum_{t \in C} \nabla \ell_t (h_t)^2 + \sum_{t \notin C} \frac{\nabla \ell_t (h_t)^2}{q_t} \right) \\ &\leq \frac{||h^*||^2}{2\eta} + \eta \left(Te + \mathbb{E} \sum_{t \notin C} \nabla \ell_t c_t \sqrt{\frac{T^2\theta^2}{B^2} - \beta} \right) \\ &\leq \frac{||h^*||^2}{2\eta} + \eta \left(Te + \frac{T^2\theta^2}{B} \sqrt{1 - \frac{\beta B^2}{T^2 \theta^2 M}} \right) \\ &< O\left(\max\left\{ \sqrt{T}, \frac{T\theta}{\sqrt{B}} \sqrt{1 - \frac{\beta B^2}{T\theta^2 M}} \right\} \right), \end{aligned}$$

and thus acquire the desired result. ■

B. Budget Minimization for Certain Quality Level

This section considers the scenario where the purpose of the mechanism is to achieve a given error between $\mathcal{L}(\overline{h})$ and $\mathcal{L}(h^*)$. We design another mechanism to pursue a minimum expectation of the money the buyer will pay to achieve the given bound of error. Recall that the difference betweeen the $\mathcal{L}(\overline{h})$ and $\mathcal{L}(h^*)$ is at most the average regret bound $R_0 = R(h^*)/T$. We thus only need to consider the constraint on the average bound of regret which makes the problem feasible.

However, we also encounter the challenge that the objective function of this situation is formed as an integral, which is not a simple task to be resolved with classical optimization methods. Unlike the method we used in Theorem 3, in this case, the constraint of the problem that $\frac{1}{T} \sum_t \frac{\nabla \ell_t^2}{q_t} \leq R_0$ is of a relatively simple form and we observe that the expectation of cost can be approximated by $\sum_t q_t c_t$. Thus we first consider a relatively simple version of the problem with objective function $\min_{q_t} \sum_t c_t q_t$, where $q_t \in [0, 1]$ is the probability of the mechanism to perchase the data in round t. Then we generalize the form between q_t and c_t to obtain the form of CDF for our original problem.

Theorem 5: The optimal q_t for the minimum $\sum_t c_t q_t$ in the budget minimization scenario is in the form of:

$$q_t = \min\left\{ 1, \sqrt{\frac{\lambda}{c_t}} \nabla \ell_t \right\}, \tag{23}$$

Proof: Considering that the objective function is convex, we can derive the corresponding Lagrangian function

$$L = \sum_t c_t q_t - \lambda \left(-\sum_t \frac{\nabla \ell_t^2}{q_t} + R_0 T \right) - \sum_t \mu_t (1 - q_t). \tag{24}$$

The optimal K-T condition of the problem is

$$\frac{\partial L}{\partial q_t} = c_t - \lambda \left[\frac{\nabla \ell_t^2}{q_t^2} \right] + \mu_t = 0. \tag{25}$$

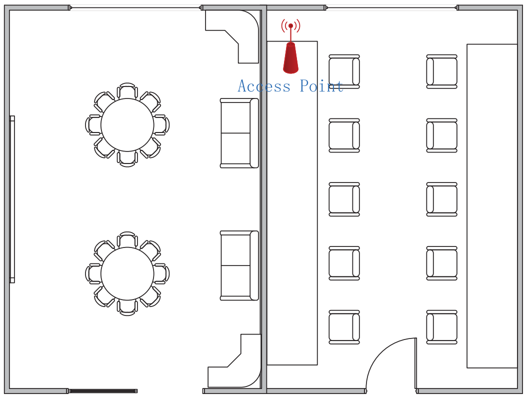


Fig. 2. Experiment location.

In light of complementary slackness, it is clear that when $q_t = 1$, we get $\mu_t \neq 0$ and when $q_t \neq 1$, we get $\mu_t = 0$. Thus based on (25), prove the theorem.

