Trellis Coded Super Unitary Differential Space-Time Modulation

Meixia Tao
Department of Electrical and Computer Engineering
National University of Singapore
Singapore 117576
Email: mxtao@nus.edu.sg.

Abstract—We introduce a new trellis coded differential unitary space-time modulation scheme for multiple-antenna wireless systems. In the new scheme, the constellation expansion of unitary space-time signal set is executed using a super unitarity technique so that the number of available unitary matrices is increased but the transmitted symbol alphabet is kept the same. A novel set partitioning strategy is then applied to the expanded matrix set. This scheme avoids the rate-loss problem suffered by traditional code design method. Accordingly, it enables a transmission rate that is no lower than that of the underlying differential unitary space-time modulation. New codes using two transmit antennas are provided. Simulation results show that, besides having lower transmitter and receiver complexity, they also provide improvements in coding gain over existing codes.

I. INTRODUCTION

Recently, much research effort has been invested in the design of non-coherent signaling schemes to exploit the non-coherent capacity of multi-input multi-output (MIMO) systems, such as unitary space-time modulation (USTM) [1], differential unitary space-time modulation (DUSTM) [2], [3], and differential space-time block codes (DSTBC) [4], [5]. These schemes waive the channel state information (CSI) acquisition at the receiver, but still attain full antenna diversity advantages. To further overcome the fading effect in wireless channels, numerous trellis coding schemes have been proposed based on USTM or DUSTM, such as the concatenated USTM with bit-interleaved trellis coded modulation scheme [6], the trellis-coded differential unitary space-time modulation (TC-DUSTM) scheme [7] and the trellis-coded unitary space-time modulation (TC-USTM) scheme [8]. These existing coding schemes are all based on the classic Ungerboeck’s concepts of constellation expansion and set partitioning. Therefore, due to redundancy introduced by the outer trellis encoder, the overall transmission rate is reduced.

In this paper, we introduce a unitary transformation technique, called super unitarity, and apply it to the design of TC-DUSTM schemes to enhance the transmission rate as well as coding gains without increasing system complexity. Our super unitarity integrates antenna permutation and phase rotation. We show that the super unitarity can expand the total number of available unitary signal matrices without expanding the constellation alphabet size of transmitted signals. The expanded matrix set, called super unitary set, does not necessarily have full diversity. We then propose a novel set-partitioning strategy and a set of trellis code design rules so that a full diversity is always ensured as well as a good coding gain. This work is inspired by a similar transformation technique applied previously in [9] and [10] for designing concatenated space-time block codes and trellis codes when CSI is known at the receiver.

II. SYSTEM MODEL

In this section, we briefly review the system model and design criteria of both DUSTM and TC-DUSTM in non-coherent signaling schemes.

Let \( \mathcal{V} = \{ \mathbf{V}_l, 0 \leq l \leq L - 1 \} \) denote a unitary space-time signal set for use in the DUSTM scheme, each element of which is an \( M \times M \) unitary matrix. To send a data matrix \( \mathbf{V}_l(\tau) \in \mathcal{V} \) using \( M \) transmit antennas at the \( \tau \)-th time block, each time block consisting of \( M \) time slots, \( \mathbf{V}_l(\tau) \) is differentially encoded in the transmitted signal matrix \( \mathbf{S}_\tau \) as \( \mathbf{S}_\tau = \mathbf{V}_l(\tau)\mathbf{S}_{\tau-1} \). The error performance of a set \( \mathcal{V} \) at high signal-to-noise ratio (SNR) is dominated by its minimum squared determinant distance defined as [7]

\[
D_{\text{min}}^2 = \min_{0 \leq l \neq l' < L} D^2(\mathbf{V}_l, \mathbf{V}_{l'}) = \min_{0 \leq l \neq l' < L} \left| \det(\mathbf{V}_l - \mathbf{V}_{l'}) \right|^{2/M}.
\]

Any set \( \mathcal{V} \) with \( D_{\text{min}}^2 > 0 \) is said to have full diversity, and a larger \( D_{\text{min}}^2 \) yields better performance.

