Outage Performance Analysis of Two-Way Relay System with Multi-Antenna Relay Node

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Abstract—This paper presents an analytical study on the outage performance of amplify-and-forward (AF) two-way relay system with multi-antenna relay node (RN). Two major bidirectional protocols, i.e., two time slots multiple access broadcast (MABC) protocol and three time slots time division broadcast (TDBC) protocol, are considered. For both considerations, we first assume that instantaneous channel-state-information (CSI) is unavailable at RN, thus RN just simply uses the fixed relay gain derived from statistical CSI to scale the received signals before forwarding. We then consider the scenario where RN can obtain the instantaneous CSI to perform the zero-forcing (ZF) relay precoding. The closed-form expressions of outage probability are derived for all cases. Based on these expressions, the diversity-multiplexing tradeoff (DMT) is further obtained for the MABC protocol. The analytical results show that, for non-precoding MABC scheme, the diversity order is only 1, which is independent to the relay antenna number $M$. While for the ZF-precoding case, the diversity order of $M-1$ can be obtained.

I. INTRODUCTION

Relay-assisted cooperative transmission has been shown able to offer significant benefits, such as throughput enhancement, coverage extension and power reduction, in wireless communications. However, due to the half-duplex constraint, the spectral efficiency of traditional one-way relaying is highly constrained. Recently, two-way relaying has been proposed as a more efficient protocol to complete the bidirectional transmission of two users by applying the physical layer network coding (PLNC). Based on the required time slots to complete the bidirectional communication, two categories of protocols have been widely studied in the literature. The first one is called multiple access broadcast (MABC) which consists of two time slots. In the first time slot, two sources send their signals to the relay node (RN) simultaneously. Then in the second time slot, the RN broadcasts the mixed signals to two destinations. The second one is referred to as the time division broadcast (TDBC) where a round of information exchange is completed within three time slots. In the first and second time slot, two sources transmit their signals to the RN separately. Then, the RN broadcasts the mixed signals to two destinations in the third time slot. Note both transmission protocols have their own merits. The MABC is more spectral efficient for needing less time slots, while TDBC can achieve more reliable transmission by exploiting extra direct-path link.

A few analytical studies have been reported to evaluate the promising gain of the two-way relaying. For example, the authors in [1] compare the performance of amplify-and-forward (AF) bidirectional transmission over different schemes. Besides the studies on two-way relaying with single RN, the multiple RNs case, which can be used to increase the transmission diversity, has recently drawn significant attention, especially combined with relay selection technique. In [2], the relay selection is investigated based on max-min criterion to minimize the outage probability under AF strategy. In [3], the relay selection is studied based on both max-min and max-sum criteria under decode-and-forward (DF) strategy.

An alternative approach to combat the fading is to use the multi-antenna RN. In [4], the authors analyze the diversity-multiplexing tradeoff (DMT) at finite signal-to-noise ratio (SNR) for multi-antenna RN (MAR) two-way relay system under DF strategy. In this work, we aim to provide an analytical study for the MAR two-way relay system under AF strategy. Here the AF strategy is chosen for its low complexity in implementation. The two major bidirectional transmission protocols, i.e., MABC and TDBC, are considered. For both considerations, we first assume that the RN is unable to obtain the required CSI to perform complex processing and just uses the fixed relay gain based on the statistical CSI to scale the received signals to satisfy the power constraint at RN in long-term. Such approach can reduce the relay computational complexity and system feedback overhead. Second, we consider a more complicated scenario where the instantaneous CSI is available at the RN. Thus the zero-forcing (ZF) relay precoding is conducted to improve the system performance. The closed-form expressions of outage probability are derived for all cases. Based on the derived expressions, the diversity-multiplexing tradeoff is obtained for the MABC protocol.