Since (23) holds for any c_t, and the cost c_t in our model is arbitrarily given, we may make a generalization of the relation between q_t and c_t, that is, for any $c \in (\lambda\nabla\ell_t^2, M]$, the probability of the data to be acquired is $\sqrt{\frac{\lambda}{c}}\nabla\ell_t$. Thus the cumulative distribution function of the budget saving pricing mechanism is of the form

$$G_t(c) = \begin{cases} 1 - \sqrt{\frac{\lambda}{c}}\nabla\ell_t & c \in (\lambda\nabla\ell_t^2, M], \\ 0 & \text{else}. \end{cases} \tag{26}$$

Similar to the problem in (16), it is difficult for the mechanism to determine the parameter λ in each round t without prior knowledge of ∇f_t and c_t. According to our constraint on regret and that the optimal condition in (25), λ can not equal zero, we can get an approximation of the $\sqrt{\lambda^{(t)}}$ through simple calculations

$$\sqrt{\lambda^{(t)}} = \frac{\sum_t \sqrt{c_t}\nabla\ell_t}{R_0 T}. \tag{27}$$

After acquiring the form of the G_t, we can now make a relatively more precise estimation of the budget B

$$E(B) = \sum_t \int_{c_t}^{M} dG_t(c) = \frac{T^2}{R_0}\theta\phi, \tag{28}$$

where $\phi = \sum_t \frac{1}{T}\nabla f_t(\sqrt{M} - \sqrt{c_t})$ and $\theta = \sum_t \frac{1}{T}\nabla f_t\sqrt{c_t}$.

VII. Experimental Results

We conduct experiments to validate our analysis. First, we set up a Wi-Fi transmitter in the indoor space and the devices held by volunteers automatically report the observed RSS values to the server when they reach into the range of Wi-Fi as shown in Fig. 2. Empirically, we set the $\nabla\mu(\vec{r}) = 1$ and $\eta = 1/\sqrt{2T}$ to minimize the regret bound through experiments.

We sample 1000 RSS values at each location $\vec{r}$ and simulate costs of data through a normal distribution with mean value of 0.5 and variance of 1, and the maximum of the cost is bounded by $M = 1$. We first evaluate the impact of the parameter choice on our mechanism. We run the RPM with fixed budget $B = 100$, range λ from 10 to 100 and β from 0 to 100. We use the risk of the final hypothesis $\overline{h}$, $\mathcal{L}(\overline{h})$ defined in Lemma 1 to measure the performance of the mechanism. For each λ and β, we make 100 repeated trails and take the average of the results to diminish the stochastic error. The results shown in Fig. 3 indicate that for each λ, there exists a best β that achieves the minimum risk. If the value of β goes greater and that of λ decreases, the performance result of the mechanism tends to be that of the naive mechanism, which is to purchase as many as data with all budget at disposal. This is in accordance with our analysis in Section VI-A.

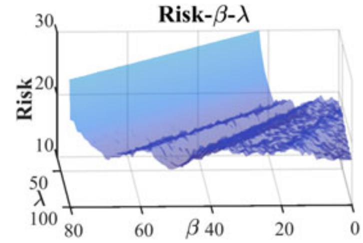


Fig. 3. Risk-β-λ.

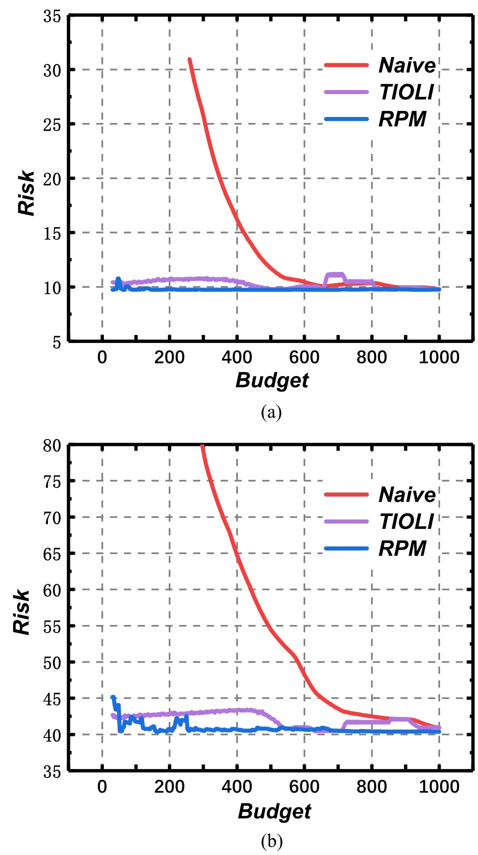


Fig. 4. Risk-budget on different sequences. (a) Low var. sequence. (b) High var. sequence.