In TC-DUSTM, a rate \( m/(m+1) \) trellis encoder takes \( m \) input information bits and generates \( m + 1 \) coded bits at each trellis transition. The \( m + 1 \) encoded bits at the \( \tau \)-th trellis transition are then used to map an element \( \mathbf{V}_l(\tau) \) from a given unitary signal set \( \mathcal{V} \) with size \( L = 2^m+1 \), based upon certain design rules. After that, the coded matrix sequence \( \mathcal{V} = \{ \mathbf{V}_l(\tau) \} \) is differentially encoded as \( \mathbf{S} = \{ \mathbf{S}_\tau \} \), and transmitted over \( M \) antennas. It is shown in [7] that, in a slowly time-varying channel, full antenna diversity can always be achieved if the employed \( \mathcal{V} \) has full diversity, and that a coding advantage, denoted as \( \delta \), of

\[
\delta = \min_{\mathcal{V} \neq \mathcal{V}} \left[ \frac{\det((\mathbf{V} - \hat{\mathbf{V}})^H(\mathbf{V} - \hat{\mathbf{V}}))}{\sum_{\tau=1}^K D^2(\mathbf{V}_l(\tau), \mathbf{V}_l(\tau))} \right]^{1/M}.
\]

\[
= \min_{\mathcal{V} \neq \mathcal{V}} \left[ \sum_{\tau=1}^K D^2(\mathbf{V}_l(\tau), \mathbf{V}_l(\tau)) \right] = D_{\text{free}}^2.
\]
is achievable, where $D_{\text{free}}^2$ is called the minimum free squared determinant distance. Hence, to maximize the coding advantage, the minimum free squared determinant distance between any pair of distinct coded matrix sequences must be made as large as possible.

III. SUPER UNITARY SPACE-TIME SIGNAL SETS

A. Group Constellations

The high rate TC-DUSTM considered here is subject to the constraints of minimizing the size of the baseband transmitted signal alphabet and minimizing the peak-to-average power ratio of the transmitted signals. The former is to reduce the complexity of the baseband signal processor, and the latter is to reduce the cost of power amplifier devices in radio frequency front-ends. Therefore, only unitary space-time group sets are applicable. With group structure, the signal matrix after differential encoding still belongs to the given set and, accordingly, there is no expansion on the size of the transmitted signal alphabet nor increase in PAPR. A counterexample is the Alamouti-code based differential space-time block codes, shown in Fig. 1, in which the original QPSK constellation, represented by circles, is expanded after differential encoding to a 13-point constellation, represented by crosses, and PAPR is significant increased.

![Constellation expansion due to non-group structure in Alamouti-code based differential space-time block codes. “o” stands for the original QPSK constellation points, “x” stands for the constellation points after differential encoding.](image)

Of existing group sets the diagonal cyclic groups proposed in [2] and [3] are of particular interest. Let $G_{M,L}$ denote a diagonal cyclic group set for $M$ transmit antennas and with $L$ elements. It can be expressed as $G_{M,L} = \{ V_1 \} \triangleq \{ I, V_1, \ldots, (V_1)^{L-1} \}$, where $V_1$ is the group generator of the diagonal form $V_1 = \text{diag}\{ e^{i(2\pi/L)}u_1, \ldots, e^{i(2\pi/L)u_M} \}$, with $u_m$ relatively prime to $L$. Note that the values of the signal elements in $G_{M,L}$ are taken from the $L$-PSK constellation. It is shown by Hughes in [11] that every fully diverse unitary space-time group set with $L = 2^k$ elements is equivalent to $G_{M,L}$ for odd $M$, and is equivalent to either $G_{M,L}$ or a dicyclic group for even $M$. We thus employ the diagonal cyclic groups $G_{M,L}$ in all our examples for the purpose of illustration.

B. Super Unitarity

The number of available unitary matrices in a given set $\mathcal{V}$ can be expanded using unitary transformation. Here, we consider the following $M \times M$ unitary transformation matrix:

$$\Psi = P \Phi,$$

where $P$ is a permutation matrix and $\Phi$ is a diagonal phase rotation matrix given by $\Phi = \text{diag}\{ e^{i\theta_1}, \ldots, e^{i\theta_M} \}$. The transformation is performed as $V \Psi$, where $V \in \mathcal{V}$. Such transformation swaps the order of transmit antennas and rotates the phase of the signals on the $m$-th antenna by $\theta_m$ radians, for $m = 1, \ldots, M$. Therefore, the transformed signal matrix is still unitary. A point to note is that the super orthogonality introduced in [9], [10] considers phase rotation only.