II. SYSTEM MODEL

Consider a two-way relay system where two single-antenna source nodes, denoted as $S_1$ and $S_2$, want to exchange messages through a RN, denoted as $R$ and equipped with $M$ antennas. Let $x_i$, with $E[|x_i|^2] = E_i$, denote the transmit signal from $S_i$. The channel vector from $S_i$ to $R$ and $R$ to $S_i$ are denoted by $h_i$ and $g_i$, respectively. They are modeled as
denotes the transpose. Thus, the received signal-to-noise-ratio (SNR) at the RN is denoted as $x_{R,i} = f_i x_i + n_{R,i}$, where $n_{R,i}$ denotes the additive noise vector at RN with $n_{R,i} \sim \mathcal{CN}(0, \sigma^2_{n,R})$. The transmit signal vectors from RN is expressed as

$$x_{R,i} = f_i h_i x_i + n_{R,i},$$

where $f_i$ denotes the linear processing for $y_{R,i}$ as in (2). Then, the in the third time slot, the RN broadcasts the superimposed signal and the received signal at destination node can be expressed as, after subtracting the self-interference

$$y_{R,i} = g_i f_i h_i x_i + g_i f_i n_{R,i} + n_{i,i},$$

where $n_{i,i}$ denotes the noise at $S_i$ in the third time slot with $n_{i,i} \sim \mathcal{CN}(0, \sigma^2_{n,S})$. Since two copies of signal $x_i$ can be exploited at $S_i$ as expressed in (4) and (6), the maximal-ratio combining (MRC) technique is used to combine them and the resultant SNR at $S_i$ is depicted as

$$SNR_{R}^{MA} = \frac{E_i |g_i f_i|^2}{\sigma^2_{n,R} + \sigma^2_{n,S} |g_i|^2 + \sigma^2_{n,R} |f_i|^2 + \sigma^2_{n,i}},$$

where $n_{R,i}$ denotes the additive noise vector at RN with $n_{R,i} \sim \mathcal{CN}(0, \sigma^2_{n,R})$. The transmit signal vectors from RN is expressed as

$$x_{R,i} = f_i h_i x_i + n_{R,i}, \quad y_{R,i} = h_{22} x_2 + n_{R,2},$$

where $f_i$ denotes the linear processing for $y_{R,i}$ as in (2). Then, the in the third time slot, the RN broadcasts the superimposed signal and the received signal at destination node can be expressed as, after subtracting the self-interference

$$y_{R,i} = g_i f_i h_i x_i + g_i f_i n_{R,i} + n_{i,i},$$

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$$SNR_{R}^{MA} = \frac{E_i |g_i f_i|^2}{\sigma^2_{n,R} + \sigma^2_{n,S} |g_i|^2 + \sigma^2_{n,R} |f_i|^2 + \sigma^2_{n,i}},$$

C. Outage Performance Metric

Based on the SNR given in (3) and (7), the corresponding rate is yielded as

$$\gamma_{i}^{MA} = \frac{1}{2} \log_2 (1 + SNR_{R}^{MA}), \quad \gamma_{i}^{TD} = \frac{1}{3} \log_2 (1 + SNR_{R}^{TD}).$$

Here, the pre-log factors $\frac{1}{2}$ and $\frac{1}{3}$ result from the fact that the MABC is operated in two time slots, while TDBC in three time slots. Since the two-way relay system involves two users, we define the system in outage if any user is in outage, Thus, the outage probability of two-way relay system can be defined as

$$P_{out}^{MA/TD} = \Pr[\gamma_{1}^{MA/TD} < R_1 \text{ or } \gamma_{2}^{MA/TD} < R_2],$$

where $R_1$ and $R_2$ are the pre-set rate thresholds. This definition can also be equivalently written as

$$P_{out}^{MA/TD} = \Pr[SNR_{R}^{MA/TA} < \tau_{1}^{MA/TA} \text{ or } SNR_{R}^{MA/TA} < \tau_{2}^{MA/TA}],$$

where $\tau_{i}^{MA} = 2^{R_i} - 1$ and $\tau_{i}^{TD} = 2^{3^{R_i}} - 1$, $i = 1, 2$.