We also compare our proposed mechanism RPM with the take-it-or-leave-it (TIOLI) mechanism in [8], [9]. The TIOLI is an incentive model for encouraging participation in the survey, where the value of the private information the subject provides is closely related to the accuracy of the information. Since the subject is always trying to protect their privacy, the provided information's quality varies; however, the TIOLI scheme can guarantee that if the subject provides inaccurate information, then the corresponding reward to the subject is very small. The quality awareness in TIOLI is very similar with the RPM scheme

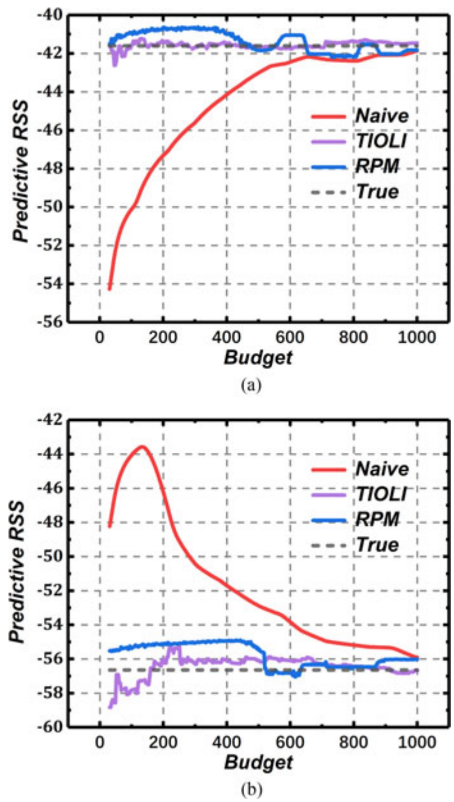


Fig. 5. Predictive RSS values on differnent sequences. (a) Low var. sequence. (b) High var. sequence.

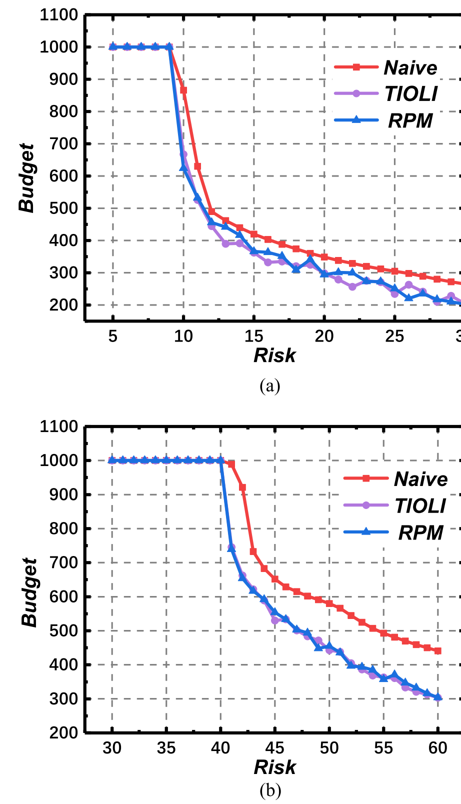


Fig. 6. Budget-risk on different sequences. (a) Low var. sequence. (b) High var. sequence.

proposed in this paper. Nevertheless, TIOLI must be applied in verifiable scenarios, where the ground truth is available; moreover, TIOLI processes data in batch instead of online as the RPM scheme in this paper. We choose TIOLI as one of the benchmarks because it can buy the most accurate data with the lowest cost. Although the in-batch processing and the verifiable scenario assumption in TIOLI may lead to its advantageous performance, if the proposed RMP can present performance close to TIOLI without those favored processing pattern and assumption, then we can say the RMP has an outstanding performance.

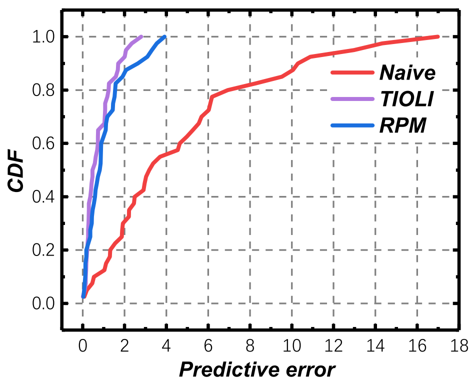
We utilize the samples in the database to evaluate the 3 kinds of mechanisms: the naive method, the TIOLI method, proposed RPM with $\beta = 0$. With the naive method, the mechanism randomly purchases data in an online manner with maximum cost M until the budget is consumed up. With the RPM method, we fix β and adjust λ iteratively according to (21), and use the risk $\mathcal{L}(\overline{h})$ to judge the quality of our final hypothesis. We run each algorithm 100 times and present the average of the results.

The value of RSS fingerprints may vary due to the noise in the indoor environment, which could be even worsened by slightly shaking the sensing devices or blockage of other people. In order to make a reasonable assessment on the mechanism, we analyze collected RSS fingerprint sequences according to how they vary in amplitude. We select two kinds of sequences to conduct the following experiments, where the first kind is the sequence with high variation and the second is with low variation.