Evidently, the resulting signal elements are also members of $L$-PSK constellations if we let $V \in G_{M,L}$ and pick $\theta_m \in \{ 2\pi k/L, k = 0, 1, \ldots, L - 1 \}$. Let $G_M(L)\{ \Psi \}$ denote the transformed $G_{M,L}$ via the transformation matrix $\Psi$, i.e., $G_M(L)\{ \Psi \} = \{ V \Psi; V \in G_{M,L} \}$. The union of all the sets obtained by different $\Psi$’s is referred to as a super unitary space-time signal set, or simply a super set. Mathematically, we can represent a super cyclic group set by $G_{M,L} = \{ V_1 \Psi; l = 0, 1, \ldots, L - 1, \Psi \in \mathcal{T} \}$, where $\mathcal{T}$ is the set of all possible $\Psi$’s. Hereafter, we shall call the original set the primal set and the others derived from unitary transformation the dual sets. Thus all the elements in the super set $G_{M,L}$ must be in either the primal set $G_{M,L}$ or the dual sets $G_{M,L}(\Psi)$.

As an example, we consider the diagonal cyclic group $G_{4,4}$ with $u_1 = u_2 = 1$, the optimal unitary group set at rate 1 bit/s/Hz for two transmit antennas. In the phase rotation matrix $\Phi$, we have $\theta_1, \theta_2 \in \{ 2\pi k/4, k = 0, \ldots, 3 \}$. Without loss of generality, we keep $\theta_1 = 0$ and let $\theta_2 = \pi$ vary from $0, \pi/2, \pi$, and $3\pi/2$. The permutation matrix $P$ in (2) has two choices:

$$P \in \left\{ P_0 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, P_1 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \right\}. \quad (3)$$

Hence, there are eight different transformation matrices, including the identity matrix. This yields eight sets in the super set $G_{2,4}$. For instance, the dual set with $P = P_0$ and $\theta = \pi$ can be expressed as

$$G_{2,4}(P_0, \theta = \pi) = \left\{ \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \right\} \left[ \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}, l = 0, \ldots, 3 \right\}.$$

Several observations can be made from this example. First, all the signal elements in $G_{2,4}$ are members of the QPSK constellation. Second, the determinant distance profile of each dual set is the same as that of the primal set. In other words, the unitary transformation does not alter the distance profile. Each dual set can be used alone as the unitary space-time constellation in DUSTM or in traditional TC-DUSTM and achieves the same performance as when the primal set is used. Third, the super set is a group. But each individual dual set may not be a group. Lastly, the inter-set determinant distance between $G_{2,4}$ and dual sets $G_{2,4}(P_1, \theta = \pi/2)$ and $G_{2,4}(P_1, \theta = 3\pi/2)$ is non-zero, whereas the inter-set distance between $G_{2,4}$ and other duals is zero. The last observation shall be utilized in set partitioning and code design. The first three observations can be generalized to an arbitrary $G_{M,L}$.

The last one may differ for different $G_{M,L}$’s.
IV. CODE CONSTRUCTION

A. SET PARTITIONING

The proposed set partitioning of a super unitary set involves two aspects. In one aspect, each primal set and dual sets are partitioned individually in a traditional way based upon their determinant distance profile. Specifically, one should maximize the intra-subset distance at each partitioning level; the subsets should exhibit similar distance properties and provide maximum symmetry. It follows a top-down approach. In the other aspect, the subsets from the primal set and dual sets are grouped together level by level into virtual sets or virtual subsets. A rule of thumb is to group those subsets that can maximize the minimum squared determinant distance of each virtual set/subset at each grouping level. The virtual sets/subsets should also exhibit similar distance properties and provide maximum symmetry. Such grouping procedure follows a down-to-top approach.

In the following we give two examples that demonstrate the proposed set-partitioning procedure. We shall emphasize the virtual set grouping process, where novelty resides.

We first consider the super set G_{2,4}. As all the dual sets have the same distance profile as that of the primal set, the top-down partitioning is identical for all sets. For simplicity we shall use the integral l to denote the l-th matrix element in each set. Fig. 2 shows the resulting partitioning tree, where only the primal set G_{2,4} and the dual sets G_{2,4}(P_0, \theta = \pi), G_{2,4}(P_1, \theta = \pi/2), and G_{2,4}(P_1, \theta = 3\pi/2) are included (the reason for omitting other duals will be explained shortly). The left side of the figure shows the intra-subset or intra-set determinant distance at each partitioning or grouping level. The solid lines represent the top-down partitioning of G_{2,4} and the duals. The root of the partitioning tree is the super set, whose $D_{\text{min}}^2$ equals zero. The dashed lines represent the down-to-top grouping of the subsets crossing the primal and its duals. In specific, at the first level of grouping, the inter-subset distance between all the subsets $S_0$’s and $S_1$’s from different sets is examined. Then, the subsets $S_0$ from G_{2,4} and $S_1$ from G_{2,4}(P_0, \theta = \pi) are grouped into a virtual set A, whose $D_{\text{min}}^2$ is 2, the same as that of the primal set G_{2,4}. Virtual sets B, C, and D are constructed in a similar way at this level and all exhibit the same distance profile. At the next level, the inter-set distance between the primal set, dual sets and all the virtual sets constructed from the previous level is examined. As a result, the set G_{2,4} and G_{2,4}(P_1, \theta = \pi/2) are grouped into a virtual set E, whose $D_{\text{min}}^2$ is $\sqrt{2} \approx 1.4142$. Similar is the virtual set F. As the inter-set distance between the virtual sets A, B, C, and D is zero, no further grouping is made. Other dual sets are not included in the partitioning tree because similar virtual sets like A, B, C, and D at level one or E and F at level two that have the same distance profile and non-zero intra-set distance do not exist.