III. OUTAGE PERFORMANCE ANALYSIS

In this section, two different relaying schemes, i.e., non-precoding scheme and ZF-precoding scheme, are considered for each protocol. In the first scheme, the instantaneous CSI is unavailable at RN. Thus, the “blind” relay just scales the received signals based on the statistical CSI to satisfy the power constraint in a long term. Note this scheme can significantly reduce the relay computational complexity and feedback overhead. While in the ZF-precoding scheme, the RN is assumed to have the knowledge of instantaneous CSI which will be used to perform the ZF-precoding at RN. Although this scheme needs the RN to have high processing ability and more feedback overheads, we will see that the relay precoding can significantly improve the system performance.
A. Non-precoding Scheme for MABC

Here, $F$ in (2) should be expressed as $\alpha^{MA}I_M$ with $\alpha^{MA} = \frac{(E|\mathbf{h}_1|^2 + \| \mathbf{h}_2 \|^2)^{0.5}}{(M(E|\mathbf{h}_1|^2 + \| \mathbf{h}_2 \|^2)^{0.5}} + \frac{E_R}{2\gamma^2}.\) Here we have used the fact that $\mathbf{h}_1, \mathbf{h}_2, \mathbf{g}_1$ and $\mathbf{g}_2$ are mutually independent.

Theorem 1: The outage probability of the MABC AF two-way relaying protocol without precoding can be expressed as

$$P_{out}^{MA} = \Pr[\rho_1 < \tau_1^{MA}] + \Pr[\rho_2 < \tau_2^{MA}] - \Pr[\rho_1 < \tau_1^{MA}]\Pr[\rho_2 < \tau_2^{MA}],$$

(9)

where $\rho_i = \frac{\alpha^{MA}E |\mathbf{g}_i|^2}{\alpha^{MA}E |\mathbf{h}_1|^2 + \| \mathbf{h}_2 \|^2}$. Here we have used the fact that $\mathbf{h}_1, \mathbf{h}_2, \mathbf{g}_1$ and $\mathbf{g}_2$ are mutually independent.

Corollary 1: The achievable DMT for each user in MABC AF two-way relaying without precoding can be expressed as

$$d(r) = 1 - 2r, \quad 0 \leq r \leq 1/2.$$  

Corollary 1.1: The outage performance of the system in terms of DMT as follows.

$$P_{out}^{MA} = \exp(-\tau_1^{MA}\sigma_{h_1}^2 (E|\mathbf{h}_1|^2)^{0.5} (M(E|\mathbf{h}_1|^2 + \| \mathbf{h}_2 \|^2)^{0.5}})$$

(8)

where $\tau_1^{MA}$ is the Gamma function and $\zeta_{M}(\cdot)$ is the Bessel function of the second kind with order $M$ [5].

Based on Theorem 1, we further obtain the asymptotic performance of the system in terms of DMT as follows.

B. Non-precoding Scheme for TDBC

For this scheme, the matrices $F_1$ and $F_2$ are reduced to $\alpha^{TD}I_M$ and $\alpha^{TD}I_M$, respectively, and $\alpha^{TD} = \frac{E|\mathbf{h}_1|^2}{M(E|\mathbf{h}_1|^2 + \| \mathbf{h}_2 \|^2)^{0.5}}$, where $a_1$ and $a_2$ are the relay power allocation parameters for $x_1$ and $x_2$, respectively. The optimal $a_i$ is expected to minimize the final system outage probability. Here we simply choose $a_1 = a_2 = 0.5^{2}$.

Theorem 2: The outage probability of the TDBC AF two-way relaying protocol without precoding is given as

$$P_{out}^{TD} = \exp(-\tau_1^{TD}\sigma_{h_1}^2 (E|\mathbf{h}_1|^2)^{0.5} (M(E|\mathbf{h}_1|^2 + \| \mathbf{h}_2 \|^2)^{0.5}} - \exp(-\tau_2^{TD}\sigma_{h_2}^2 (E|\mathbf{h}_2|^2)^{0.5} (M(E|\mathbf{h}_1|^2 + \| \mathbf{h}_2 \|^2)^{0.5}}),$$

(10)

where $\tau_i^{TD}$ is the Gamma function and $\zeta_{M}(\cdot)$ is the Bessel function of the second kind with order $M$ [5].