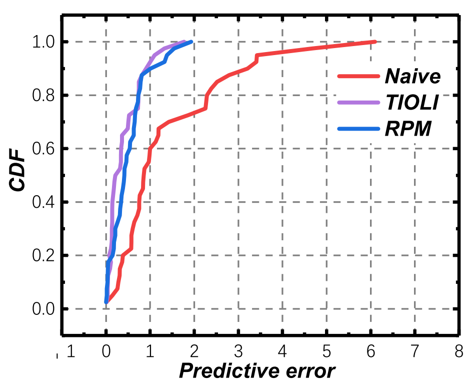
We first examine the risk of final hypothesis output by different mechanisms with different budget limits, where the results are shown in Fig. 4. It is apparent that the curves of all those mechanisms gradually converge as budget mounts, verifying the online-to-batch conversion we mentioned before. It can be seen that the final risk of RPM outperforms naive method in a large amount while the budget limit is at a median level (100-600 approximately). The difference between RPM and naive method is getting smaller as the budget is increasing. It demonstrates that while the budget is insufficient, RPM works much better than the naive method. There is a remarkable fact that RPM makes prediction according to the data sample received in each round, while TIOLI could obtain all the information from data sequence. The results demonstrate that the performance of RPM catches up with that of TIOLI, indicating RPM is an effective algorithm under the online learning framework.

We then conduct experiments to estimate the mean value of RSS distribution with different budget limits. According to Theorem 1, we approximate the true value of fingerprint by averaging reported data sequence with large amount of RSS samples, and compare it with the prediction generated by different mechanisms. As shown in Fig. 5, the difference between RPM and TIOLI is very small and both of them can predict RSS closely to the true value even though the budget is very low, while naive method predicts well only if the budget is relatively high (800 or more). Considering the variation factor on different sequences, our RPM shows its robustness during the data procurement process, while naive method may generate wrong prediction trend in high-variance data sequence.

We also consider the derivative problem about minimizing the budget with given risk. We compare the least needed budget output by different mechanisms when the risk is required at different levels. The experimental results evince that once the risk is fixed, the budget required by RPM and TIOLI are close



(a)



(b)

Fig. 7. Cumulative distribution function of error. (a) Budget = 200. (b) Budget = 600.

to each other and less than naive method, indicating that RPM can save much more money to reach a certain risk threshold, as shown in Fig. 6. The gap becomes larger when the variation is relatively high, indicating the stability of RPM in crowdsensing based data collection process.

Moreover, we perform measurements on the mean value of RSS fingerprints at different locations. Volunteers randomly walk around the test region to collect data sequences at 100 different locations. The predictive error is defined as the difference between true value and the predicted value generated by mechanisms. We evaluate different mechanisms by their statistical distribution, which is shown in Fig. 7. Although our RPM method has no prior knowledge about the collected data sequence, it still makes accurate estimations on the mean value and achieves a comparative error rate with TIOLI. In practice, the whole fingerprint data set is not always available in advance, and the prediction may vary with the characteristics of arriving data, our online learning based mechanism definitely shows superiority in such practical scenarios. It is also observed that the difference between naive method and RPM is large with low budget($B = 200$) and gets smaller when the budget is getting higher($B = 600$), verifying the effectiveness of our proposed mechanism when the budget is not ample.

VIII. Conclusion

In this paper, we have presented a data pricing scheme dedicated to MCSed fingerprints by enhancing the online learning technique. We have revealed the principle of fingerprints quality assessment for accurate localization. Based on the principle, we have designed corresponding loss and regret function, which is able to reflect values of the fingerprints with respect to localization accuracy. We have presented an online pricing scheme for MCSed data, which results in that the worker's payoff is a random variable following an optimal probability density function (PDF) leading to the minimum expected regret. Further, we have extended our scheme to application scenarios with different budget settings, where the pricing strategies for the scenarios of regret minimization with fixed budget and budget minimization for certain fingerprints quality level have been investigated. Experimental results have been presented to verify our theoretical analysis.

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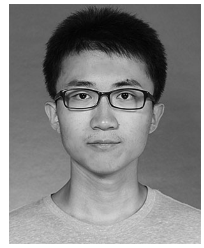

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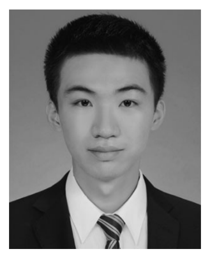

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