Fig. 3 depicts another example based on the super set G_{2,16}, whose primal set is the diagonal cyclic group G_{2,16} with $u_1 = 1$ and $u_2 = 7$, the optimal group unitary space-time signal set at rate 2 bits/Hz for two transmit antennas [2], [3]. Due to similar reasons as in G_{2,4}, we only display the results for G_{2,16} and G_{2,16}(P_1, \theta = \pi/2) (the inter-set distance between G_{2,16} and other dual sets is either zero or not larger than the inter-set distance between G_{2,16} and G_{2,16}(P_1, \theta = \pi/2)). Here, virtual sets are constructed by a 3-level down-to-top grouping. Note that both virtual sets A and B at level two have the same number of elements as G_{2,16} but their intra-set distance is greater than that of G_{2,16}. As a stand-alone unitary signal set, A or B will have better error performance than G_{2,16} in DUSTM transmission. Virtual set C at the third level is constructed by grouping A and B. Compared with G_{2,16}, the size of C is doubled, yet, $D_{\text{min}}^2$ is kept the same.

Both Figs. 2 and 3 only reveal part of the set partitioning of G_{2,4} and G_{2,16}, respectively. They are intended to demonstrate the proposed set-partitioning method, not to illustrate the complete partitioning trees. In addition, as shown in Section IV-B, not all the dual sets are needed for the code construction.

B. DESIGN RULES

Here we start with two code examples and then present the general design rules.

Fig. 4 illustrates a rate 1-bit/s/Hz 4-state code constructed using the super set G_{2,4}. Here, G_{2,4}(k, \theta) represents the dual set G_{2,4}(P_k, \theta). The one with $k = \theta = 0$ reduces to the primal set. Only the elements from G_{2,4} and G_{2,4}(P_0, \theta = \pi) in the set partitioning of G_{2,4} in Fig. 2 are used. Each trellis branch consists of 2 parallel transitions, and is assigned with subsets $S_0$’s or $S_1$’s from Fig. 2. The branches that diverge from states zero and two are assigned with the elements from the primal set G_{2,4}, whereas the branches diverging from states one and three are with the dual set G_{2,4}(P_0, \theta = \pi). The error path that has the shortest length is the parallel transition, i.e., there is $p = 1$ trellis transition. The coding gain associated with the parallel transitions, denoted as $\delta_{p1}$, equals the minimum squared determinant distance of each subset. Hence, we have $\delta_{p1} = 4$. Due to the structure of the trellis, the error path with length of two does not exist, and the second shortest error length is three. The minimum free squared determinant distance associated with $p = 3$ trellis transitions, $D_{\text{free}}^2$, is $2 + 0 + 2 = 4$. According to the inequality in (1), the coding advantage associated with this path, $\delta_{3S}$, is thus lower bounded by 4 (It can be verified that $\delta_{3S} = 6$). Similarly, the coding advantages associated with error paths that have $p > 3$ trellis transitions are all lower bounded by 4. Hence, the minimum coding advantage of this code is $\delta = 4$. Comparing this code with the rate 1-bit/s/Hz 4-state code using G_{2,8} in [7, Fig. 5a], we find that the two codes achieve the same analytical coding advantage but the signal constellation of the new code is QPSK instead of 8PSK, which results in lower transmitter complexity.