C. ZF-precoding Scheme for MABC

We rewrite (1) in vector form as $y_{ZF-M}^{HF} = \mathbf{H}x + \mathbf{n}_R$, where $\mathbf{H} = [\mathbf{h}_1, \mathbf{h}_2]$ and $x = [x_1, x_2]^T$ with $E_x(H^Hx) = \mathbf{E} = \text{Diag}(E_1, E_2)$. Superscript $(\cdot)^H$ denotes the conjugate transpose. The transmit signal at RN is denoted as $x_{ZF-M} = \alpha_{ZF-M} \mathbf{G}^{H}x + \alpha_{ZF-M} \mathbf{F} \mathbf{n}_R$. Similarly, the received signals at two destination nodes can be expressed as

$$y_{ZF-M} = \alpha_{ZF-M} \mathbf{G}^{H}x + \alpha_{ZF-M} \mathbf{F} \mathbf{n}_R + \mathbf{n},$$

(11)

where $y_{ZF-M} = [y_2, y_1]$, $\mathbf{G} = [\mathbf{g}_2, \mathbf{g}_1]^T$ and $\mathbf{n} = [n_2, n_1]^T$ is the destination noise vector with $E(\mathbf{n}^H) = \text{Diag}(\sigma_{g_2}^2, \sigma_{g_1}^2)$. Note that the ZF relay precoder aims to eliminate the multi-user interference. Thus, $F$ can be expressed as

$$F = \mathbf{G}^H \mathbf{H}^H,$$

(12)

where $\mathbf{G}^H = \mathbf{G}^H (\mathbf{G}^H)^{-1}$ and $\mathbf{H}^H = (\mathbf{H}^H \mathbf{H}^H)^{-1} \mathbf{H}^H$.

Next, we try to determine the power scalar $\alpha_{ZF-M}$. Although the instantaneous CSI is available at RN, we assume that $\alpha_{ZF-M}$ is still decided based on the statistical CSI as in [6]. Such decision approach can reduce the computational complexity at RN since it does not need to compute the scalar for each channel realization. On the other hand, it is helpful in deriving the closed-form analytical results. From (11), it is not hard to obtain $\alpha_{ZF-M} = (E_1|\mathbf{G}_1|^2 + E_2|\mathbf{G}_2|^2)^{0.5} = (E_1|\mathbf{G}_1|^2 + E_2|\mathbf{G}_2|^2)\sigma_{g_1}^2 \sigma_{g_2}^2$. Then, $\phi$ denotes the trace operator. To obtain the explicit value of $\alpha_{ZF-M}$, we have the following Lemma.

Lemma 1: For two-way relaying channel, we have

$$\phi[\mathbf{E}(\mathbf{G}^H (\mathbf{G}^H)^{-1} \mathbf{E})] = \frac{E_1|\mathbf{G}_1|^2 + E_2|\mathbf{G}_2|^2}{E_1|\mathbf{G}_1|^2 + E_2|\mathbf{G}_2|^2}.$$  

Let $\varphi = \phi[\mathbf{E}(\mathbf{H}^H \mathbf{G}^H (\mathbf{G}^H)^{-1} \mathbf{E})]$, if $\sigma_{h_1}^2 \sigma_{h_2}^2 = \sigma_{h_1}^2 \sigma_{h_2}^2 = \sigma_{h_2}^2$, we have

$$\varphi = \sigma_{h_2}^2 \tau_{i,j} \prod_{i=1}^{M}(M-2j)(M-2i)$$

$$\prod_{i=1}^{M}(M-2j)(M-2i)$$

$$D_{i,j} = \begin{cases} (M-2-i)(M-2-j)(M-2-i)(M-2-j), & j = k \\ (M-2-i)(M-2-j)(M-2-i)(M-2-j), & j \neq k \end{cases}$$
While if \( \sigma_{h_1}^2 \neq \sigma_{g_2}^2 \), we have 
\[ \varphi = \frac{\sigma_{g_2}^2 - \sigma_{h_1}^2}{(M-2)(\sigma_{h_1} - \sigma_{g_2})}, \]
where \( \sigma_1 = \max\{\sigma_{h_1}^2, \sigma_{g_2}^2\} \) and \( \sigma_2 = \min\{\sigma_{h_1}^2, \sigma_{g_2}^2\} \).