We provide in Fig. 5 a rate 2-bit/s/Hz 4-state code using G_{2,16}. The set-partitioning in Fig. 3 is used. In this figure $S_{ij}$ denotes the subset $S_{ij}$, for $i, j \in \{0, 1\}$, from the dual set G_{2,16}(P_1, \theta = \pi/2). Each trellis branch consists of 4 parallel transitions and is assigned with elements from subsets $S_{ij}$’s. In designing such a code, we assign the branches that diverge from a same state with elements from virtual sets A
or $B$, rather than from $G_{2,16}$ or $G_{2,16}(P_1, \theta = \pi/2)$. The reason is that the minimum determinant distance of the virtual sets is greater than that of $G_{2,16}$ or $G_{2,16}(P_1, \theta = \pi/2)$. The minimum coding advantage of this code is associated with the path having 2 trellis transitions and equals 1.8089. Compared with the rate 2-bit/s/Hz 16-state code using $G_{2,32}$ in [7, Fig. 5b], the new code not only achieves larger analytical coding gain (the coding advantage of [7, Fig. 5b] is 1.7631) but also reduces the number of trellis states. Moreover, the transmitted signal constellation is 16PSK rather than 32PSK.

Codes with different number of trellis states and at different transmission rates can be systematically designed using the proposed super set and set partitioning. The general rules to guarantee full transmit diversity and large coding advantage are the following. The trellis branches that diverge from or converge to a common state should be assigned with elements from a same set, where the set can be either the primal, the dual or the virtual. Virtual sets are preferred when they exhibit a greater minimum squared determinant distance than that of the primal set. It is thus assured that any path diverging from or converging to a correct path differs by at least one signal matrix that contributes a non-zero portion, also made as large as possible, to the minimum free squared determinant distance. Parallel transitions should be assigned with elements from subsets with the greatest intra-subset distance. Neighboring states should be usually assigned with elements from different sets. Depending on the number of dual sets involved, codes with different transmission rates can be designed from a same super set. Tradeoff between performance and complexity can be made by using different number of trellis states.

V. Simulation Results

In this section we present the simulation results for our new codes. In all simulations, two transmit antennas and one receive antenna are used. The channel is modeled as time-varying flat Rayleigh fading. The time variation of channel coefficients over each transmission frame is given by the Jakes model with autocorrelation function $J_0(2\pi f_d T_s t)$,
where $J_0(\cdot)$ is the zeroth order Bessel function of the first kind. The normalized Doppler frequency $f_d T_s$ is set to 0.0025. For a carrier frequency of 900 MHz and a symbol rate of 30000 bauds, a normalized Doppler frequency of 0.0025 corresponds to a vehicle’s speed of 90 km/hr. Each transmission frame consists of 102 symbol intervals, in which two symbol intervals are used in the beginning to send a known unitary matrix to facilitate differential modulation. Neither the transmitter nor the receiver knows the instantaneous channel coefficients. The Viterbi-algorithm based suboptimal differential decoder in [7] is employed. The performance is measured using frame error rate (FER) versus the total transmit SNR.

Fig. 6 presents the results for the code from Fig. 4. For comparison, we also provide the results for $G_{2,4}$ and the 4-state code using $G_{2,8}$ from [7, Fig. 5a], both having a rate of 1 bit/s/Hz. As expected, the performance of the new code using $G_{2,4}$ is almost the same as that of the code designed previously using $G_{2,8}$. As the signals from the new code use QPSK constellation and the previous code uses 8PSK, the transmitter hardware complexity of the new scheme is lower.

In Fig. 7, we present the results for the code from Fig. 5, in comparison with the rate 2-bit/s/Hz 16-state code using $G_{2,32}$ from [7, Fig. 5b]. One can observe that the new 4-state code provides more than 0.5 dB improvement over the existing 16-state code. This is expected because the new code has a larger analytical coding advantage. In addition, the decoding complexity, in terms of the average number of branch metric calculations per information bit, of the new code using 4 states and with 4 parallel transitions is only half of that of the existing code using 16 states and with 2 parallel transitions.

VI. CONCLUSIONS

A new TC-DUSTM scheme using the super unitarity technique is proposed. The original unitary space-time constellation is expanded to a super unitary set by performing antenna permutation and phase rotation. The number of available unitary matrices is greatly increased without increasing the modulation level of transmitted signals. The new set partitioning strategy tailored for the super sets differs from the traditional method with its down-to-top grouping of the so-called virtual sets. Compared with the traditional TC-DUSTM, the new TC-DUSTM not only reduces the transmitter and receiver complexity, but also achieves larger coding gains. The proposed super unitarity technique and set partitioning method are also applicable in designing high rate TC-USTM scheme. As there is no differential encoding in TC-USTM, the group structure requirement may not be necessary.

REFERENCES