Substituting (12) into (11), we have
\[ y_{ZF-M} = \alpha_{ZF-M} x + \alpha_{ZF-M} H_i n_R + n. \]

Thus, the SNR at each destination is yielded as
\[ \zeta_i = \frac{\alpha_{ZF-M}^2 E_i}{\alpha_{ZF-M}^2 \sigma_R^2 (H H H^H)^{-1}} + \sigma_i^2, \quad i = 1, 2 \]
where \([\cdot]_{i,i}\) denotes the \(i\)-th diagonal entry of a matrix. Then, the outage probability of ZF-precoding MABC is obtained as
\[ P_{out}^{ZF-M} = \Pr\{\zeta_1 < \tau_1^{MA}\} \text{ or } \zeta_2 < \tau_2^{MA}\}. \]

To compute (13), we need to know the relationship between \([H H^H]^{-1}\) and \([H H H^H]^{-1}\) and \([H H H]^{-1}\). Next we assume that \([H H H^H]^{-1}\) and \([H H H]^{-1}\) are independent with each other to simplify the derivation. As confirmed by the simulation, this approximation almost leads no gap between analytical and Monte-Carlo results.

**Theorem 3**: When \( \tau_1^{MA} < \frac{\alpha_{ZF-M}^2 E_i}{\sigma_i^2} \) and \( \tau_2^{MA} < \frac{\alpha_{ZF-M}^2 E_i}{\sigma_i^2} \) (otherwise \( P_{out}^{ZF-M} = 1 \)), the outage probability of MABC two-way relaying with ZF-precoding can be approximated as \( P_{out}^{ZF-M} = P_{out_1}^{ZF-M} + P_{out_2}^{ZF-M} - P_{out_1}^{ZF-M} P_{out_2}^{ZF-M} \) with 
\[ P_{out_1}^{ZF-M} = \frac{\gamma}{\Gamma(M-1)} \left( \frac{\sigma_1^2 \alpha_{ZF-M}^2 \sigma_R^2}{\sigma_i^4 (E_i \alpha_{ZF-M} - \tau_1^{MA} \sigma_i^2)} \right)^{M-1}, \]
where \( \gamma(\cdot) \) is the incomplete gamma function [5].

**Corollary 2**: The achievable DMT for each user in MABC AF two-way relaying with ZF-precoding can be expressed as 
\( d(\rho) = (M - 1)(1 - 2\rho) \), \( 0 \leq \rho \leq 1/2 \).

**Corollary 2** implies that, different from the non-precoding case, the ZF-precoding at relay node can benefit from increasing the relay antenna number, and the diversity order is linear with the number of relay antennas.

**D. ZF-precoding Scheme for TDBC**

For ZF-precoding TDBC protocol, we rewrite (5) as 
\[ y_{ZF-T} = H x + n_R, n_R = n_{R1} + n_{R2}. \]
Since \( x_1 \) and \( x_2 \) are received separately in different time slots at RN, the relay precoding matrix \( F_1 \) and \( F_2 \) can be designed differently to improve the performance. Here for simplicity, we let \( F_1 = F_2 = F \) as presented in (12), and the power allocation parameter \( \alpha_{ZF-T} \) is also determined as \( \alpha_{ZF-M} \) only by replacing \( \sigma_R^2 \) with \( 2\sigma_R^2 \). Therefore, we have 
\[ x_{ZF-T} = \alpha_{ZF-T} F H x + \alpha_{ZF-T} F (n_{R1} + n_{R2}). \]
By combining the direct-path link presented in (5), we obtain the destination SNR after using the MRC technique as
\[ \kappa_i = \frac{E_i}{\sigma_i^2} |h_{ii}|^2 + \frac{\alpha_{ZF-T}^2 E_i}{\sigma_R^2 (H H H^H)^{-1}} + \sigma_i^2. \]
Due to the independence of \( h_{12} \) and \( h_{21} \), we can approximate the outage probability as
\[ P_{out} = \Pr\{\kappa_1 < \tau_1^{TD}\} + \Pr\{\kappa_2 < \tau_2^{TD}\} - \Pr\{\kappa_1 < \tau_1^{TD}\}\Pr\{\kappa_2 < \tau_2^{TD}\}. \]

**Theorem 4**: The outage probability of TDBC two-way relaying with ZF-precoding can be approximated as \( P_{out}^{ZF-T} = P_{out_1}^{ZF-T} + P_{out_2}^{ZF-T} - P_{out_1}^{ZF-T} P_{out_2}^{ZF-T} \) with \( P_{out}^{ZF-T} \) being given in the following two cases:

**Case 1**: when \( \tau_1^{TD} \geq \frac{\alpha_{ZF-T}^2 E_i}{\sigma_i^2} \), 
\[ P_{out_1}^{ZF-T} = \exp\left(-\frac{\tau_1^{TD} E_i}{\sigma_i^2}\right) - 1 \]
and 
\[ P_{out_2}^{ZF-T} = \exp\left(-\frac{\tau_2^{TD} E_i}{\sigma_i^2}\right) - 1. \]

**Case 2**: when \( \tau_1^{TD} < \frac{\alpha_{ZF-T}^2 E_i}{\sigma_i^2} \),
\[ P_{out_1}^{ZF-T} = 1 - \exp\left(-\frac{\tau_1^{TD} E_i}{\sigma_i^2}\right) \]
and 
\[ P_{out_2}^{ZF-T} = 1 - \exp\left(-\frac{\tau_2^{TD} E_i}{\sigma_i^2}\right). \]

**Remark 3**: Similar to Theorem 2, although \( P_{out}^{ZF-T} \) contains the infinite number of \( l \), it converges very fast and setting the maximal \( l \) as 20 is enough.

IV. NUMERICAL RESULTS AND DISCUSSIONS

In the numerical evaluation, all the channels are set to be Rayleigh fading. We let \( \rho_R = \frac{E_i\sigma_i^2}{\sigma_R^2} \) and \( \rho_R = \frac{E_i\sigma_i^2}{\sigma_R^2} \) define
the SNR from source $S_1$ to RN $R$, and RN $R$ to destination node $S_i$, respectively, and let $\rho_{ii} = \frac{E_i \sigma_i^2}{\sigma^2}$ define the SNR from $S_i$ to $S_i$ in the TDBC scheme. For simplicity, we also assume $\rho_1 = \rho_{1R} = \rho_{2R} = \rho_{R1} = \rho_{R2}$ and $\rho_2 = \rho_{12} = \rho_{21}$.

In Fig. 2, we show both the analytical and simulation results on the outage performance for all the schemes when $M = 4$ and $R = R_1 = R_2 = 0.08$. It is seen that the derived analytical results match the Monte-Carlo results very well, although we have used the approximation for the ZF precoding scheme. Meanwhile, it is observed that precoding at RN can significantly improve the system performance.

Fig. 3 illustrates the outage performance comparison when the number of relay antennas changes. We find that increasing the antenna in non-precoding MABC and TDBC schemes does not bring any notable performance difference. However, for the ZF-precoding two-way relaying scheme, the performance is enhanced significantly. Fig. 3 also shows that the non-precoding MABC scheme obtains the diversity order of one and it is irrelevant to relay antenna number, which confirms our conclusions derived in Corollary 1. Moreover, compared to the non-precoding MABC scheme, the diversity of 2 can be achieved for non-precoding TDBC scheme due to the existence of direct-path link. While for the ZF-precoding MABC scheme, we see that the diversity order is increased with the increasing of relay antenna number which is consistent with our conclusion derived in Corollary 2. Similarly, the obtained diversity order of ZF-precoding TDBC scheme is also increased with the antenna number. However, different from the non-precoding scheme, 2 more diversity order can be achieved compared to the ZF-precoding MABC scheme due to the existence of direct-path link.

In Fig. 4, the outage probability is shown as the function of $\rho_2$ for fixed $\rho_1$ at 0 and 3dB. We see that TDBC can always outperform MABC when $\rho_2$ exceeds some thresholds and usually such threshold is increased if ZF-precoding is applied at the relay node.

V. CONCLUSIONS

This paper presented the study on the outage performance of AF two-way relaying with multi-antenna RN. Both two time slots MABC protocol and three time slots TDBC protocol were considered. We derived the closed-form expressions of outage probability for the two protocols under both non-precoding and ZF-precoding schemes. Based on the derived outage probabilities, the DMT was also discussed for MABC protocol. Out analytical results implied that, with the ZF relay precoding, the system can benefit from the increased diversity order by increasing the number of relay antennas.

REFERENCES